A UNIFIED APPROACH TO DYNAMICAL DISCONTINUOUS FEEDBACK CONTROL OF A DOUBLE EFFECT EVAPORATOR

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Abstract. In this article a unified approach is proposed for the design of dynamical discontinuous feedback controllers leading to a continuous (i.e. chattering-free) stabilization of a double effect evaporator. The proposed controllers correspond to Pulse Frequency Modulation, Pulse Width Modulation and Sampled Sliding Modes strategies. Simulations are performed which validate the proposed approach.

Key Words. Discontinuous Feedback Control, Pulse Frequency Modulation, Pulse Width Modulation, Sliding Modes

1. INTRODUCTION

In this article Fiess's Generalized Observability Canonical Form (GOCF) (Fliess 1990) is shown to naturally allow for a unified approach in dynamical disconitnuous feedback compensator design for nonlinear systems. The proposed chattering-free controllers are based on Pulse Frequency Modulation (PFM) strategies, Pulse Width Modulation (PWM) policies and Sampled Sliding Modes (SSM). Natural integration of the system nonlinearities in the GOCF result in sufficiently smooth controller outputs in spite of the underlying discontinuous feedback control policies. The obtained control inputs are substantially smoothed with respect to what can be achieved by means of high-gain substitution techniques (Slotine and Li 1991). The design method does not resort to traditional aproximation schemes, based on average models, nor it requires exact or approximate discretization procedures. Section 2 contains the derivation of the GOCF of a double effect evaporator and it also proposes, in a unified manner, the discontinuous feedback controllers of the PWM, PFM and SSM types. Section 3 presents some computer simulations depicting the performance of the proposed controllers. Finally, section 4 contains the conclusions and some suggestions for further research.

2. GOCF OF A DOUBLE EFFECT EVAPORATOR

The following double effect evaporator model is taken from Montano and Silva (Montano and Silva 1991)

$$\begin{aligned} \dot{x}_1 &= \delta_1 F_0 (c_0 - x_1) + \delta_2 x_1 u \\ \dot{x}_2 &= \delta_3 F_0 (x_1 - x_2) \\ &\quad + (\delta_4 x_1 + \delta_5 x_2) u \\ y &= x_2 - x_{2d} \end{aligned} \tag{1}$$

where x_1 represents the product concentration in the first stage of the evaporator, x_2 is the output concentration of the product at the second stage and u is the positive control input representing the steam fow provided by the boiler. The outupt y of the system is the concentration error $x_2 - x_{2d}$, with x_{2d} representing the desired output concentration. The parameters in (2) are assumed to be known and constant.

It may be shown that an input-dependent state coordinate transformation takes the system into the following GOCF

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= (\delta_5 u - \delta_1 F_0)\ z_2 \\ &+ \dot{u} \left[\frac{\delta_3 F_0 (\delta_4 + \delta_5)(z_1 + x_{2d}) + \delta_4 z_2}{\delta_3 F_0 + \delta_4 u} \right] \\ &+ c_0 \delta_1 F_0 (\delta_3 F_0 + \delta_4 u) \\ &+ (\delta_2 u - \delta_1 F_0) \\ &\quad [\ (\delta_3 F_0 - \delta_5 u)\ (z_1 + x_{2d}) + z_2] \\ y &= z_1 \end{aligned} \tag{2}$$

The *zero dynamics* assocated to (2) is simply obtained as

$$\dot{u}\left[\frac{\delta_3 F_0(\delta_4+\delta_5)(z_1+x_{2d})+\delta_4 z_2}{\delta_3 F_0+\delta_4 u}\right]$$
$$+\quad c_0\delta_1 F_0(\delta_3 F_0+\delta_4 u)$$
$$+\quad (\delta_2 u-\delta_1 F_0)$$
$$[(\delta_3 F_0-\delta_5 u)\ x_{2d}]=0 \tag{3}$$

