ADAPTIVE CHATTERING-FREE SLIDING MODE CONTROL OF NONLINEAR UNCERTAIN SYSTEMS*

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Abstract In this article, an adaptive dynamical sliding mode based feedback strategy is presented for asymptotic output stabilization of nonlinear controlled systems exhibiting parametric uncertainty. A dynamical feedback controller, nominally acheving output stabilization via exact linearization, is obtained by resorting to generalized observability canonical forms and sliding mode control stabilization. The adaptive version of the dynamical variable structure controller is then obtainable via standard, direct, overparametrized adaptive control techniques available for linearizable systems through static state feedback. An illustrative example from the chemical process control area is provided.

1. INTRODUCTION

Asymptotic output stabilization for parametric uncertain nonlinear systems constitutes a most important problem in control systems design. Contributions, from the differential geometric viewpoint, were given by Isidori and Sastry [1], Kanellakopoulos *et al* [2],[3], Taylor *et al* [4], Campion and Bastin [5], Teel *et al* [6] and many others. For enlightening details, and general results, the reader is referred to the books by Sastry and Bodson [7], and Narendra and Annaswamy [8]. Research trends are contained in the collection of lectures edited by Kokotovic [9]. For other contributions to the area, the reader is referred to the reprint book edited by Narendra *et al* [10].

In this article, using the results of [1], an adaptive asymptotic output stabilization scheme is proposed for dynamical sliding-mode-based exactly linearizing controllers, obtained by repeated output differentiation. The scheme is restricted to the class of nonlinear systems which exhibit linear parameter dependence in their defining vector fields. Overparametrization [5] and availability of the dynamical controller state variables are the key issues that allow application of direct adaptive control techniques, available for statically input-output linearizable systems, to dynamical controlled systems. A chemical process control application example is presented.

In Section 2 of this paper, the adaptive dynamical variable structure control stabilization scheme is presented (see Sira-Ramirez [11]-[12] for the non adaptive case). Section 3 deals with a chemical process control example including computer simulations. Concluding remarks, and proposals for further research, are collected in Section 4.

2. ADAPTIVE OUTPUT STABILIZATION OF DYNAMICALLY LINEARIZABLE NON LINEAR SYSTEMS

2.1 Linearization by Discontinuous Dynamical Feedback Control.

Consider the following n-dimensional *state space realization* of a single-input single-output nonlinear system :

$$\dot{x} = f(x,\theta) + g(x,\theta)u \qquad y = h(x,\theta) \tag{2.1}$$

where $f : R^{n+p} \to R^n$ and $g: R^{n+p} \to R^n$ are, for fixed θ in R^p, C^∞ vector fields globally defined on R^n, and $h : R^{n+p} \to R$ is a C^∞ function. It is assumed that the system has *strong relative degree* $r < n$ (Isidori [13]). The parameter vector θ is assumed to be constant and f,g and h are linear functions of θ.

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The i-th time derivative of the output function may be written, in terms of the state vector x and the control input u, as

$$\begin{aligned} y^{(i)} &= b_i(x,\theta) \quad \text{for } i < r \quad ; \text{with } b_0(x,\theta) = h(x,\theta) \\ y^{(i)} &= b_i(x,\theta,u,u^{(1)},\ldots,u^{(i-r-1)}) + a(x,\theta)u^{(i-r)} \quad \text{for } r \le i \le n \end{aligned} \tag{2.2}$$

In particular, the n-th time derivative of y may be obtained as :

$$y^{(n)} = b_n(x,\theta,u,u^{(1)},\ldots,u^{(n-r-1)}) + a(x,\theta)u^{(n-r)} \tag{2.3}$$

We assume that the "observability " matrix, constituted by the (row vector) gradients, with respect to x, of $y^{(i)}$ (i=0,1,...,n-1) is full rank n. i.e.,

$$\text{rank}\frac{\partial(y,y^{(1)},\ldots,y^{(n-1)})}{\partial x} = \text{rank}\frac{\partial(y,y^{(1)},\ldots,y^{(n)})}{\partial x} = n \tag{2.4}$$

This assumption implies that (2.1) can be described by an n-th order input-output scalar differential equation (see Conte *et al* [14], Diop [15]). The implicit function theorem allows one to locally solve for x, from (2.2), in terms of u and its time derivatives, as well as in terms of the derivatives of y. In other words, there exist a set of n independent functions ϑ_i, implicitly defined by (2.2), such that :

$$x_i = \vartheta_i(y,y^{(1)},\ldots,y^{(n-1)},u,u^{(1)},\ldots,u^{(n-r-1)}) \quad ; \quad i=1,2,\ldots,n \tag{2.5}$$

In general, one locally obtains a representation of (2.1) in the form :

$$y^{(n)} = c(y,y^{(1)},\ldots,y^{(n-1)},\theta,u,u^{(1)},\ldots,u^{(n-r)}) \tag{2.6}$$

Definition 2.1 (Fliess [16]) Let the output y be identically zero for an indefinite amount of time. The *zero dynamics*, associated with (2.1), is defined as :

$$c(0,\theta,u,u^{(1)},\ldots,u^{(n-r)}) = 0 \tag{2.7}$$

We assume that (2.7) is locally asymptotically stable to a constant operating point, u = U. In such a case we say (2.1) is locally *minimum phase* around the equilibrium point of interest.

