

DYNAMICAL VARIABLE STRUCTURE CONTROL OF A HELICOPTER IN VERTICAL FLIGHT

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Abstract In this article, a dynamical multivariable discontinuous feedback control strategy of the sliding mode type is proposed for the altitude stabilization of a nonlinear helicopter model in vertical flight. While retaining the basic robustness features associated to sliding mode control policies, the proposed approach also results in smoothed out (i.e., non-chattering) input trajectories and controlled state variable responses.

1. INTRODUCTION

Using techniques derived from the *differential algebraic* approach to control theory (see Fliess [1]-[2]), dynamical sliding mode control of nonlinear systems has been recently introduced for the *chattering-free* variable structure control regulation of nonlinear single-input single-output systems (see Sira-Ramírez [3]-[5] and, for seminal ideas, see Fliess and Messager [6]). In Sira-Ramírez *et al* [7], the technique was shown to possess particularly desirable features for the robust solution of stabilization and tracking problems defined on mechanical systems, such as flexible joint robotic manipulators.

In this paper we propose a dynamical sliding mode controller for the attitude control of a multivariable nonlinear helicopter under model. The main rotor collective pitch and the engine throttle input were used as control variables for height stabilization around a desired constant reference. The nonlinear dynamical system equations, which describe the vertical motions of the helicopter, were identified from an experimental flight control facility which consists of an X-Cell 50 radio-controlled model helicopter, powered by a 0.5 in³ two-cycle Webra gasoline engine (see Pallet *et al* [8]). A state coordinate transformation of the nonlinear system dynamics into Isidori's Normal Canonical Form is shown to yield a, decouplable, exactly linearizable multivariable system. On such a transformed system, *static sliding mode* control techniques should not be directly applied, as they result in undesirable chattering of the collective pitch and engine throttle inputs. Input chattering would also result in unnecessary excitation of unmodelled dynamics and high frequency vibrations of the airframe and propulsion systems. The advantageous robustness features of the sliding mode control approach are made compatible with the mechanical limitations of the system through an *extended*

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system model (see Nijmeijer and Van der Schaft [9]) on which an *auxiliary static sliding mode controller* design is performed via well-known techniques (Sira-Ramírez [10]). The obtained static design is then re-interpreted, in terms of the original control input variables, as a *dynamical sliding mode feedback controller*. The chattering state responses and chattering inputs trajectories, otherwise characteristic of sliding mode control techniques, are thus entirely confined to the state space of the dynamic controller and effectively eliminated from the system state space, and control inputs. As a result, the generated input signal and the corresponding state trajectory response are sufficiently smoothed by the inherent integration.

Modern linear controller design methodologies have been used in the past for helicopter altitude regulation problems. Such techniques include H_∞ optimal control, linear quadratic regulator design and eigenvalue-eigenvector assignment techniques. Reviews of such approaches are contained in Garrad and Low [11] and in Mannes *et al* [12] where the reader is referred for more thorough details on results. The work of Pallet *et al* [13] served as the basis for our understanding of the helicopter model.

Section 2 presents a nonlinear helicopter model and the corresponding dynamical sliding mode controller design for altitude and the rotor pitch angle regulation. In this section, computer generated simulations are presented and discussed. Section 3 collects the conclusions and suggestions for further research.

2. DYNAMICAL SLIDING MODE CONTROL OF AN HELICOPTER IN VERTICAL FLIGHT.

2.1 The helicopter model [8]

We consider a miniature helicopter mounted on a stand (see Figure 1) which places it sufficiently high above the ground (over one rotor blade diameter). The stand is equipped with conveniently located pistons which offset the weight of the stand while the helicopter is in motion. The following set of differential equations describes the vertical motions of the X-Cell 50 model miniature helicopter:

$$\ddot{z} = K_1(1+G_{eff})C_T\omega^2 - g - K_2\dot{z} - K_3\dot{z}^2 - K_4 \quad (2.1)$$

where:

$$C_T = \left[-K_{C1} + \sqrt{K_{C1}^2 + K_{C2}\theta_c} \right]^2 \quad (2.2)$$

and

$$\dot{\omega} = -K_5\omega - K_6\omega^2 - K_7\omega^2 \sin\theta_c + K_8u_{th} + K_9 \quad (2.3)$$

$$\ddot{\theta}_c = K_{10} (-0.00031746 u_{0c} + 0.5436 - \theta_c) - K_{11}\dot{\theta}_c - K_{12}\omega^2 \sin \theta_c \quad (2.4)$$

The above variables are defined as:

z : height above the ground (m).
 ω : rotational speed of the rotor blades (rad/s).
 g : gravitational force (m/s²).
 θ_c : collective pitch angle of the rotor blades (rad).
 u_{th} : input to the throttle.
 u_{0c} : input to the collective servomechanisms (rad).