The equilibrium points of the zero dynamics (4) are given by the real solutions of the resulting quadratic algebraic equation. It is relatively easy to see, although quite tedious, that the physically interesting solution corresponds to a *minimum phase* equilibrium point of the zero dynamics. The results are therefore valid for trajectories that stay away from the second obtained equilibrium solution which is, indeed, unstable and, hence, *non-minimum phase*. The design of a dynamical discontinuous feedback control policy requires the definition of an *auxiliary output* function s (Sira–Ramírez 1993). This auxiliary output funciton exhibits the property that when it is forced to zero (by means of discontinuous feedback control actions) the remaining dynamics of the system asymptotically and autonomously stabilize also to its (stable) equilibrium point. For the case of the transformed system 2 such a function is given by

$$s(z)=z_2+\gamma_1 z_1 \tag{4}$$

where γ_1 is a positive scalar representing a desirable rate of exponential decay. Evidently, when $s(z)$ is forced to be zero, the resulting transformed dynamics is asymptotically stable to zero and hence y approaces its desired equilibrium value.

Several discontinuous feedback control designs are achieved by imposing on the dynamics of the regulated auxiliary output function the following discontinuous scalar dynamics:

$$\dot{s}=-W\ \mathrm{DDC}\ (s) \tag{5}$$

where DDC stands for a PWM, PFM or SSM control policy (see Sira–Ramírez 1993). and W is any positive constant parameter. For instance, a SSM control policy would be specified, during any inter–sampling interval $t_k < t \leq t_k+T$ by:

$$\mathrm{DDC}\,[s(t)]=\mathrm{sign}\,[s(t_k)] \tag{6}$$

A PWM control policy, on the other hand, would cause DDC to be set to

$$\mathrm{DDC}(s)=PWM_{\tau,\,T}\,[s(t)]= \tag{7}$$

$$\begin{cases}\mathrm{sign}\,[s(t_k)] & \text{for } t_k\leq t<t_k+\tau\,[s(t_k)]\,T\\ 0 & \text{for } t_k+\tau\,[s(t_k)]\,T<t\leq t_k+T\end{cases}$$

where $\tau\,[s(t_k)]$ is the *duty ratio* function, which specifies the pulse width at each sampling instant according to the value of the auxiliary output function $s(t_k)$.

Finally, in the PFM case the function DDC is defined as

$$\mathrm{DDC}(s)=PFM_{\tau,\,T}\,[s(t)]= \tag{8}$$

$$\begin{cases}\mathrm{sign}\,[s(t_k)] & \text{for } t_k\leq t<t_k+\tau\,[s(t_k)]\,T\,[s(t_k)]\\ 0 & \text{for } t_k+\tau\,[s(t_k)]\,T\,[s(t_k)]\\ & \quad <t\leq t_k+T\,[s(t_k)]\end{cases}$$

where now the width of the sampling period T , known as the *duty cycle* function, is also dependent upon the sampled values of the auxiliary output function $s(t)$.

The asymptotic stability of the discontinuous differential equation (5), with the function DDC(s) given by any of the cases depicted in the equations (6), (7) or (8), is fully presented, for typical prescriptions of the duty ratio and the duty cycle functions, in Sira–Ramírez and Llanes–Santiago (1992), where the reader is referred for further details.

Imposing the discontinuous dynamics given by (5) on the auxiliary output function, described in transformed coordinates by (4), and using the GOCF derived in equation (2) one obtains a dyanamical feedback controller in terms of a differential equation, for the control input u, which exhibits a discontinuous right hand side

$$\dot{u}\quad\left[\frac{\delta_3 F_0(\delta_4+\delta_5)(z_1+x_{2d})+\delta_4 z_2}{\delta_3 F_0+\delta_4 u}\right]$$
$$+(\delta_5 u-\delta_1 F_0)\ z_2$$
$$+c_0\delta_1 F_0(\delta_3 F_0+\delta_4 u)+(\delta_2 u-\delta_1 F_0)$$
$$[(\delta_3 F_0-\delta_5 u)(z_1+x_{2d})+z_2]+\gamma_1 z_2$$
$$=\quad -W\ \mathrm{DDC}\,(z_2+\gamma_1 z_1) \tag{9}$$