Proposition 2.2 Let $u^{[i]}$ denote $u,u^{(1)},\ldots,u^{(i)}$, and let μ be a strictly positive scalar quantity. Then, the following dynamical discontinous feedback controller:

$$\begin{aligned} a(x,\theta)u^{(n-r)} =& -b_n(x,\theta,u^{[n-r-1]}) - \sum_{i=1}^{r-1}\alpha_i b_i(x,\theta) \\ & -\sum_{j=r}^{n-1}\alpha_j\left[b_j(x,\theta,u^{[j-r-1]}) + a(x,\theta)u^{(j-r)}\right] \\ & -\mu\,\text{sgn}\left\{\sum_{i=1}^{r}\alpha_i b_{i-1}(x,\theta) + \right. \\ & \left.\sum_{j=r+1}^{n}\alpha_j\left[b_{j-1}(x,\theta,u^{[j-r-2]}) + a(x,\theta)u^{(j-r-1)}\right]\right\} \\ & ;\ \alpha_n = 1 \end{aligned} \tag{2.8}$$

drives the output of system (2.1) to satisfy, in finite time, a linearized dynamics of the form :

$$y^{(n-1)} + \alpha_{n-1}y^{(n-2)} + \ldots + \alpha_1 y = 0 \qquad (2.9)$$

Proof : Define the quantity : $s = y^{(n-1)} + \alpha_{n-1}y^{(n-2)} + \ldots + \alpha_1 y$, and let $s(0)$ stand for the value of s at time $t = 0$. One easily verifies that $ds/dt = -\mu\, \mathrm{sgn}(s)$. Hence the condition $s = 0$ is reached in finite time (given by : $T = \mu^{-1} |s(0)|$) and it is indefinitely sustained in a *sliding mode* fashion (Utkin [17]). □

The scalar time-varying differential equation (2.8) defines a *dynamical feedback controller* which can accomplish exponential output stabilization to zero, in a manner entirely prescribed by the set of chosen design coefficients $\{\alpha_1, \alpha_2, \ldots, \alpha_{n-1}\}$, provided that the system is minimum phase. Typically, one chooses the α's to obtain an exponentially asymptotically stable dynamics for (2.9). The set of input derivatives $u^{[n-r-1]}$, in (2.8), naturally qualifies as a state vector, for the dynamical controller, which is available for measurement. If the quantity $a(x,\theta)$ is bounded away from zero then no *impasse* points need be considered for the dynamical system representing the linearizing controller (see Fliess and Hasler [18]). This assumption is equivalent to the *strong relative degree* assumption adopted in [1].

Remark 2.3 Notice that the discontinuities associated to the underlying variable structure control strategy, imposed on the auxiliary function s, directly affect the (n-r)-th derivative of the input signal u. The output of the *dynamical controller* (2.8) is, thus, a smoothed signal. This feature thus provides a chattering-free, yet robust, control input to the regulated plant. Evidently, a simpler static sliding mode controller may also be directly obtained from (2.2), by stopping the differentiation process when $i = r$. However, our main objective is to propose a feedback regulation scheme which retains the smoothness, and robustness, inherent in (2.8) for those cases in which the vector of system parameters, θ, is unknown.

2.2 An Adaptive Regulation Scheme for Dynamical Sliding-Mode Linearizable Systems.

In this section we propose, for the class of systems described by (2.1), a chattering-free adaptive variable structure control linearization scheme for asymptotic output stabilization problems. We should stress that, eventhough, traditionally, the siding mode control technique has been specially devised to efficiently regulate systems with parametric and external uncertainty, the class of systems where the switching surface does not depend on system parameters may be quite limited. *Dynamical*, (non-adaptive) *sliding mode control* for nonlinear systems, as proposed in [12] and described above, exhibits the advantageous possibility of smoothed (i.e., chattering-free) control input signals and state responses. However, dynamical sliding modes are naturally created on suitable input-dependent sliding surfaces which generally depend, in a crucial manner, upon the system parameters. These parameters may, in turn, be imprecisely known, or, still worse, completely unknown. This fact makes the sliding surface poorly defined and switchings cannot take place, as precisely required, on the switching manifold. We address this class of discontinuous control problems from the perspective of an adaptive control viewpoint.

The effectiveness of the dynamical feedback controller (2.8) is thus highly dependent upon perfect knowledge of the involved system parameters θ. It is clear that exact cancellation of nonlinearities would not be generally possible if the dynamical controller (2.8) was computed using estimated values of such parameters, which are known to be in error with respect to their true values. In this section we assume that the components of θ are constant, but otherwise unknown, and present an adaptive approach to dynamical discontinuous feedback linearization. We denote the estimated values of the parameter vector as $\hat{\theta}$.

