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A Controller Resetting Strategy for the Stabilization of DC–to–DC Power Converters towards Non–minimum Phase Equilibria *

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Abstract

A feedback resetting strategy, excercised on the output of a purposely designed unstable dynamical feedback controller, is shown to yield a robust, and efficient, output voltage stabilization of nonlinear average models of DC–to–DC Power Converters around any non–minimum phase equilibrium point of the system. The technique guarantees, in an average sense, an asymptotically stable behavior of the controlled converter state variables towards their required equilibrium values, while the unstable dynamical compensator output is forcefully kept bounded within an arbitrarily small neighborhood of the required controller operating point

Keywords : DC–to–DC Power Converters, Non–minimum Phase Systems, Sliding Regimes.

1 Introduction

Discontinuous feedback regulation of linear, and nonlinear, dynamical systems has been traditionally studied in the context of *Sliding Regimes*, *Pulse Width Modulation* (PWM) or *Pulse Frequency Modulation* (PFM) strategies (see Utkin [1], Tsypkin [2], Skoog and Blankenship [3], Sira-Ramírez and coworkers [4]–[5] and Taylor [6]). The obtained feedback controllers are known to be robust, and rather insensitive, with respect to "matched" perturbation inputs and bounded structural perturbations affecting the controlled plant.

In a rather non–traditional context, induced controller output discontinuities have been proposed, in Abu el Atta–Dos and Fliess [7], [8], as a feasible means of circumventing singularities associated to nonlinear static controllers in Predictive Control schemes based on system inversion. The fundamental feature of this development is based on inducing calculated "jumps", or controller resettings, on the compensator output when its trajectories approach the immediate vicinity of a singularity in the control space. The corresponding controlled state trajectories of the plant, nevertheless, remain continuous. The possibility of inducing discontinuities in the control variable, or in the dynamical controller states (such as those obtained by "resetting of integrators"), has been little explored in controlling nonlinear systems around non–minimum phase equilibrium points. Recent contributions dealing with the control of non–minimum phase systems by means of an educated re–specification of the output variables has been proposed by Benvenuti *et al* in [9]. In the linear case, the same approach has been entirely justified from a Module theoretic viewpoint by Fliess and Sira-Ramírez in [10].

In this article, the robust features of *quasi–sliding mode control*, induced by the possibilities of "resetting" of the states (i.e., integrator outputs) of a dynamical linearizing controller, is shown to yield a feasible feedback stabilization procedure for the robust regulation of DC–to–DC Power Converters (see Kasakian *et al* [12]), towards non–minimum phase equilibrium points. The basic idea consists in utilizing the *Fliess's Generalized Observability Canonical Form* (FGOCF) (see Fliess [11]) associated to the DC–to–DC Power Converter (see also Sira–Ramírez and Lischinsky–Arenas [13]). Using such a generalized canonical form, the specification, by means of straightforward system inversion, of an exactly linearizing, yet, unstable first order dynamical feedback controller is rather direct. The resulting closed loop state trajectories naturally seek the desired constant equilibrium under the influence of the unstable controller output. A switching logic, based on *active resetting* of the nonlinear controller's integrator output, may then be devised such that a *quasi sliding*

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regime is formed on a small "band" centered around the unstable equilibrium point of the dynamically generated control input signal. By forcing the controller to exhibit a small amplitude, quasi-periodic, oscillation around the required equilibrium value, the controlled system state trajectory achieves, in an average sense, asymptotic convergence towards its corresponding equilibrium point.

Section 2 is devoted to apply the proposed controller resetting stabilization procedure for the direct regulation of output voltage variables, around non-minimum phase equilibrium points, in average models of PWM controlled DC-to-DC Power Converters. Simulations are shown which demonstrate the advantageous features of the proposed control strategy. Section 3 contains the conclusions and suggestions for further research.