3. SIMULATION RESULTS

Simulations were carried out to assess the performance of all three dynamical discontinuous feedback controllers. The parameter values of the double effect evaporator plant were taken from Montano and Silva (1991).

$$F_0=2.525\ [\mathrm{Kg/min}]\ ,\ c_0=0.04\ ,\ \delta_1=0.0105$$

$$\delta_3=9.52310^{-3}\ ,\ \delta_4=-7.69910^{-3}\ ,$$
$$\delta_5=10.30510^{-3}$$

with these parameter values, the physically meaningful equilibrium point is computed to be

$$x_1=0.7\quad ;\quad x_2=x_{2d}=0.0939$$

The state responses obtained from the dynamical PFM control policy are shown in Figure 1. The plant state trajectories are seen to converge towards the desired equilibrium point. The control input generated by the dynamical PFM controller is clearly seen to be sufficiently continuous, as shown in Figure 2. Figures 3 and 4 depict the same state and input variables trajectories which result from the use of the dynamical PWM control policy. The qualitative features of the controlled behaviour is practically identical with that obtained from the dynamical PFM controller. Finally, the controlled state responses and generated input signal trajectory, obtained from the SSM policy, is depicted in Figures 5 and 6. The first order discontinuous input obtained in this case can only be decreased in amplitude through faster sampling. However, such a chattering input represents a considerable improvement over "bang-bang" input signals obtained otherwise by means of classical sliding mode control policies.

4. CONCLUSIONS

In this article a unified approach has been presented for the design of dynamical discontinuous feedback controllers in double effect evaportator systems. The approach has heavily relied on *Fliess's Generalized Observability Canonical Form* which, for systems with relative degrees higher than one, naturally exhibits input signal derivatives in its structure. This feature implicitly entitles integration of the discontinuous feedback inputs prescribed to the dynamical controller and, hence, smoothed signals are obtained from the controller's output. In this article such discontinuous inputs were chosen as PWM, PFM or SSM alternatives but other policies are possible. The approach generally results in sufficiently smoothed compensator outputs while still retaining some of the insensitivity features of discontinuous control policies to small parameter variations and external perturbation inputs. The performance of the proposed controllers was illustrated by means of digital computer simulations. Further research is needed into the possibilities of building discontinuous observers for nonlinear uncertain systems.

5. REFERENCES

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FIGURES

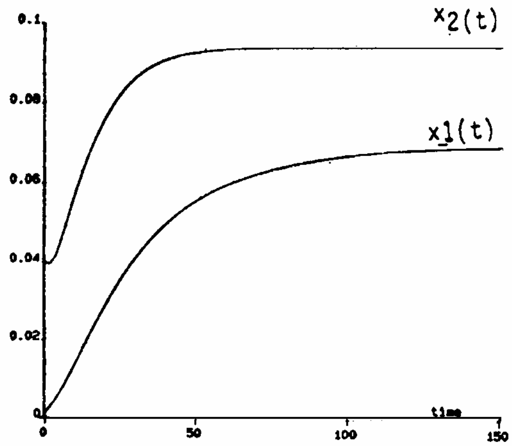

Fig. 1. State response of the double effect evaporator to dynamical PFM controller action.