Remark 2.4. It may be verified that the linearity of f, g and h with respect to θ implies that the quantities $b_i(x,\theta)$ ($i=0,1,\ldots,n-1$) and $a(x,\theta)$, in (2.2), are *multilinear* functions of the components θ_i of θ. Hence, if one defines a large dimensional vector Θ containing, as individual components, all possible ordered homogeneous multinomial expressions in the θ_i's, of degree smaller than n, then the expressions for b_i ($i=0,1,\ldots,n-1$) and a are indeed linear functions of Θ. This observation and the involved process, known as "overparametrization" [5], allows us to extend recently proposed adaptive control techniques [1], developed for systems linearizable by *static* feedback, to systems linearizable by *dynamical* feedback (see Fliess [19] , and also [11]). □

Consider the time derivative of the quantity s, defined in the proof of proposition 2.1:

$$\dot{s} = \sum_{i=1}^{r-1} \alpha_i b_i(x,\theta) + \sum_{j=r}^{n} \alpha_j \left[b_j(x,\theta,u^{[j-r-1]}) + a(x,\theta)u^{(j-r)} \right] \qquad (2.10)$$

Let $\hat{s}$, the estimate of the sliding surface coordinate function, be defined as :

$$\hat{s} = \sum_{i=1}^{r} \alpha_i b_{i-1}(x,\hat{\theta}) + \sum_{j=r+1}^{n} \alpha_j \left[b_{j-1}(x,\hat{\theta},u^{[j-r-2]}) + a(x,\hat{\theta})u^{(j-r-1)} \right] \qquad (2.11)$$

We explicitly assume that the originally specified sliding surface is "robust" with respect to small parametric perturbations, in the sense that motions constrained to its estimated value $\hat{s}$ do not result in unstable constrained dynamics. This assumption means that small parametric perturbations do not result in large discrepancies between the actual and the perturbed sliding suface coordinate functions. If the imprecision of system parameters is so large that estimated values of the sliding surface do not, somehow, guarantee stability of the corresponding *ideal sliding dynamics*, then, surely, the method here proposed is not applicable.

Define also the following dynamical discontinuous feedback controller, based on estimates of the system parameters :

$$\begin{aligned} a(x,\hat{\theta})u^{(n-r)} = & -b_n(x,\hat{\theta},u^{[n-r-1]}) - \sum_{i=1}^{r-1} \alpha_i b_i(x,\hat{\theta}) \\ & - \sum_{j=r}^{n-1} \alpha_j \left[b_j(x,\hat{\theta},u^{[j-r-1]}) + a(x,\hat{\theta})u^{(j-r)} \right] \\ & - \mu\, \mathrm{sgn} \left\{ \sum_{i=1}^{r} \alpha_i b_{i-1}(x,\hat{\theta}) + \sum_{j=r+1}^{n} \alpha_j \left[b_{j-1}(x,\hat{\theta},u^{[j-r-2]}) + a(x,\hat{\theta})u^{(j-r-1)} \right] \right\} \end{aligned} \qquad (2.12)$$

Then, if a dynamical controller of the form (2.12) is used to regulate the evolution of ds/dt, the expression (2.10) is found to be, after some manipulations :

$$\begin{aligned} \dot{s} = & -\mu\, \mathrm{sgn}\, \hat{s} + \sum_{i=1}^{r-1} \alpha_i \left[b_i(x,\theta) - b_i(x,\hat{\theta}) \right] \\ & + \sum_{j=r}^{n-1} \alpha_j \left\{ b_j(x,\theta,u^{[j-r-1]}) - b_j(x,\hat{\theta},u^{[j-r-1]}) + \left[a(x,\theta) - a(x,\hat{\theta}) \right] u^{(j-r)} \right\} \\ & + b_n(x,\theta,u^{[n-r-1]}) - b_n(x,\hat{\theta},u^{[n-r-1]}) + \left[a(x,\theta) - a(x,\hat{\theta}) \right] u^{(n-r)} \end{aligned} \qquad (2.13)$$

By virtue of Remark 2.4, expression (2.13) can be written as a linear function of the parameter estimation error $\Theta - \hat{\Theta} := \phi$

$$\dot{s} = -\mu\, \mathrm{sgn}\, \hat{s} + (\Theta - \hat{\Theta})^T W(x,u^{[n-r]}) = -\mu\, \mathrm{sgn}\, \hat{s} + \phi^T W(x,u^{[n-r]}) \qquad (2.14)$$

where W is the nonlinear state-dependent *regressor vector*, dependent also upon the "state" of the dynamical controller, represented by u and the derivatives of u up to order n-r-1, and $u^{(n-r)}$ as given by (2.12). Thus, the regressor vector W is actually of the form $W(x,\hat{\Theta},u^{[n-r-1]})$, but we prefer to use the simpler form: $W(x,u^{[n-r]})$. The following assumption is quite standard in adaptive control schemes.

Assumption 2.5 We assume that the regressor vector $W(x,u^{[n-r]})$ is well defined and it is a bounded function for bounded values of all its arguments.

It is easy to see that the switching surface coordinate estimation error $s-\hat{s}$ is given by :

$$s - \hat{s} = \sum_{i=1}^{r} \alpha_i\left[b_{i-1}(x,\theta) - b_{i-1}(x,\hat{\theta})\right] +$$

$$\sum_{j=r+1}^{n} \alpha_j\left\{b_{j-1}(x,\theta,u^{[j-r-2]}) - b_{j-1}(x,\hat{\theta},u^{[j-r-2]}) + \left[a(x,\theta)-a(x,\hat{\theta})\right]u^{(j-r-1)}\right\}$$

$$= (\Theta - \hat{\Theta})^T W_s(x,u^{[n-r-1]}) = \phi^T W_s(x,u^{[n-r-1]}) \qquad (2.15)$$

where $W_s(x,u^{[n-r-1]})$ is a *switching surface regressor vector*, which does not depend on the parameter estimates. The following assumption is not very restrictive.

