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Chattering-free pulse-width-modulation feedback control strategies in the regulation of robotic manipulators

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Abstract

In this article, a differential algebraic approach is presented to deal, in full generality, with the problem of designing chattering-free, dynamical multivariable Pulse-Width-Modulation (PWM) feedback control strategies for the regulation of rigid robotic manipulators.

1. INTRODUCTION

A differential algebraic approach is presented for the design of multivariable Pulse-Width-Modulation (PWM) feedback control strategies for nonlinear systems such as rigid robotic manipulators. A major difficulty associated to PWM controllers lies in the "chattering" responses due to the bang-bang nature of the involved feedback signals. A dynamical PWM controller is proposed as a means of effectively circumventing the discontinuities associated to the PWM control strategies and relate PWM control to the differential algebraic approach to control theory (see Fliess [3]). PWM control schemes, for nonlinear systems, were presented in Sira-Ramirez [1]-[2] (see also the references therein).

Section 2 of this article deals with the differential algebraic approach to the PWM control of nonlinear multivariable systems. A characterization, through invertibility, is proposed for multivariable systems which are capable of sustaining PWM controlled trajectories on the joint zero level set of a number of independent stabilizing auxiliary output functions. Section 3 presents the implications of the obtained results in the PWM controller design for a multivariable rigid-joint manipulator system. Simulation examples are also presented. The conclusions of this work are collected in Section 4.

2. A DIFFERENTIAL ALGEBRAIC APPROACH TO MULTIVARIABLE PWM CONTROL IN NONLINEAR SYSTEMS

Definition 2.1 [3] Let k be a differential ground field and let u be a set of differential transcendent elements over k. An input-output system consists of: a) a given set of (independent) inputs : $u = (u_1,...,u_m)$, b) a set of outputs $y = (y_1,..., y_m)$, belonging to a universal field extension U, such that the components of y are differentially algebraic over $k<u>$. Notice that we only deal with *square* systems. The field $k<y,u>$ is differentially algebraic over $k<u>$

Definition 2.2. [3] Let U be a universal differential field and let $k\{y,u\}$ denote the *differential ring* generated by the components of y and u. A differential k-specialization is a mapping φ: $k\{y,u\} \to U$ which leaves the field k invariant. The differential transcendence degree of the extension, over k, of the differential quotient field $Q(\varphi(k\{y,u\})$ is nonnegative and it is never higher than the differential transcendence degree of $k<u>/k$ (i.e., diff tr d° $Q(\varphi(k\{y,u\})/k \leq$ diff tr d° $k<u>/k = m$). Notice that $Q(\varphi(k\{y,u\}) = k<\varphi y,\varphi u>$.Frequently φ is the identity mapping. We only deal with cases in which: diff tr d° $k<\varphi y,\varphi u>/k = 0$.

Definition 2.3 [3] A *closed loop control* is a differential k-specialization $\varphi : k\{y,u\} \to U$ such that diff tr d° $k<\varphi y, \varphi u>/k = 0$. We refer to such feedback loops as *pure feedback loops*. In such a case the set of specialized elements $\varphi u_1, ..., \varphi u_m$, $\varphi y_1, ..., \varphi y_m$ satisfiy a set of ordinary algebraic differential equations. Whenever diff tr d° $k<\varphi y>/k$ is smaller than m, the closed loop is said to be *degenerate*.

Definition 2.4 An *auxiliary stabilizing surface* is a differential k-specialization ϕ, mapping $k\{y\} \to U$, such that diff tr d° $k<\phi y>/k = 0$. The elements $\sigma \in k<\phi y>/k$ are referred to as *average PWM controlled dynamics*.

Definition 2.5 Let σ be an element of $k<y>/k$ such that $\sigma = 0$ represents a desirable average PWM controlled dynamics. A *PWM feedback controlled motion* is said to exist on σ, for the system $k<y,u>/k<u>$, if there exists a differential k-specialization $\varphi : k\{y,u\} \to U$, which represents a pure feedback loop, known as the *average PWM control*, such that $\sigma \in k<\varphi y>/k$ and diff tr d° $k<\varphi y>/k = 0$.

Definition 2.6 A square input-output system $k<y,u>/k<u>$ is *invertible* if u is differentially algebraic over $k<y>$, i.e., if diff tr d° $k<u,y>/k<y> = 0$. (See [3])

Proposition 2.7 A PWM controlled motion exists on an element $\sigma \in k<y>/k$, if and only if the square system $k<y,u>/k<u>$ is invertible.

Proof (see [4]).

Proposition 2.8 The scalar state s of the following PWM controlled system is asymptotically stable to zero if and only if $\rho WT < 2$

$$\dot{s} = v$$
$$v = -w \; pwm_\tau [s(t_k)]$$
$$pwm_\tau[s(t_k)] = \begin{cases} \text{sign}[s(t_k)] & \text{for } t_k \le t < t_k+\tau[s(t_k)]T \\ 0 & \text{for } t_k+\tau[s(t_k)]T \le t < t_k+T \end{cases} \quad k = 0,1,\ldots \tag{2.1}$$
$$\tau[s] = \begin{cases} 1 & \text{for } |s| \ge 1/\rho \\ \rho|s| & \text{for } |s| < 1/\rho \end{cases}$$

Proof (see [5]).