The first term in equation (2.1) is the main thrust term, taken to be proportional to the square of the rotational speed of the rotor blades, ω , and dependent also upon the ground effect term, $G_{eff}(z)$, which we will assume to be zero for sufficiently high initial conditions of the altitude variable. A damping term is present in equation (2.1) just to account for the piston mounted on the stand. Equation (2.1) also includes parasitic and constant drag forces as third and fourth terms respectively. Equation (2.2) is a modification of that found in Johnson [14] and it relates the thrust constant C_T to the collective pitch angle θ_c . The two stroke engine, and its effect on the rotational velocity of the rotor blades, is modelled by equation (2.3). This equation includes a damping term, two air foil drag losses terms and a linear approximation of the combustion engine, as well as the throttle servo input, u_{th} , to the rotational speed, ω . Finally, equation (2.4) represents the collective pitch servo response to the input u_{0c} . The first terms of (2.4) represent a linear approximation of the relationship between the servo input and the resulting collective pitch in steady state. The last two terms represent the damping of the servo system due to the servo motor and linkages and a torque drag term.

Nominal values of the parameters, taken from Pallet *et al* [8], are given as :

$$K_1 = 0.25m, K_2 = 0.10s^{-1}, K_3 = 0.1 m^{-1}, K_4 = 7.86 m/s^2, \\ K_5 = 0.70 s^{-1}, K_6 = 0.0028, K_7 = 0.005, K_8 = -0.1088s^2, \\ K_9 = -13.92s^{-2}, K_{10} = 800.00 s^{-2}, K_{11} = 65.00s^{-1}, \\ K_{12} = 0.1, K_{C1} = 0.03259, K_{C2} = 0.061456.$$

Model (2.1)-(2.4) may be written in terms of a state variable representation as follows:

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 \quad (2.5) \\ y = Cx$$

where:

$$x = \begin{bmatrix} z & \dot{z} & \omega & \theta_c & \dot{\theta}_c \end{bmatrix}^T, u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T \\ = \begin{bmatrix} K_8 u_{th} & -0.00031746 K_{10} u_{0c} \end{bmatrix}^T \\ f(x) = \begin{bmatrix} x_2 \\ x_3^2(a_1 + a_2x_4 - \sqrt{a_3 + a_4x_4}) + a_5x_2 + a_6x_2^2 + a_7 \\ a_8x_3 + a_{10}x_3^2 \sin x_4 + a_9x_3^2 + a_{11} \\ x_5 \\ a_{13}x_4 + a_{14}x_3^2 \sin x_4 + a_{15}x_5 + a_{12} \end{bmatrix};$$

$$g_1(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}; g_2(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} x$$

The parameters a_1 through a_{15} are given by:

$$a_1 = 5.31 \times 10^{-4}, a_2 = 1.5364 \times 10^{-2}, \\ a_3 = 2.82 \times 10^{-7}, a_4 = 1.632 \times 10^{-5} \\ a_5 = -K_2, a_6 = -K_2, a_7 = -g-K_4, \\ a_8 = -K_5, a_9 = -K_6, a_{10} = -K_6 \\ a_{11} = K_9, a_{12} = 0.5436 K_{10}, a_{13} = -K_{10}, \\ a_{14} = -K_{12}, a_{15} = -K_{11}$$

The "extended" system equations for the helicopter model (2.5) are of the form :

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 \\ \dot{u}_1 = v_1 \\ \dot{u}_2 = v_2 \\ y = Cx \quad (2.6)$$

The following (input-dependent) invertible state coordinate transformation:

$$\zeta_{11} = x_1 \\ \zeta_{12} = x_2 \\ \zeta_{13} = x_3^2(a_1 + a_2x_4 - \sqrt{a_3 + a_4x_4}) + a_5x_2 + a_6x_2^2 + a_7 \\ \zeta_{14} = 2x_3(a_1 + a_2x_4 - \sqrt{a_3 + a_4x_4}) (a_8x_3 + a_{10}x_3^2 \sin x_4 \\ + a_9x_3^2 + a_{11} + u_1) \\ + (a_5 + 2a_6x_2)[x_3^2(a_1 + a_2x_4 - \sqrt{a_3 + a_4x_4}) \\ + a_5x_2 + a_6x_2^2 + a_7] \\ + x_3^2[a_2x_5 - \frac{1}{2}a_4x_5(a_3 + a_4x_4)^{-1/2}] \\ \zeta_{21} = x_4 \\ \zeta_{22} = x_5 \\ \zeta_{23} = a_{13}x_4 + a_{14}x_3^2 \sin x_4 + a_{15}x_5 + a_{12} + u_2 \quad (2.7)$$