Assumption 2.6 We assume that the switching surface regressor vector $W_s(x,u^{[n-r-1]})$ has bounded first order partial derivatives, with respect to all its arguments, for bounded values of its arguments.

Lemma 2.7 Suppose ϕ and its time derivative $\dot{\phi}$ are bounded functions. Assume also that u and all its time derivatives up to order n-r are bounded. Then, the time derivative of the estimate of the sliding surface coordinate function, $d\hat{s}/dt$ is also bounded. Hence, $\hat{s}$ is uniformly continuous.

Proof From (2.14), the assumption in the lemma about the boundedness of ϕ, and assumption 2.5, it follows that $\dot{s}$ is bounded. Using now (2.15), and the previous assumption 2.6, it readily follows from the fact that ϕ is assumed to be bounded, that $d\hat{s}/dt$ is also bounded, provided the control input u and its involved time derivatives are bounded. It is well known that a *sufficient condition* for a function to be uniformly continuous in time is that its time derivative be bounded (see [20], pp 125). Hence $\hat{s}$ is uniformly continuous. The lemma is established.

Let K be a known positive definite matrix. Consider the Lyapunov function given by :

$$V(s,\phi) = \frac{1}{2}s^2 + \frac{1}{2}\phi^T K \phi \qquad (2.16)$$

The time derivative of such a Lyapunov function is obtained, after use of (2.14) and (2.15), as:

$$\dot{V}(s,\phi) = s\dot{s} + \phi^T K \dot{\phi} = -\mu\, s \,\mathrm{sgn}\,\hat{s} + \phi^T\left[s\, W(x,u^{[n-r]}) + K\dot{\phi}\right]$$

$$= -\mu|\hat{s}| + \phi^T\left[W(x,u^{[n-r]})\left(\hat{s} + \phi^T W_s(x,u^{[n-r-1]})\right) - \mu W_s(x,u^{[n-r-1]})\,\mathrm{sign}\,\hat{s} + K\dot{\phi}\right]$$

Choosing the variations of the parameter adaptation error according to the law :

$$\dot{\phi} = -\dot{\hat{\Theta}} =$$

$$-K^{-1}\left\{\left[\hat{s}+\phi^T W_s(x,u^{[n-r-1]})\right] W(x,u^{[n-r]}) - \mu W_s(x,u^{[n-r-1]})\,\mathrm{sign}\,\hat{s}\right\} \qquad (2.17)$$

one, hence, obtains:

$$\dot{V}(s,\phi) = -\mu\,\hat{s}\,\mathrm{sgn}\,\hat{s} = -\mu|\hat{s}| \le 0 \qquad (2.18)$$

The Lyapunov function (2.16) decreases along the trajectories of the controlled system and, therefore, both, the sliding surface coordinate s and the estimation error ϕ are bounded. The boundedness of s and ϕ implies, by integration of both sides of (2.18), that the estimated sliding surface coordinate function $\hat{s}$ is absolutely integrable. Notice, moreover, that, from the definition of s, a bounded s implies bounded values for y and for all its time derivatives, up to order n-1. This, together with the minimum phase assumption means, by virtue of the full rank condition in (2.4), that the state vector x is bounded. This in turn implies, by assumptions 2.5 and 2.6, that the regressor vector W is bounded and that the partial derivatives of the switching regressor vector W_s are also bounded. Since $\dot{\phi}$ is bounded, it then follows by virtue of lemma 2.7, in conjunction with the demonstrated boundedness of ϕ, that $d\hat{s}/dt$, the time derivative of $\hat{s}$, is bounded and, hence, that $\hat{s}$ is, indeed, uniformly continuous. Evidently, this result implies that the absoulte value of $\hat{s}$ is also uniformly continuous. The following "Lyapunov like" lemma, based on Barbalat's lemma (see Slotine and Li [20], pp. 125-127) guarantees then the convergence of $|\hat{s}|$ to zero.

Lemma 2.8 [20] If the scalar function $V(s,\phi)$ is lower bounded, and its firt order time derivative $\dot{V}(s,\phi)$ is negative semidefinite and uniformly continuous in time, then $\dot{V}(s,\phi)$ tends to zero as time goes to infinity.

Evidently, the Lyapunov function (2.16) satisfies all the assumptions of Lemma 2.8 and , therefore, $|\hat{s}|$ asymptotically approaches zero as time goes to infinity.

Let $|\hat{s}|$ asymptotically approach zero. Then, the linearized dynamics (2.9) will not be exactly satisfied and, instead, the following output dynamics, obtained from (2.15) and the definition of s, will be valid, when $\hat{s} = 0$:

$$y^{(n-1)} + \alpha_{n-1}y^{(n-2)} + \ldots + \alpha_1 y = \phi^T W_s(x,u^{[n-r-1]}) \qquad (2.19)$$

The choice of the α's in s is such that the system (2.19) is exponentially stable when the right hand side is set to zero. It follows, by a well known bounded input-bounded oputput theorem for linear systems (see Brockett [21], pp. 196), that system (2.19) is then uniformly bounded input bounded output. This means that y, and all its time derivatives, are uniformly bounded, whenever the scalar input $\phi^T W_s(x,u^{[n-r-1]})$ is uniformly bounded. Moreover, y and all its time derivatives approach zero if the bounded scalar input is known to converge to zero, as t approaches infinity.

