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A Dynamical Sliding Mode Control Approach to Predictive Control by Inversion *

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Abstract

In this article a dynamical discontinuous feedback control strategy, of the sliding mode type, is proposed for robust predicitive control schemes based on system inversion.

1 Introduction

The Model Based Predictive Control technique has been the topic of sustained research eversince first introduced by Richalet *et al* [1], in 1978. The technique has been further developed by Clark *et al* [2], Bitmead *et al* [3], and by Ronald and Stoeterboek [4]. Extensions to the nonlinear case are due to Abu el Ata *et al* [5].
In this article we propose an approach that uses an advantageous combination of dynamical sliding mode control (Sira-Ramirez [6]) and input-output system inversion (Fliess [7]). These two techniques naturally blend together to yield a robust solution to the nonlinear output tracking problem associated to any predictive control scheme defined within a prespecified prediction interval.

2 Robust Predictive Control via a Sliding Mode Strategy

Consider a nonlinear n-dimensional single-input single-output dynamical system, expressed in Generalized Observability Canonical Form [7]:

$$\begin{aligned} \dot{\eta}_i &= \eta_{i+1} \quad i=1,\ldots,n-1 \\ \dot{\eta}_n &= c(\eta,u,\dot{u},\ldots,u^{(\alpha)})+\nu \\ y &= \eta_1 \end{aligned} \tag{2.1}$$

where the scalar signal function $\nu \leq N$ represents bounded external perturbation signals and an assessment of possible modeling errors. We assume $\alpha > 0$. Let $y_R(t)$ be a prescribed reference output function, assumed to be sufficiently smooth and defined over a given prediction interval $[0, Tp]$. Such an interval will be determined below.
Define a tracking error function $e(t)$ as $e(t) = y(t) - y_R(t)$ and an error vector $\mathbf{e} = \text{col}\ (e_1, e_2, \ldots, e_n)$. We then have :

$$\begin{aligned} \dot{e}_i &= e_{i+1} \quad i=1,\ldots,n-1 \\ \dot{e}_n &= c(\mathbf{e}+\xi_R(t),u,\dot{u},\ldots,u^{(\alpha)}) - y_R^{(n)}(t)+\nu \\ e &= e_1 \end{aligned} \tag{2.2}$$

with $\xi_R(t) = \text{col}\ \left(y_R(t), y_R^{(1)}(t), \ldots, y_R^{(n-1)}(t)\right)$.
A (desired) system output tracking error dynamics is prescribed on $[0, Tp]$ by means of a reduced order, asymptotically stable, linear dynamics,

$$\begin{aligned} \dot{e}_i &= e_{i+1} \quad i=1,\ldots,n-2 \\ \dot{e}_{n-1} &= -m_{n-1}e_{n-1} - \cdots - m_1 e_1 \\ e &= e_1 \end{aligned} \tag{2.3}$$

with a suitably prescribed set of real (stable) coefficients $\{m_1, \ldots, m_{n-1}\}$. We denote by μ the smallest, in absolute value, of the real parts of the (stable) eigenvalues associated to (2.3). This parameter is used in the computation of the prediction interval as indicated below.

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We also define a *sliding surface coordinate function* as the auxiliary output function

$$w = e_n + \sum_{i=1}^{n-1} m_i e_i \tag{2.4}$$

Note that if w is driven to zero, in finite time, by means of a suitable control action, then the desired error dynamics, specified in (2.3), is accomplished.
An implicit dynamical discontinuous controller, inducing a robust sliding motion on $w = 0$, is immediately found from the above equations as

$$c(\xi_R + e, u, \dot{u}, \ldots, u^{(\alpha)}) = y_R^{(n)} - \sum_{i=1}^{n-1} m_i e_{i+1} - W \text{ sign } \left(e_n + \sum_{i=1}^{n-1} m_i e_i \right) \tag{2.5}$$

The above controller determines the following evolution of the auxiliary output function w:

$$\dot{w} = \nu - W \text{ sign } w \tag{2.6}$$

For sufficiently high values of W, the controlled values of w go to zero in finite time in spite of the bounded values of ν. A *sliding regime* can, thus, be indefinitely sustained on $w = 0$.
After convergence to zero of the output tracking error, the dynamical controller exhibits the following *remaining dynamics* :

$$c(\xi_R, u, \dot{u}, \ldots, u^{(\alpha)}) = y_R^{(n)} \tag{2.7}$$

It is assumed that this nonlinear time-varying dynamics is stable for the given output reference function $y_R(t)$. The dynamics (2.7) is evidently coincident with the *zero dynamics* whenever $y_R(t) = 0$. In such cases the assumption implies the given system is *minimum phase*.
When the proposed predictive dynamical discontinuous controller, is used on the actual system, one may generally obtain, at the end of the prediction horizon, a nonzero value for the sliding surface. This value is evidently a function of the model mismatch and of the various unmodelled uncertainties affecting the system. The predictive control technique proposes a number of procedures for obtaining an error improvement for the next prediction interval $[T_p, T'_p]$ (see [5]).
A reasonable choice for the setting of the new prediction horizon $[Tp, T'p]$ may be devised as:

$$T'_p = \frac{|w(T_p)|}{W - N} + \frac{2}{\mu} \tag{2.8}$$

i.e. the new prediction interval is comprised of the predicted reaching time to the sliding surface, $w = 0$, computed from the previously obtained sliding surface value, plus *twice* the slower time constant of the impossed linear error dynamics. This guarantees, at the end of the new prediction horizon, a theoretical decrease of the slowest tracking error mode to about 13 % of its initial value at the hitting of the newly proposed sliding surface.

3 Conclusions

In this article, a model based predictive control scheme has been proposed which combines the advantages of sliding mode control robustness, and its traditional high performance features, with the conceptual simplicity of nonlinear system inversion. This combination efficiently deals with the associated output tracking problem present in every predictive feedback control scheme.

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