ADAPTIVE PWM REGULATION SCHEMES IN SWITCHED CONTROLLED SYSTEMS[1]

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Abstract Adaptive discontinuous feedback regulators, of static nature, are proposed for the stabilization of switch-mode controlled systems. An average pulse-width-modulation (PWM) model controlled by a piecewise smooth, and limited, duty ratio function is used as the basis for a static adaptive controller design which locally stabilizes the system motions, through imposed linearization, towards a constant equilibrium point.

Keywords Discontinuous feedback control, Adaptive feedback regulation, Pulse-width-modulation.

1. INTRODUCTION

Asymptotic output stabilization for uncertain nonlinear systems constitutes a most important problem in control theory. Contributions, were given by Isidori and Sastry (1989), Campion and Bastin (1990), and many others. Research trends in the area are contained in Kokotovic (1991)

Pulse-Width-Modulation (PWM) feedback control has been extensively treated, specially in work related to power electronics applications (see Sira-Ramírez et al, 1993a). Most of the available regulation results deal with the assumption of known parameter values for the given nonlinear plant. However, this fundamental assumption is sometimes invalid due to imprecise knowledge of such parameters. Non-adaptive feedback control schemes for switch controlled systems, relying on nominal or manufaturer-povided parameter values, may lead to performance degradation, when applied on an actual system. In general terms,such parametric variations crucially influence those nonlinear controller design schemes, performed on the basis of exact linearization, attainable via static or dynamical controllers specification.

The results in Sira-Ramirez *et al* (1993b) show that an adaptive feedback strategy leads to a definite performance improvement in the steady state regulation characteristics of PWM feedback controllers developed on the basis of average PWM models. This fact constitutes a second prevailing reason for resorting to adaptation on the nominally designed controller for switched regulated plants.

Section 2 of this article deals with some generalities regarding an adaptive PWM feedback control scheme for standard (infinite frequency) average models of PWM controlled systems. The adaptive controller is based on a slight extension of well established results developed for the adaptive control of linearizable systems (as in Sastry and Isidori, 1989). Section 3 presents the conclusions and some suggestions for further research in this area.

2. AN ADAPTIVE PWM FEEDBACK CONTROLLER SCHEME FOR SWITCH-REGULATED LINEARIZABLE SYSTEMS

2.1 An Average Model for PWM Switch-Controlled Nonlinear Systems

Consider the n dimensional state representation of a single-input single-output PWM controlled nonlinear system containing a parameter vector θ, with constant but unknown components :

$$\dot{x} = f(x,\theta) + ug(x,\theta)$$
$$y = h(x,\theta) \tag{2.1}$$

[1] This work was supported by the Consejo de Desarrollo Científico, Humanístico and Tecnológico of the Universidad de Los Andes under Research Grants I-358-91 and I-362-91.

where the control input variable u takes values in the discrete set $\{0,1\}$ representing the values of a switch position function, determining one particular feedback structure among two possible (nonlinear) feedback paths. The values of u are specified according to the PWM controlled law :

$$u = \begin{cases} 1 & \text{for } t_k \le t < t_k + \mu[x(t_k)]T \\ 0 & \text{for } t_k + \mu[x(t_k)]T < t \le t_k + T \end{cases} \quad t_k + T = t_{k+1} \; ; \; k = 0,1,2,\dots \qquad (2.2)$$

where the mapping $\mu : R^n \to R$, represents the *duty ratio* function, which is usually regarded as a state dependent quantity (i.e., as a truly feedback law) that specifies, at each sampling instant t_k, the state dependent width $\mu[x(t_k)]T$ of the control input pulse during the upcoming inter-sampling interval of fixed duration T (known also as the *duty cycle*, or, simply, the *sampling period*). It is easy to see, from (2.2), that the duty ratio function μ is evidently limited to non-negative values, which do not exceed the upper bound of 1, i.e, $\mu \in [0, 1]$. The scalar function $y = h(x,\theta)$ represents an output error which is to be stabilized to zero and $L_g h \neq 0$, locally.

It has been shown in Sira-Ramirez (1989) that a piece-wise smooth *average* model of the PWM controlled system (2.1) can be obtained by assuming an infinitely large sampling frequency 1/T. This assumption results in an model of (2.1) in which the discrete-valued control input function u is replaced by the limited piecewise smooth (feedback) duty ratio function μ. i.e., :

$$\dot{z} = f(z,\theta) + \mu g(z,\theta) \qquad y = h(z,\theta) \qquad (2.3)$$

where z denotes the averaged state vector. Such an average model represents a crude smooth approximation to the actual PWM controlled system behavior, yet, it has proven to be quite useful in PWM controller design strategies for nonlinear systems in which finite, but relatively large, sampling frequencies are used.