Remark 2.10 It follows from (2.15) and the previous considerations that, if the parameter estimation error ϕ converges to zero then the actual value of the surface coordinate function s will, indeed, converge to zero. However, convergence of the estimation error ϕ to zero is very much attached to a condition of *persistency of excitation* (see also [7],[8]). This condition may be derived, in this case, as follows: Consider that $\hat{s} = 0$. Then, one may rewrite (2.17) as:

$$\dot{\phi} = -K^{-1}\, W(x,u^{[n-r]})W_s^T(x,u^{[n-r-1]})\,\phi \qquad (2.20)$$

i.e. the parameter update law is represented by a time-varying linear differential equation with solution given by :

$$\phi(t) = \left[\exp\left(-K^{-1}\int_0^t W(x,u^{[n-r]})W_s^T(x,u^{[n-r-1]})\,dt\right)\right]\phi(0) \qquad (2.21)$$

It is well known that if the regressor vectors are *persistently exciting* i.e., if there exist a_1, a_2 and δ, all positive, constant, quantities such that, for all t :

$$a_1 I \ge \int_t^{t+\delta} W(x,u^{[n-r]})W_s^T(x,u^{[n-r-1]})\,dt \ge a_2 I \qquad (2.22)$$

then, both, s and ϕ exponentially converge to zero. Condition (2.22), however cannot be verified *a priori* due to the fact that both regressor vectors W and W_s are functions of the state x of the system, and of the state $u^{[n-r-1]}$ of the dynamical feedback controller. ❑

As it is standard in nonlinear adaptive control theory, it should be stressed that equation (2.17) must be regarded as a set of simultaneous, coupled, time-varying (discontinuous) nonlinear ordinary differential equations in the unknown components of, both, the parameter estimation error vector ϕ, and the parameter estimate vector $\hat{\Theta}$. i.e.,

$$\dot{\phi} = -K^{-1}\left\{\left[\hat{s} + \phi^T W_s(x,u^{[n-r-1]})\right] W(x,u^{[n-r]}) - \mu \dot{W}_s(x,u^{[n-r-1]}) \operatorname{sign} \hat{s}\right\}$$

$$\dot{\hat{\Theta}} = K^{-1}\left[\hat{s} + \phi^T W_s(x,u^{[n-r-1]})\right] W(x,u^{[n-r]}) - \mu W_s(x,u^{[n-r-1]}) \operatorname{sign} \hat{s} \tag{2.23}$$

Initial conditions for the coupled system (2.23) are usually arbitrarily chosen for the unknown components of the composite vector $[\phi^T, \hat{\Theta}^T]^T$. The parameter update equations are, hence, assumed to be solved on line, and their generated solution trajectories immediately delivered to the dynamical adaptive controller (2.12).

3. AN APPLICATION EXAMPLE IN CHEMICAL PROCESS CONTROL

3.1 A Continuously Stirred Tank Reactor Model. (Kravaris and Palanki [22]).

Consider the following simple nonlinear dynamical model of a controlled CSTR in which an isothermal, liquid-phase, multi-component chemical reaction takes place :

$$\begin{aligned}
\dot{x}_1 &= 1 - (1+D_{a1})\,x_1 + D_{a2}x_2^2 \\
\dot{x}_2 &= D_{a1}\,x_1 - x_2 - (D_{a2}+D_{a3})x_2^2 + u \\
\dot{x}_3 &= D_{a3}x_2^2 - x_3 \\
y &= x_3 - Y
\end{aligned} \tag{3.1}$$

Where x_1 represents the normalized (dimensionless) concentration C_A/C_{AF} of a certain species A in the reactor, with C_{AF} being the feed concentration of the species A measured in $mol.m^{-3}$. The state variable x_2 represents the normalized concentration C_B/C_{AF} of the species B. The state variable x3 represents the normalized concentration C_C/C_{AF} of a certain species C in the reactor. The control variable u is defined as the ratio of the per-unit volumetric molar feed rate of species B, denoted by N_{BF}, and the feed concentration C_{AF}. i.e., $u = N_{BF}/(FC_{AF})$ where F is the volumetric feed rate in $m^3\,s^{-1}$. The constants D_{a1}, D_{a2} and D_{a3} are respectively defined as k_1V/F, k_2VC_{AF}/F and k_3VC_{AF}/F with V being the volume of the reactor, in m^3, and k_1, k_2 and k_3 are the first order rate constants. in s^{-1}. Y represents a desired total concentration value.

It is desired to regulate the normalized concentration C_C/C_{AF} to a prescribed set-point vaue specified by the constant Y. It is assumed that the control variable u is naturally bounded in the closed interval $[0,U_{max}]$ reflecting the physical limits of molar feed rate of the species B.