Consider now a general multivariable nonlinear system:

$$y_i^{(\alpha_i)} = c_i(y,\dot{y},\ldots,y^{(\alpha_i-1)},\; u,\dot{u},\ldots,u^{(\beta_i)}) ; \quad i=1,2,\ldots,m \tag{2.2}$$

where $y^{(\alpha_i)}$ means that the components of the vector y have derivatives which are smaller than, or equal, to α_i. The system (2.2) is assumed to be square and invertible. Moreover, it will be required that, locally, the matrix: $\partial c_i/\partial u_j^{(\beta_j)}$ is non-singular.

Without any loss of generality, it may be assumed that the desired set of average output dynamics is given by:

$$y_i^{(\alpha_i-1)} - s_i(y,\dot{y},\ldots,y^{(\alpha_i-2)}) = 0 \; ; \quad i=1,2,\ldots,m \tag{2.3}$$

i.e., each output component dynamics is only of order α_i-1.

Let the set of auxiliary output functions σ_i be defined as:

$$\sigma_i = y_i^{(\alpha_i-1)} - s_i(y,\dot{y},\ldots,y^{(\alpha_i-2)}) \quad ; \quad i=1,2,\ldots,m \tag{2.4}$$

then $\sigma_i = 0$ (i=1,2,..,m) represent the desirable average dynamics to be imposed, by means of discontinuous feedback control, on the output components of the system.

Imposing on each one of the elements σ_i of (2.3) the dynamics specified by (2.1) one obtains:

$$\dot{\sigma}_i = y_i^{(\alpha_i)} - \sum_{j=1}^{m} \sum_{k=0}^{\alpha_i-2} \frac{\partial s_i}{\partial y_j^{(k)}} (y,\dot{y},\ldots,y^{(\alpha_i-2)}) y_j^{(k+1)} = -W_i \; PWM_{\tau_i}[\sigma_i] ; \quad i=1,2,\ldots,m \tag{2.5}$$

Using now the dynamics (2.2) on the expression (2.5) one obtains an implicit dynamical PWM feedback controller which asymptotically accomplishes the desired average dynamics:

$$c_i(y,\dot{y},\ldots,y^{(\alpha_i-1)},\; u,\dot{u},\ldots,u^{(\beta_i)}) = \sum_{j=1}^{m} \sum_{k=0}^{\alpha_i-2} \frac{\partial s_i}{\partial y_j^{(k)}} (y,\dot{y},\ldots,y^{(\alpha_i-2)}) y_j^{(k+1)} - W_i \; PWM_{\tau_i}[\sigma_i] \tag{2.6}$$

$i = 1,2,\ldots,m$

Invertibility of the original system (2.2) implies that one can solve for each control input component, from the above set of equations, in the sense of obtaining a, possibly explicit, coupled set of ordinary differential equations for the control inputs u_i $(i=1,2,...,m)$.

Let Y be a constant equilibrium value, for the output vector y, of the autonomous differential system (2.3). It is explicitly assumed that the set of coupled autonomous differential equations:

$$c_i(Y,0,...,0,\ u,\dot{u},...,u^{(\beta_i)}) = 0 \quad ; \quad i=1,2,...,m \tag{2.7}$$

exhibits an asymptotically stable behavior toward a constant equilibrium vector given by $u = U$. This assumption corresponds to a particular definition of a stable *zero dynamics* or of a *minimum phase* multivariable system.

3. A MULTIVARIABLE PWM FEEDBACK CONTROL APPROACH FOR THE STABILIZATION OF ROBOTIC MANIPULATORS

We consider the following general input-output model of a rigid joint robotic manipulator, taken from [6, pp. 1-4]:

$$H(q)\ddot{q} + C(q,\dot{q})\,\dot{q} + g(q) = u \quad ; \quad y = q \tag{3.1}$$

where $y = q$ stands for the n-dimensional angular position vector, H(q) is is the n x n positive definite inertia matrix of the manipulator, the term $C(q,\dot{q})\dot{q}$ is a n- dimensional vector of centripetal and Coriolis torques and g(q) is the vector of gravitational torques. The vector u is the vector of applied torques.

We consider *the extended system model* of (3.1) (see [6, p. 190]):

$$\dot{q} = \omega \ ; \quad H(q)\dot{\omega} + C(q,\omega)\omega + g(q) = u \quad ; \quad \dot{u} = v \tag{3.2}$$

where v is an auxiliary independent input. Systems (3.1),(3.2) are trivially invertible.