2.2 An Adaptive PWM Feedback Control Scheme for Nonlinear Systems.

We assume, in the spirit of Sastry and Isidori (1989), that the parameter-dependent average vector fields $f(z,\theta)$ $g(z,\theta)$ and the nonlinear output function $h(x,\theta)$ exhibit the following linear dependence on the components of the parameter vector θ :

$$f(z,\theta) = \sum_{i=1}^{n_1} \theta_i^1 f_i(z) \; ; \; g(z,\theta) = \sum_{j=1}^{n_2} \theta_j^2 g_j(z)$$

$$h(z,\theta) = h_0(z) + \sum_{k=1}^{n_3} \theta_k^3 h_k(Z) \qquad (2.4)$$

where $f_i(z)$, $g_j(z)$, $h_0(z)$ and $h_k(Z)$ are smooth functions of their arguments. Z is an n-dimensional constant vector, possibly representing the steady state value of the average state vector z.

The following result represents a slight extension of that found in Sastry and Isidori (1989) :

Theorem 2.1 Let $\theta_1 = (\theta_1^1, \dots, \theta_{n_1}^1)$, $\theta_2 = (\theta_1^2, \dots, \theta_{n_2}^2)$, $\theta_3 = (\theta_1^3, \dots, \theta_{n_3}^3)$ and $\theta = (\theta_1, \theta_2, \theta_3)$ denote, respectively, the parameter vectors associated with the vector fields f, g, the nonlinear output function h, and the composition of the three parameter vectors. Let ϕ be the parameter estimation error defined as $\phi = \theta - \hat{\theta}$, with $\hat{\theta}$ being an estimate of the actual (unknown) value of the composite vector θ. Given a relative degree one nonlinear system of the form (2.3), with the vector fields $f(z,\theta)$ and $g(z,\theta)$, defined as in (2.4), then the feedback control law :

$$\hat{\mu}(z) = -\frac{\left[\sum_{i=1}^{n_1} \hat{\theta}_i^1 L_{f_i} h_0(z) + \alpha\left(h_0(z) + \sum_{k=1}^{n_3} \hat{\theta}_k^3 h_k(Z)\right)\right]}{\sum_{j=1}^{n_2} \hat{\theta}_j^2 L_{g_j} h_0(z)} \qquad (2.5)$$

where $\hat{\theta}$ evolves according to the following *parameter update law* :

$$\dot{\phi} = -\dot{\hat{\theta}} = -y W = -y \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = -y \begin{bmatrix} L_{f_1} h_0(z) \\ \dots \\ L_{f_{n_1}} h_0(z) \\ [L_{g_1} h_0(z)]\, \hat{\mu}(z) \\ \dots \\ [L_{g_{n_2}} h_0(z)]\, \hat{\mu}(z) \\ \alpha h_1(Z) \\ \dots \\ \alpha h_{n_3}(Z) \end{bmatrix} \qquad (2.6)$$

locally yields a bounded asymptotic convergence of the output y to zero, with bounded state variables responses, provided the quantity $L_g h = L_g h_0$ is bounded away from zero.

Proof: Suppose system (2.3) is relative degree one. It is desired to impose, in order to obtain an asymptotic approach of the scalar output error to zero, the following asymptotically stable dynamics on the controlled output error : $\dot{y} = -\alpha y$, with $\alpha > 0$. Rewriting this expression as: $L_f h(x,\theta) + u L_g h(x,\theta) = -\alpha h(x,\theta)$ one obtains the following non adaptive nonlinear controller :

$$u = -\frac{\alpha h(x,\theta) + L_f h(x,\theta)}{L_g h(x,\theta)}$$

Using, on the above controller expression, the definitions provided in (2.4) and by virtue of the fact that the parameter vector θ is unknonw, the above control law is modified, using the "certainty equivalence" principle, to its adaptive version given in (2.5). Substituting now (2.5) in (2.3) and after some straightforward manipulations, one obtains:

$$\dot{y} = -\alpha y + \left(\theta - \hat{\theta}\right)^T W = -\alpha y + \phi^T W$$

with W being the "regressor vector" implicitly defined in (2.6).