together with its inverse transformation, given by :

$$x_1 = \zeta_{11} \\ x_2 = \zeta_{12} \\ x_3 = \sqrt{\frac{\zeta_{13} - a_5\zeta_{12} - a_6\zeta_{12}^2 - a_7}{(a_1 + a_{12}\zeta_{21} - \sqrt{a_3 + a_4\zeta_{21}})}} =: \beta_1 \\ x_4 = \zeta_{21} \\ x_5 = \zeta_{22} \\ u_1 = \frac{[\zeta_{14} - (a_5 + 2a_6\zeta_{12})]}{2\beta_1(a_1 + a_{12}\zeta_{21} - \sqrt{a_3 + a_4\zeta_{21}})} \\ \frac{[\beta_1^2(a_1 + a_{12}\zeta_{21} - \sqrt{a_3 + a_4\zeta_{21}}) + a_5\zeta_{12} + a_6\zeta_{12}^2 + a_7]}{}$$

$$\begin{aligned}
& \frac{\beta_1^2 [a_2 \zeta_{22} + \frac{1}{2} a_4 \zeta_{22} (a_3 + a_4 \sin \zeta_{21})^{-1/2}]}{2\beta_1 (a_1 + a_{12} \zeta_{21} - \sqrt{a_3 + a_4 \zeta_{21}})} \\
& - a_8 \beta_1 - a_{10} \beta_1^2 \sin \zeta_{21} - a_9 \beta_1^2 - a_{11} \\
u_2 = & \zeta_{23} - a_{15} \zeta_{22} - a_{12} \\
& - a_{13} \zeta_{21} - a_{14} \left[\frac{\zeta_{13} - a_5 \zeta_{12} - a_6 \zeta_{12}^2 - a_7}{(a_1 + a_2 \zeta_{21} - \sqrt{a_3 + a_4 \zeta_{21}})} \right] \sin \zeta_{21}
\end{aligned} \quad (2.8)$$

take the extended system (2.6) into Isidori's normal canonical form:

$$\begin{aligned}
\dot{\zeta}_{11} &= \zeta_{12} \\
\dot{\zeta}_{12} &= \zeta_{13} \\
\dot{\zeta}_{13} &= \zeta_{14} \\
\dot{\zeta}_{14} &= [2\beta_2^2 + 2\beta_1 (a_8 \beta_2 + 2a_{10} \beta_1 \beta_2 \sin \zeta_{21} + \\
& a_{10} \beta_1^2 \zeta_{22} \cos \zeta_{21} + 2a_9 \beta_1 \beta_2)] \\
& (a_1 + a_2 \zeta_{21} - \sqrt{a_3 + a_4 \zeta_{21}})^{-1/2} \\
& + 4\beta_1 \beta_2 (a_2 \zeta_{22} + \frac{1}{2} a_4 \zeta_{22} (a_3 + a_4 \zeta_{21})^{-1/2} \\
& + (a_5 + 2a_6 \zeta_{12}) \zeta_{14} + 2a_6 \zeta_{13}^2 + \beta_1^2 [a_2 \zeta_{23} \\
& + a_4 + \frac{1}{2} a_4^2 \zeta_{22}^2 (a_3 + a_4 \zeta_{21})^{-3/2}] \\
& + 2\beta_1 (a_1 + a_2 \zeta_{21} - \sqrt{a_3 + a_4 \zeta_{21}}) v_1 \\
\dot{\zeta}_{21} &= \zeta_{22} \\
\dot{\zeta}_{22} &= \zeta_{23} \\
\dot{\zeta}_{23} &= a_{13} \zeta_{22} + 2a_{14} \beta_1 \beta_2 \sin \zeta_{21} \\
& + a_{14} \beta_1^2 \zeta_{22} \cos \zeta_{21} + a_{15} \zeta_{23} + v_2 \\
y_1 &= \zeta_{11} \\
y_2 &= \zeta_{21}
\end{aligned} \quad (2.9)$$

with:

$$\begin{aligned}
\beta_2 = & \frac{\zeta_{14} - (a_5 + 2a_6 \zeta_{12}) \zeta_{13}}{2\beta_1 (a_1 + a_2 \zeta_{21} - \sqrt{a_3 + a_4 \zeta_{21}})} \\
& - \frac{\beta_1^2 [a_2 \zeta_{22} + \frac{1}{2} a_4 \zeta_{22} (a_3 + a_4 \zeta_{21})^{-1/2}]}{2\beta_1 (a_1 + a_2 \zeta_{21} - \sqrt{a_3 + a_4 \zeta_{21}})}
\end{aligned} \quad (2.10)$$