The set of input-output differential equations relating the output vector y to the auxiliary input vector v is easily obtained as:

$$H(y)y^{(3)} + \left[\dot{H}(y) + C(y,\dot{y})\right]\ddot{y} + \left[\dot{C}(y,\dot{y}) + \frac{\partial g(y)}{\partial y}\right]\dot{y} = v \tag{3.3}$$

We propose the following average decoupled second order asymptotically stable dynamics on the output vector y, obtained by the given average control:

$$\ddot{y} + 2\Xi\Omega_n\dot{y} + \Omega_n^2(y-Y) = 0 \ ; \ u_{av} = c(y,\dot{y})\dot{y} + g(y) - 2H(y)\Xi\Omega_n\dot{y} - H(y)\Omega_n^2(y-Y) \tag{3.4}$$

where Ξ and Ω_n are diagonal constant matrices appropriately chosen. Then, according to the results in Section 2, we let the vector of auxiliary output functions become:

$$\sigma = \ddot{y} + 2\,\Xi\,\Omega_n \dot{y} + \Omega_n^2 (y-Y) \tag{3.5}$$

Forcing (3.5) to satisfy a set of decoupled autonomous asymptotically stable PWM dynamics of the same form as in (2.1) one obtains:

$$\dot{\sigma} = y^{(3)} + 2\Xi\Omega_n \ddot{y} + \Omega_n^2\, \dot{y} = -W\, PWM_\tau[\sigma(t_k)] \tag{3.6}$$

where PWM stands for a vector whose components are constituted by the pwm functions of the components of the argument and W is a diagonal matrix.

The following dynamic discontinuous feedback controller of the PWM type, obtained by substitution of (3.3) into (3.6), forces the vector y to satisfy the set of (decoupled) autonomous dynamics specified by (3.4):

$$\begin{aligned} \dot{u} = &\left[\dot{H}(y) + C(y,\dot{y}) - 2H(y)\,\Xi\,\Omega_n\right]\ddot{y} + \left[\dot{C}(y,\dot{y}) + \frac{\partial g(y)}{\partial y} - H(y)\,\Omega_n^2\right]\dot{y} \\ &- H(y)\, W\, PWM_\tau[\ddot{y} + 2\,\Xi\,\Omega_n\,\dot{y} + \Omega_n^2 (y-Y)] \end{aligned} \tag{3.7}$$

Simulations were run for a 2 degree of freedom robotic manipulator (see [6, pp. 1-4] with: $l_1 = l_2 = 1$ [m] , $m_1 = m_2 = 1$ [kg]. The desired position vector components were set to be:

$Y_1 = q_{1d} = 0.9$ [rad] ; $Y_2 = q_{2d} = -0.6$ [rad]

The PWM controller parameters were chosen as :

$$W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \rho = \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix},\ \Xi = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix},\ \Omega_n = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$$

Figure 1a shows the PWM controlled position and velocity responses of the robotic manipulator when governed by a dynamical feedback controller of the form (3.7). Figure 1b depicts the smoothed components of the applied torque input vector as generated by the dynamical PWM policy (3.7). Figure 1c and 1d shows the evolution trajectories of the auxiliary output functions along with the corresponding duty ratios, and the involved pwm signals, for each input component. The sampling time, for both inputs, was set to 0.1 seconds.

4. CONCLUSIONS

Smoothed PWM control of nonlinear multivariable input-output systems, through a dynamical feedback strategy, constitutes an advantageous, practical, possibility with theoretical foundations directly found on the differential algebraic approach to control theory. Exactly linearizable input-output systems do not naturally exhibit such a smoothing possibilities. By resorting to a prolongation of the system, the low pass filtering effects may be robustly recovered on the associated "extended" PWM controller.

5. REFERENCES

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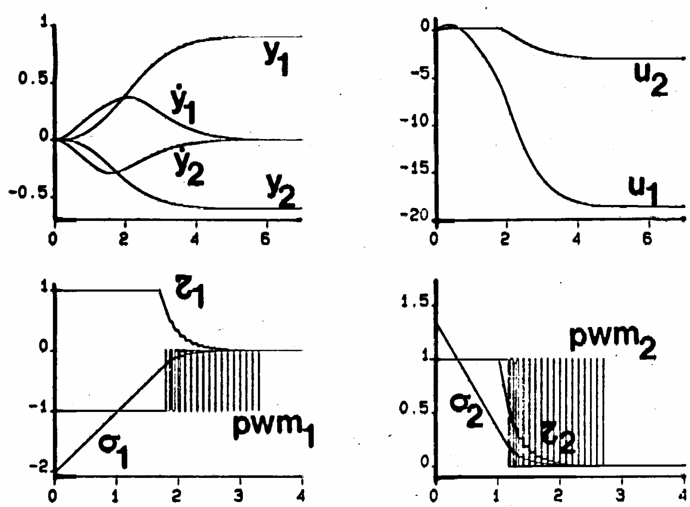

Figure 1. Dynamical PWM controlled response of two-degree-of-freedom robotic manipulator.