2 Direct Output Capacitor Voltage Stabilization in DC-to-DC Power Supplies

2.1 The Boost Converter

Consider the Boost converter model shown in figure 1. This circuit is described by the following bilinear state equation model

$$\begin{aligned} \dot{x}_1 &= -(1-u)\,\omega_0\,x_2 + b \\ \dot{x}_2 &= (1-u)\,\omega_0\,x_1 - \omega_1\,x_2 \\ y &= x_2 \end{aligned} \tag{2.1}$$

where $x_1 = I\sqrt{L}$ and $x_2 = V\sqrt{C}$ represent normalized input inductor current and normalized output capacitor voltage variables, respectively. The positive quantity $b = E/\sqrt{L}$ is the normalized external input voltage and, the constants ω_1 and ω_0 are, respectively, the RC-output circuit time constant and the LC-input circuit natural oscillation frequency. The variable u denotes the switch position function, acting as a control input, and taking values in the discrete set $\{0, 1\}$. The output y of the system is represented by the normalized output capacitor voltage x_2.

The *average pulse width modulation* model, associated to the above switch regulated system, is simply obtained by replacing the switch position function u by a piecewise smooth function, μ, representing the *duty ratio function* which is naturally bounded by the closed interval $[0, 1]$. The averaged normalized state variables are denoted by z_1 and z_2. We still, abusively, denote the average output voltage by y.

$$\begin{aligned} \dot{z}_1 &= -(1-\mu)\,\omega_0\,z_2 + b \\ \dot{z}_2 &= (1-\mu)\,\omega_0\,z_1 - \omega_1\,z_2 \\ y &= z_2 \end{aligned} \tag{2.2}$$

The equilibrium point of the system dynamics, corresponding to a constant value $0 < U < 1$ of the duty ratio function μ, is given by

$$\mu = U \ ; \ Z_1(U) = \frac{b\omega_1}{\omega_0^2(1-U)^2} \ ; \ Z_2(U) = \frac{b}{\omega_0(1-U)} \tag{2.3}$$

The equilibrium values $Z_1(U)$ and $Z_2(U)$ are, both, positive quantitities.

An approximate linearization of the average dynamics around the obtained equilibrium point yields the following scalar transfer function, relating the incremental output voltage $z_{2\delta} = z_2 - Z_2(U)$ to the incremental duty ratio input $\mu_\delta = \mu - U$:

$$G_U(s) = \frac{z_{2\delta}(s)}{\mu_\delta(s)} = \tag{2.4}$$
$$-\omega_0 Z_1(U)\frac{s - \frac{b}{Z_1(U)}}{s^2 + \omega_1 s + (1-U)^2\omega_0^2}$$

The input-output representation of the linearized system is, evidently, non-minimum phase, as the quantity $b/Z_1(U)$ is assumed to be positive.

Consider now the FGOCF of the average pulse width modulated controlled Boost Converter (see [13]), computed on the basis of the average normalized output voltage error $\eta_1 = z_2 - Z_2(U)$:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= -(1-\mu)^2\,\omega_0^2\,[\eta_1 + Z_2(U)] - \omega_1\,\eta_2 \\ &\quad +\omega_0\,(1-\mu)\,b - \dot{\mu}\left[\frac{\eta_2 + \omega_1\,(\eta_1 + Z_2(U))}{1-\mu}\right] \end{aligned} \tag{2.5}$$

The *zero dynamics*, associated to the stabilization towards the value zero of the average normalized output voltage error η_1 , is given by

$$\dot{\mu} = -\frac{\omega_0^2}{\omega_1}(U-\mu)\,(1-\mu)^2 \tag{2.6}$$

The equilibrium points of the zero dynamics are clearly given by $\mu = U$ and $\mu = 1$. Both equilibria are unstable as it easily follows from the phase diagram of figure 2.