System (3.1) is of the form:

$$\dot{x} = \theta_1 f_1(x) + \theta_2 f_2(x) + \theta_3 f_3(x) + \theta_4 f_4(x) + \theta_5 f_5(x) + \theta_6 g_1(x)u$$
$$y = h(x) \tag{3.2}$$

with:

$$f_1(x) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};\ f_2(x) = \begin{bmatrix} -x_1 \\ -x_2 \\ -x_3 \end{bmatrix};\ f_3(x) = \begin{bmatrix} -x_1 \\ x_1 \\ 0 \end{bmatrix};$$

$$f_4(x) = \begin{bmatrix} x_2^2 \\ -x_2^2 \\ 0 \end{bmatrix};\ f_5(x) = \begin{bmatrix} 0 \\ -x_2^2 \\ x_2^2 \end{bmatrix};\ g_1(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$h(x) = x_3 - Y$$

and :

$$\theta_1 = 1\,;\ \theta_2 = 1\,;\ \theta_3 = D_{a1}\,;\ \theta_4 = D_{a2}\,;\ \theta_5 = D_{a3}\,;\ \theta_6 = 1$$

It is easy to verify that for the given system (3.1), the rank of the following 3 by 3 matrix :

$$\frac{\partial(y, y^{(1)}, y^{(2)})}{\partial x} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2D_{a3}x_2 & -1 \\ 2D_{a1}D_{a3}x_2 & \begin{matrix} 2D_{a1}D_{a3}x_1 - 6D_{a3}x_2 \\ -6D_{a3}(D_{a2}+D_{a3})x_2^2 + 2D_{a3}u \end{matrix} & 1 \end{bmatrix} \tag{3.3}$$

is everywhere equal to 3, except on the line $x_2 = 0$. Natural physical considerations lead us to restricting x_2 to values grater than zero. Negative values of x_2 have no physical significance.

A stable constant equilibrium point for this system is given by :

$$u = U\,;$$

$$x_1(U) = \frac{1 + D_{a2}[x_2(U)]^2}{(1+\theta_3)}\,;$$

$$x_2(U) = (1+\theta_3)\frac{\left[-1 + \sqrt{1 + 4\left(U + \frac{\theta_3}{1+\theta_3}\right)\left(\frac{\theta_4 + \theta_5 + \theta_3\theta_5}{1+\theta_3}\right)}\right]}{2(\theta_4+\theta_5+\theta_3\theta_5)}\,;$$

$$x_3(U) = \theta_5[x_2(U)]^2$$

The zero dynamics associated to system (3.1) is obtained from (3.5) by letting $y = y^{(1)} = y^{(2)} = 0$, in the input output representation which is ommitted here in the interest of simplicity . Such a zero dynamics is:

$$\begin{aligned}
&\left\{\left[-2\theta_3\theta_5(1+\theta_3) - 6\theta_3\theta_5 - 8\theta_3\theta_5(\theta_4+\theta_5)\sqrt{\frac{Y}{\theta_5}}\right]\sqrt{\frac{Y}{\theta_5}}\right. \\
&\left. + 4\theta_3\theta_5 u + \theta_3\left[2Y\sqrt{\frac{\theta_5}{Y}} + 2(\theta_4+\theta_5)\,Y + 2\theta_5\,u\right]\right\} \\
&\left[\frac{2\,Y + 2(\theta_4+\theta_5)\,Y + 2\theta_5\,u}{2\theta_3\theta_5}\right] \\
&+ \left[2\theta_3\theta_4 + 10(\theta_4+\theta_5) + 6(\theta_4+\theta_5)^2\sqrt{\frac{Y}{\theta_5}}\right] Y\sqrt{\frac{Y}{\theta_5}} \\
&+ \left[2\theta_3\theta_5 - 6\,\theta_5 u + 2\theta_5\dot{u} - 8\theta_5(\theta_4+\theta_5)\sqrt{\frac{Y}{\theta_5}}\,u\right]\sqrt{\frac{Y}{\theta_5}} \\
&+ 2\theta_5 u^2 = 0
\end{aligned} \tag{3.6}$$

It can be verified after tedious but straightforward manipulations that the system is minimum phase around the physically meaningful equilibrium point of (3.6), given by the largest solution, $u = U > Y$, of the resulting quadratic equilibrium equation. This solution coincides with the one obtained from (3.4) under the equilibrium condition : $Y = X_1(U)+X_2(U)$.

3.2 Non Adaptive Linearizing Sliding Mode Controller for Continuously Stirred Tank Reactor Model.

Imposing on the output y of (3.1) the following linear asymptotically stable dynamics:

$$y^{(2)} + \alpha_2\, y^{(1)} + \alpha_1 y = 0 \quad ; \quad \alpha_1\,,\ \alpha_2 > 0 \tag{3.7}$$

one readily obtains, using the result of proposition 2.2 above, the following stabilizing discontinuous dynamical feedback controller:

$$\dot{u} = \frac{1}{2\theta_5 x_2} \Bigg[\theta_5\big(8x_2u - (7+\alpha_1)x_2^2 - 2u^2 - 4\alpha_2 x_2 u + 6\alpha_2 x_2^2 \big) + \theta_3\theta_5\big((10-4\alpha_2)x_1x_2 - 2x_2 - 4x_1u\big) + \theta_3^2\theta_5\big(2x_1x_2 - 2x_1^2\big) + \theta_3\theta_4\theta_5\big(8x_1x_2^2 - 2x_2^3\big) + \theta_3\theta_5^2\big(8x_1x_2^2\big) + \theta_4\theta_5\big(8x_2^2u - 12x_2^2 + 4\alpha_2 x_2^3\big) + \theta_5^2\big(8x_2^2u - 12x_2^2 + 4\alpha_2 x_2^3\big) - 6\theta_4^2\theta_5 x_2^4 - 12\theta_4\theta_5^2 x_2^4 - 6\theta_5^3 x_2^4 + (1-\alpha_2+\alpha_1)x_3 - \mu\,\mathrm{sign}(s) \Bigg] \tag{3.8}$$