Transformation (2.8), (2.9) is everywhere invertible, except on the set of state values satisfying the condition:

$\beta_1 (a_1 + a_2 \zeta_{21} - \sqrt{a_3 + a_4 \zeta_{21}}) = 0$. The rotor blade speed $\omega = x_3 = \beta_1$ is never zero while the helicopter is in flight. It is easy to see that since $a_3 = a_1^2$ the only physically meaningful solution of this relation happens when the collective pitch ζ_{21} is zero. From a practical standpoint such a situation seldom happens since the collective pitch takes a nominal nonzero value (typically, 7 to 8 degrees) which is controlled, throughout the maneuver, to the same, or higher, operating point. However, if it is absolutely required to cross the zero collective pitch condition, in a

complex altitude maneuver, then the method exposed here fails, and large discontinuities have to be imposed, momentarily, on the control input. This topic is not addressed here in any further detail (see Fliess *et al* [15] for related details).

3.2 A dynamical sliding mode controller design for helicopter altitude stabilization

Given the mechanical nature of the helicopter system being controlled, static sliding mode controller design should be avoided, as its actions result in chattering of the throttle input and chattering collective pitch servomechanism input. The behavior of the system would be sufficiently smooth but the actuators will unnecessarily suffer the effects of excessive vibratory (bang-bang type) commands. Thus, a dynamical variable structure control design procedure will be applied to the helicopter model by designing a static variable structure controller on the extended helicopter model. For this, we define the sliding surface coordinate functions as:

$$\begin{aligned}
\sigma_1(\zeta) &= \zeta_{14} + \alpha_{13} \zeta_{13} + \alpha_{12} \zeta_{12} + \alpha_{11} (\zeta_{11} - y_{1d}) \\
\sigma_2(\zeta) &= \zeta_{23} + \alpha_{22} \zeta_{22} + \alpha_{21} (\zeta_{21} - y_{2d})
\end{aligned} \quad (2.11)$$

where y_{1d} is the desired constant height to which the helicopter is to be driven while achieving a stable hovering. The desired value y_{2d} of the collective pitch angle is usually chosen as a nominal value at which liftoff and hovering is obtained.

If a sliding regime exists on the zero level sets of the sliding surface coordinate functions, σ_1 and σ_2 , then the ideal sliding dynamics for each input-output decoupled subsystem is asymptotically stable toward the desired equilibrium condition. The output signals y_1, y_2 are then governed by the following asymptotically stable, decoupled, autonomous, time-invariant linear differential equations:

$$\begin{aligned}
\ddot{y}_1 + \alpha_{13} \dot{y}_1 + \alpha_{12} \dot{y}_1 + \alpha_{11} (y_1 - y_{1d}) &= 0 \\
\ddot{y}_2 + \alpha_{22} \dot{y}_2 + \alpha_{21} (y_2 - y_{2d}) &= 0
\end{aligned} \quad (2.12)$$

The expressions for the dynamical controllers are obtained by forcing the surface coordinate functions σ_1 and σ_2 to satisfy the following autonomous sliding mode dynamics:

$$\begin{aligned}
\dot{\sigma}_1 &= -\mu_1 [\sigma_1 + W_1 \operatorname{sgn} \sigma_1] ; \\
\dot{\sigma}_2 &= -\mu_2 [\sigma_2 + W_2 \operatorname{sgn} \sigma_2]
\end{aligned} \quad (2.13)$$

Using (2.11) and (2.13), and solving for the first order derivatives of the original control vector components, one obtains a set of two time-varying, first order, nonlinear discontinuous differential equations for the multivariable controller accomplishing output stabilization around the desired equilibrium condition. Such an expression is quite involved and may be found in all detail in Sira-Ramirez *et al* [16].

2.3 Simulation Results

Simulations were performed for the dynamical variable structure feedback controllers proposed above. From a hovering equilibrium condition, located at $y_1 = 0.75$ mt, with a nominal collective pitch of $y_2 = 0.125$ rad, the helicopter was required to rise to a height of 1.25m while simultaneously rising the collective pitch from 0.125 to a new nominal value of 0.20 to ease the throttle magnitude and at the same time obtain adequate lift force. The

generated input trajectories for the dynamical variable structure controller are quite smooth with unnoticeable chattering while exhibiting the same qualitative response for the output vector trajectories. The dynamical sliding mode controlled responses for the output variables are shown in figure 2. Figure 3 shows the dynamically generated control input trajectories for u_1 and u_2 . The values of μ , W and α 's for the dynamical sliding mode controller were set to be :

$$\begin{aligned}\mu_1 &= 10, W_1 = 2, \alpha_{11} = 30, \alpha_{12} = 25, \alpha_{13} = 8, \\ \mu_2 &= 10, W_2 = 2, \alpha_{21} = 20, \alpha_{22} = 9\end{aligned}$$

3. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

A dynamical variable structure controller scheme has been presented which achieves robust asymptotic output stabilization for nonlinear multivariable systems which are exactly linearizable. The dynamical feedback controller generates smoothed control inputs to the given system and constrains all undesirable chattering effects to the state space of the controller, thus effectively eliminating the undesirable effects of high frequency bang-bang signals on the system variables and inputs. The proposed controller requires the on-line integration of a nonlinear discontinuous system of differential equations. Such integration offers no particular difficulty for the implementation over those commonly encountered in, say, state observers. Dynamical sliding mode control opens up the possibilities of having chattering-free controlled responses in a variety of dynamical controlled systems where, traditionally, the variable structure control approach encountered natural limitations for its implementation due to unmodelled dynamics excitation. Thus, through system extension, one may directly apply this robust control technique to mechanical systems in general.

Applications of the proposed dynamical feedback variable structure regulator were carried out in this article for a non-trivial helicopter example comprising 5 states and 2 inputs. The simulation results are quite encouraging and work is presently under way leading to actual implementation in the laboratory facility of the Real Time Robot Control Systems Laboratory at Purdue University.

As topics for further research, dynamical sliding mode control strategies can be extended to nonlinear multivariable systems of the non-decouplable class. Adaptive regulation techniques for cases in which parameter uncertainty is present both at the plant, and at the sliding surface, is being presently pursued.

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FIGURES

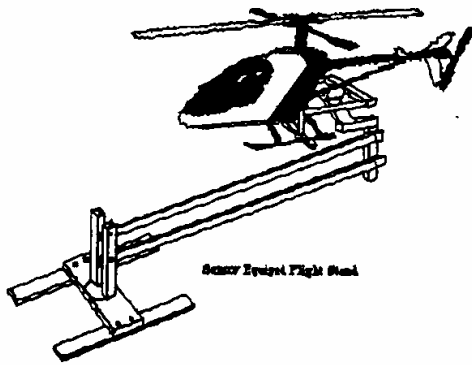


Figure 1. Miniature Helicopter and Flight Stand.

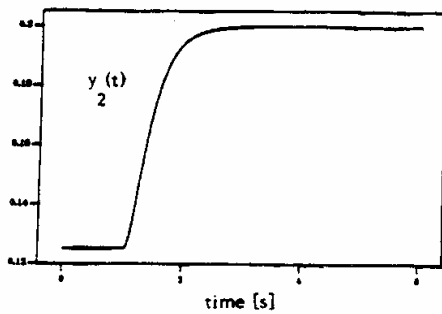
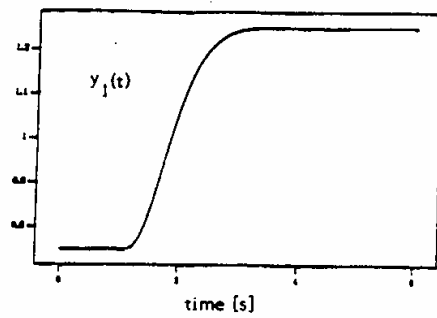


Figure 2. Dynamical Multivariable Sliding Mode Controlled Output Responses for Helicopter Altitude Stabilization Problem.

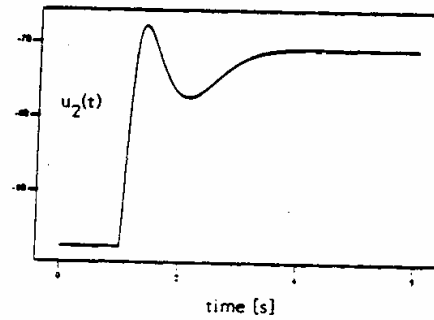
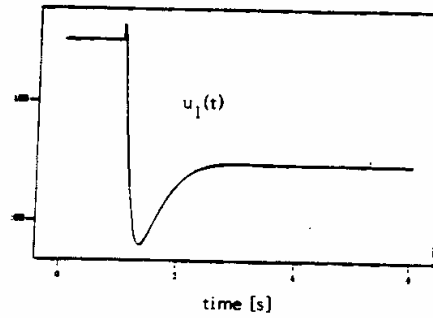


Figure 3. Control Input Trajectories of Dynamic Multivariable Sliding Mode Control Helicopter .