The corresponding unstable first order linearizing controller, achieving an asymptotically stable linear dynamics of the form:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= -2\zeta\omega_n\,\eta_2 - \omega_n^2\,\eta_1 \end{aligned} \tag{2.7}$$

is then given, in transformed variables, by

$$\dot{\mu} = \frac{1-\mu}{\eta_2+\omega_1(\eta_1+Z_2(U))}\left[(2\zeta\omega_n-\omega_1)\eta_2+\omega_n^2\eta_1 \right.$$
$$\left. -(1-\mu)^2\,\omega_0^2\,(\eta_1+Z_2(U))+\omega_0\,(1-\mu)\,b\right] \quad (2.8)$$

In terms of the original average normalized state variables z_1, z_2, the dynamical unstable controller is rewritten as:

$$\dot{\mu} = \frac{1}{\omega_0 z_1}\left\{\left[\omega_1\,(\omega_1-2\zeta\omega_n)-\omega_0^2(1-\mu)^2\right]\,z_2 \right.$$
$$-(1-\mu)(\omega_1-2\zeta\omega_n)\omega_0\,z_1+\omega_0\,(1-\mu)\,b$$
$$\left. +\omega_n^2(z_2-Z_2(U))\right\} \quad (2.9)$$

The controller exhibits a singularity at $z_1=0$. Initial conditions for the system, and the controller state, can always be appropriately set such that this singularity is conveniently avoided.

A controller resetting strategy is now proposed which achieves stabilization of the dynamically generated control input trajectory, $\mu(t)$; $t>t_0$, towards a small neighborhood of the required controller equilibrium point $\mu=U$.

Let $0<\delta<\epsilon$ be two arbitrarily small positive real numbers. Assume, furthermore, that the initial value of the duty ratio function $\mu(t_0)$ is found within an ϵ-neighborhood of the unstable controller output equilibrium value U, i.e., $|\mu(t_0)-U|<\epsilon$.

The following resetting strategy produces an oscillatory motion, or quasi-sliding motion, of the dynamically generated control input $\mu(t)$ around the desired equilibrium value $\mu=U$

$$\mu(t)=\begin{cases}\text{if, for any } t>t_0 \;;\; |\mu(t)-U|=\epsilon,\\ \text{then, set } \mu(t^+)=U-\delta\,\text{sign}\,[\dot{\mu}(t)]\\ \text{otherwise, } \mu(t) \text{ obeys equation (2.9)}\end{cases}$$

2.2 Simulation Results

Simulations were performed, for the proposed dynamical controller resetting strategy, on a typical Boost converter with parameter values: $L=0.020$ H, $C=20\ \mu$F $R=30\ \Omega$ and $E=15$V. The desired unstable equilibrium value for the duty ration function, generated by the dynamical controller, was set to be $\mu=U=0.6$. The corresponding equilibrium point for the normalized average input inductor current and output capacitor voltage was found to be $Z_1(0.6)=0.4419$, $Z_2(0.6)=0.1677$. The linearized closed loop dynamics is characterized by the design parameters $\zeta=0.85$ and $\omega_n=700$.

The simulations, shown in figure 3, depict the behavior of the controlled state trajectories as well as the corresponding resetting activity excercised on the controller output trajectory $\mu(t)$ with $U=0.6$, $\delta=0.002$ and $\epsilon=0.005$. The average normalized state variables, z_1 and z_2, are shown to asymptotically converge towards the required equilibrium values with small (chattering-like) oscillations. The controller resetting strategy clearly portrays two situations. 1) When the states values are far away from their required equilibrium, the control input crosses the value $\mu=U=0.6$, either constantly growing, or constantly decreasing, and rapidly reaches one of the boundaries of the neighborhood of $\mu=0.6$ at the values for which $|\mu-0.6|=0.005$. 2) When the controlled states are close to their equilibrium values, the unstable zero dynamics (2.6) approximately describes the unstable controller behavior. The controller outupt does not, by itself, cross the value $\mu=0.6$, but only at the resetting instants. The duty ratio trajectories tend to diverge from the equilibrium value, $U=0.6$, towards one of the baoundaries specified by $|\mu-0.6|=0.005$, as the zero dynamics rightfully dictates. As a result, a *quasi sliding regime* is formed for the controller output μ which, on the average, adopts the equilibrium value $\mu=0.6$.