Consideration of the following positive definite Lyapunov function candidate: $v(y,\phi) = \frac{1}{2}\left(y^2 + \phi^2 \right)$, yields, after use of the parameter update law (2.6): $\dot{v}(y,\phi) = -\alpha y^2 \le 0$. The results follow immediately from the same considerations found in Theorem 3.1 of the work of Sastry and Isidori (1989). ◻

Given system (2.1) as:

$$\dot{x} = f(x,\theta) + u g(x,\theta) \qquad y = h(x,\theta) \tag{2.7}$$

we propose the following *adaptive PWM feedback controller* :

$$u = \begin{cases} 1 & \text{for } t_k \le t < t_k + \hat{\mu}[x(t_k)]T \\ 0 & \text{for } t_k + \hat{\mu}[x(t_k)]T < t \le t_k + T \end{cases} \qquad t_k + T = t_{k+1} \ ; \ k = 0,1,2,\ldots \tag{2.8}$$

with the *estimated duty ratio function* μ given by :

$$\hat{\mu}(x) = \mathrm{sat}_{[0,1]}\ \hat{\mu}_c(x) := \begin{cases} 1 & \text{if } \hat{\mu}_c(x) \ge 1 \\ \hat{\mu}_c & \text{if } 0 < \hat{\mu}_c(x) < 1 \\ 0 & \text{if } \hat{\mu}_c(x) \le 0 \end{cases} \tag{2.9}$$

where $\hat{\mu}_c(X)$ is the *adaptive computed duty ratio function*, directly obtained from the average designed duty ratio synthesizer given in (2.5) as:

$$\hat{\mu}_c(x) = -\frac{\left[\sum_{i=1}^{n_1} \hat{\theta}_i^1 L_{f_i} h_0(x) + \alpha\left(h_0(x) + \sum_{k=1}^{n_3} \hat{\theta}_k^3 h_k(X)\right)\right]}{\sum_{j=1}^{n_2} \hat{\theta}_j^2 L_{g_j} h_0(x)} \tag{2.10}$$

where the vector X coincides with the steady state value Z, of the average state vector z.

The parameter update law is given by :

$$\dot{\phi} = -\dot{\hat{\theta}} = -y W = -y \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = -y \begin{bmatrix} L_{f_1} h_0(x) \\ \cdots \\ L_{f_{n_1}} h_0(x) \\ \left[L_{g_1} h_0(x)\right] \hat{\mu}(x) \\ \cdots \\ \left[L_{g_{n_2}} h_0(x)\right] \hat{\mu}(x) \\ \alpha h_1(X) \\ \cdots \\ \alpha h_{n_3}(X) \end{bmatrix} \tag{2.11}$$

Due to the saturation imposed on the computed adaptive duty ratio function, the asymptotic convergence of the average output variable y to zero can only be locally guaranteed in that region of the state space where the computed duty ratio function does not exhibit such a saturation. Due to the explicit state dependence exhibited by the computed duty ratio function μ_c on the state x of the system, the region of nonsaturation may be explicitly estimated by means of its boundaries defined as : $\mu_c(x) = 0$ and $\mu_c(x) = 1$.

3. CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

In this article, a general adaptive feedback regulation scheme, of static nature, has been proposed for PWM controlled systems. Adaptive dynamical feedback regulation, based on dynamical duty ratio synthesizers, can be developed for pwm regulated systems along similar, but certainly more involved, lines (see Sira-Ramirez and Lischinsky-Arenas, 1991).

4. REFERENCES

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FIGURES

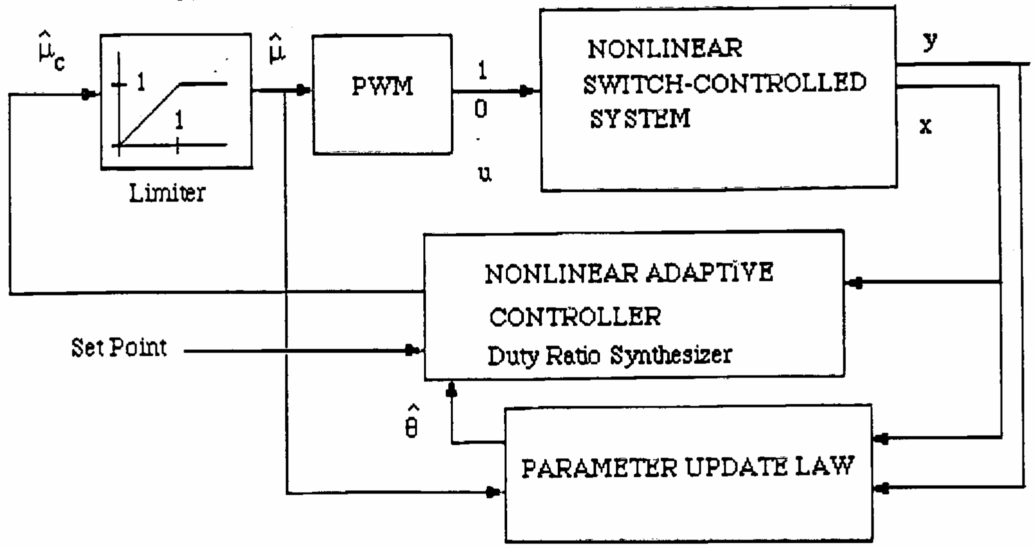

Figure 1. Adaptive feedback regulation scheme for nonlinear PWM switched-controlled systems.