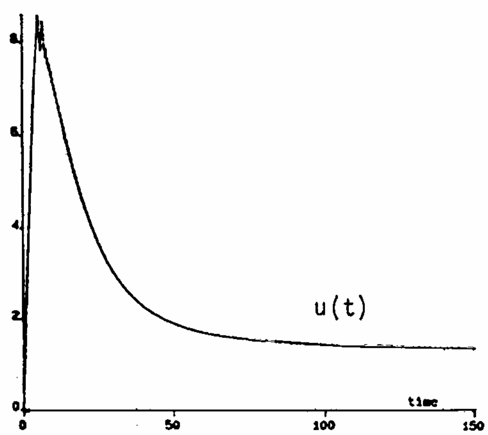

Fig. 2. Control input trajectory generated by means of dynamical PFM controller

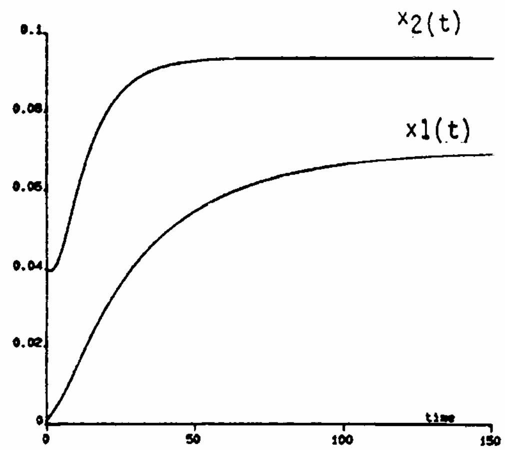

Fig. 3. State response of the double effect evaporator to dynamical PWM controller action.

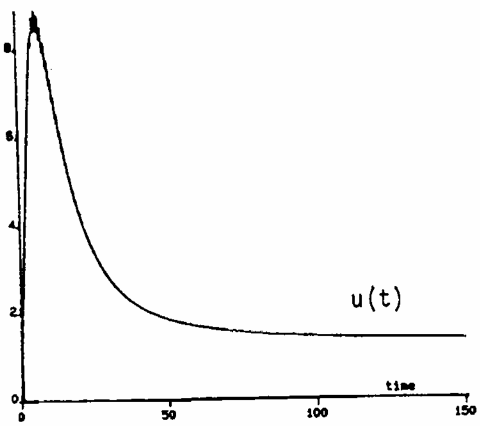

Fig. 4. Control input trajectory generated by means of dynamical PWM controller

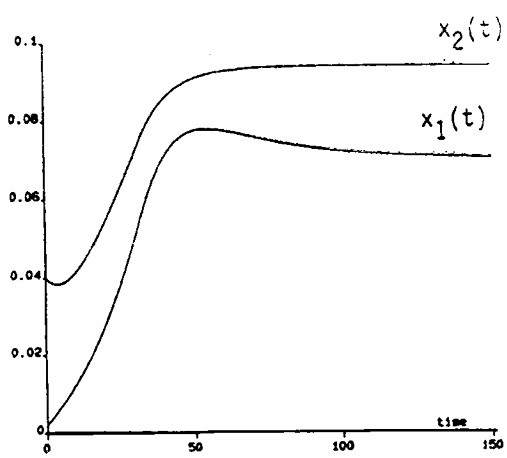

Fig. 5. State response of the double effect evaporator to dynamical SSM controller action

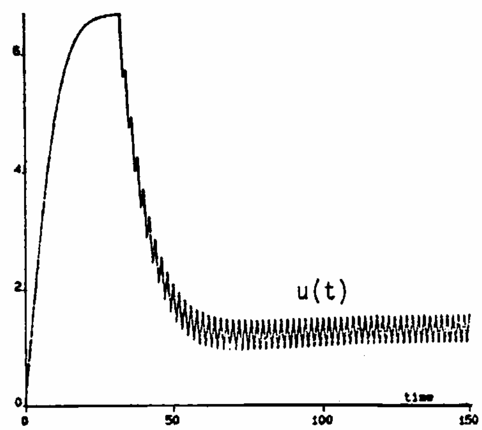

Fig. 6. Control input trajectory generated by means of dynamical SSM controller