The ideal sliding dynamics (3.7) takes place on the input-dependent sliding surface:

$$s(x,u,\theta) = -[(3-\alpha_2)x_2^2 - 2x_2u]\theta_5 + 2\theta_3\theta_5 x_1 x_2 - 2\theta_4\theta_5 x_2^3 - 2\theta_5^2 x_2^3 + (1-\alpha_2)x_3 + \alpha_1(x_3 - Y) = 0 \tag{3.9}$$

The performance of controller (3.8) is depicted in Figure 1, where the computer generated state variable trajectories $x_1(t)$, $x_2(t)$ and $x_3(t)$ are shown, along with the non-chattering control input trajectory u(t). The evolution of the sliding surface coordinate s is shown in figure 2. The variable structure controller parameters, used in the computer simulation, were : $\mu = 5$, $\alpha_2 = 5.4$, $\alpha_1 = 9$, $\theta_3 = D_{a1} = 3$, $\theta_4 = D_{a2} = 0.5$, $\theta_5 = D_{a3} = 1$ and $Y = 0.7737$. The state trajectories are seen to converge to their equilibrium values given, according to (3.4), by $x_1 = 0.3467$, $x_2 = 0.8796$ and $x_3 = 0.7737$. U_{max} was taken as 3.

3.2 Adaptive Sliding Mode Dynamical Linearizing Control for Continuously Stirred Tank Reactor Model

Due to lack of parameter knowledge, instead of the exactly linearizing controller (3.8), one uses a dynamical variable structure controller, based on estimates of the overparametrization vector components, given by:

$$\dot{u} = \frac{1}{2\hat{\Theta}_5 x_2} \Bigg[\hat{\Theta}_5\big(8x_2u - (7+\alpha_1)x_2^2 - 2u^2 - 4\alpha_2 x_2 u + 6\alpha_2 x_2^2 \big) + \hat{\Theta}_6\big((10-4\alpha_2)x_1x_2 - 2x_2 - 4x_1u\big) + \hat{\Theta}_7\big(2x_1x_2 - 2x_1^2\big) + \hat{\Theta}_8\big(8x_1x_2^2 - 2x_2^3\big) + \hat{\Theta}_9\big(8x_1x_2^2\big) + \hat{\Theta}_{10}\big(8x_2^2u - 12x_2^2 + 4\alpha_2 x_2^3\big) + \hat{\Theta}_{11}\big(8x_2^2u - 12x_2^2 + 4\alpha_2 x_2^3\big) - 6\hat{\Theta}_{12}x_2^4 - 12\hat{\Theta}_{13}x_2^4 - 6\hat{\Theta}_{14}x_2^4 + (1-\alpha_2+\alpha_1)x_3 - \mu\,\mathrm{sign}(\hat{s}) \Bigg] \tag{3.10}$$

where $\hat{\Theta}_5, \hat{\Theta}_6 \cdots \hat{\Theta}_{14}$ are, respectively, the estimates of θ_5, $\theta_3\theta_5$ ...... θ_5^3.

The sliding mode approach would then be based on an estimate of the switching surface coordinate function, given by :

$$\hat{s}(x,u,\hat{\Theta}) = -[(3-\alpha_2)x_2^2 - 2x_2u]\hat{\Theta}_5 + 2\hat{\Theta}_6 x_1 x_2 - 2\hat{\Theta}_{10}x_2^3 - 2\hat{\Theta}_{11}x_2^3 + (1-\alpha_2)x_3 + \alpha_1(x_3 - Y) = 0 \tag{3.11}$$

The results of the previous section were used and an update law of the form (2.17) was obtained, The expressions are quite complex and are ommitted.

Simulations were run to assess the performance of the adaptive dynamical sliding mode controller (3.10),(3.11),(3.14). The state variable trajectories $x_1(t)$, $x_2(t)$ and $x_3(t)$ are depicted in Figure 3, together with the non-chattering control input trajectory u(t). The state trajectories are seen to converge to their ideal equilibrium values given by $x_1 = 0.3467$, $x_2 = 0.8796$ and $x_3 = 0.7737$. The time evolutions of the sliding surface coordinate function s, and of its estimate $\hat{s}$, are shown in figure 4. Besides the small discrepancy between the two surface coordinates, it is clearly seen that $\hat{s}$ converges to zero reasonably fast while the actual sliding surface converges to zero in a much slower fashion. In figure 5, the estimated parameters are also shown to converge to constant values not coinciding with their "true" values. The variable structure controller parameters and the constants for the adaptation laws were set as : $\mu = 5$, $\alpha_1 = 9$, $\alpha_2 = 5.4$, $K_{55} = 0.5$, $K_{66} = 1.0$, $K_{77} = 20.0$, $K_{88} = 20.0$, $K_{99} = 40.0$, $K_{10\,10} = 1.0$, $K_{11\,11} = 1.0$, $K_{12\,12} = 20.0$, $K_{13\,13} = 20.0$, $K_{14\,14} = 20.0$.

4. CONCLUSIONS

In this paper, adaptive dynamical discontinuous feedback compensators were examined for a class of parametric uncertain systems linearizable by dynamical sliding mode based strategies. The results show that whenever the input-dependent sliding surface exhibits an explicit dependance on the uncertain parameters of the system, an estimate of the switching surface, which is known to be in error with respect to the exactly linearizing manifold, must be used for the generation of the controlled switchings. A Lyapunov approach shows that the estimated trajectory of the sliding surface coordinate function is asymptotically driven to zero by means of the dynamical variable structure control strategy. It should be remarked, however, that such asymptotic behavior is not, generally speaking, achieved by means of sliding motions taken place on the zero level set of the estimated value of the sliding surface. The parameter estimation error adaptation law is of the discontinuous type, with discontinuities taking place precisely on the estimated values of the sliding surface coordinate function. As it is quite standard, parameter convergence is achieved to the actual, or nominal, sliding surface if a modified version of the well known condition of persistency of excitation is verified.

The proposed adaptive dynamical sliding mode control approach to stabilization tasks benefits from the fact that the generated input trajectories, and the associated state and output responses, are non-chattering. This is due to the smoothing of the discontinuities accomplished by the integration features imbedded in the dynamical discontinuous feedbakc controller.

An illustrative chemical process control example was presented along with highly satisfactory simulations results.

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FIGURES

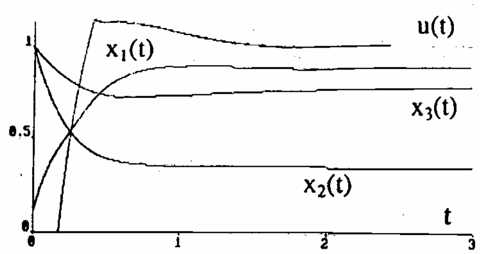

Figure 1. Time response of states and input variables for non adaptive dynamical sliding mode controlled Continuous Stirred Tank Reactor Example.

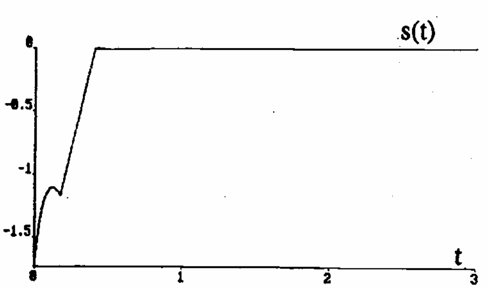

Figure 2. Sliding surface coordinate function evolution for non adaptive dynamical sliding mode controlled CSTR.

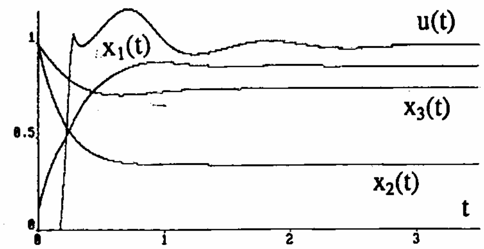

Figure 3. Time response of states and input variables for adaptive dynamical sliding mode controlled Continuous Stirred Tank Reactor Example.

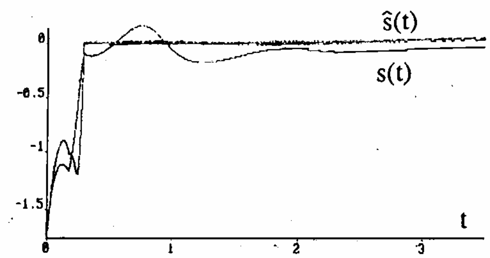

Figure 4. Estimated sliding surface and (actual) sliding surface coordinates functions evolution for adaptive dynamical sliding mode controller case.

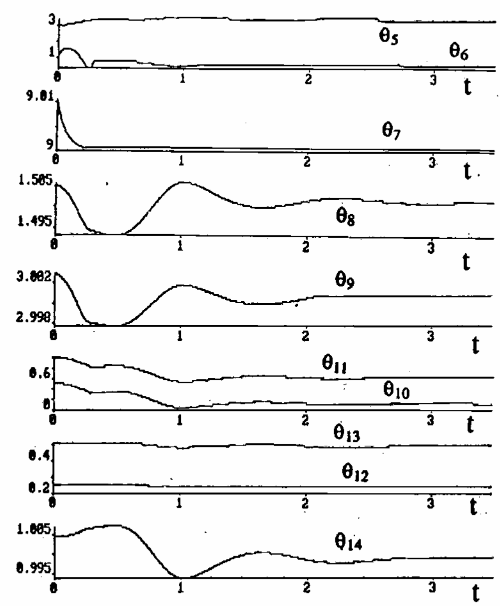

Figure 5. Evolution of estimated parameters θ_5 θ_6 θ_7 ... θ_{14} for adaptive dynamical sliding mode controlled Continuous Stirred Tank Reactor Example.