2.3 The Buck–Boost Converter

Consider the Buck–Boost converter model shown in figure 4. This circuit is described by the following nonlinear state equation model

$$\begin{aligned}\dot{x}_1 &= (1-u)\,\omega_0\,x_2+u\,b\\ \dot{x}_2 &= -(1-u)\,\omega_0\,x_1-\omega_1\,x_2\\ y &= x_2\end{aligned} \quad (2.10)$$

where, as before, x_1 and x_2 represent normalized input current and normalized output voltage variables, respectively. b is the normalized external input voltage and, ω_1 and ω_0 have identical interpretations as in the Boost example. The variable u is the switch position function, The output y is the normalized output capacitor voltage x_2.

The *average pulse width modulation* model, associated to the above switch regulated system, is simply given by

$$\begin{aligned}\dot{z}_1 &= (1-\mu)\,\omega_0\,z_2+\mu\,b\\ \dot{z}_2 &= -(1-\mu)\,\omega_0\,z_1-\omega_1\,z_2\\ y &= z_2\end{aligned} \quad (2.11)$$

The equilibrium point of the system dynamics, corresponding to a constant value U of the duty ratio function μ, is now given by

$$\mu = U \; ; \; Z_1(U) = \frac{bU\omega_1}{\omega_0^2(1-U)^2} \; ; \; Z_2(U) = \frac{-bU}{\omega_0(1-U)} \tag{2.12}$$

It may be easily verified that the input–output representation of the linearized average Buck–Boost converter is also non-minimum phase.

Consider now the FGOCF of the average pulse width modulated controlled Buck–Boost Converter (see [13]), computed on the basis of the average normalized output voltage error $\eta_1 = z_2 - Z_2(U)$:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \qquad (2.13) \\ \dot{\eta}_2 &= -(1-\mu)^2\omega_0^2(\eta_1 + Z_2(U)) - \omega_1\eta_2 \\ &\quad -\omega_0 b\mu(1-\mu) - \dot{\mu}\left[\frac{\eta_2 + \omega_1(\eta_1 + Z_2(U))}{1-\mu}\right] \end{aligned}$$

The zero dynamics, associated to the stabilization to zero of the average normalized output voltage error is given by

$$\dot{\mu} = \frac{\omega_0^2}{\omega_1 U}(\mu - U)\;(1-\mu)^2 \tag{2.14}$$

The zero dynamics is clearly unstable at the equilibrium points $\mu = U$, and $\mu = 1$, as depicted in the phase diagram of figure 5.

Similarly to the Boost case, the corresponding unstable first order linearizing controller, achieving an asymptotically stable second order linear dynamics of the form (2.7) is readily obtained, in terms of generalized phase coordinates, as,

$$\begin{aligned} \dot{\mu} &= \frac{1-\mu}{\eta_2 + \omega_1(\eta_1 + Z_2(U))}\left[(2\zeta\omega_n - \omega_1)\eta_2 + \omega_n^2\eta_1\right. \\ &\quad \left. -(1-\mu)^2\omega_0^2(\eta_1 + Z_2(U)) - \omega_0\, b\, \mu(1-\mu)\right] \end{aligned} \tag{2.15}$$

In terms of the original average normalized state variables z_1, z_2, the dynamical unstable controller is given by

$$\begin{aligned} \dot{\mu} &= \frac{1}{\omega_0 z_1}\left\{-\left[\omega_1(\omega_1 - 2\zeta\omega_n) - \omega_0^2(1-\mu)^2\right] z_2\right. \\ &\quad +(1-\mu)(\omega_1 - 2\zeta\omega_n)\omega_0\, z_1 + \omega_0\, b\,(1-\mu)\mu \\ &\quad \left. -\omega_n^2(z_2 - Z_2(U))\right\} \end{aligned} \tag{2.16}$$

A resetting strategy, similar to the one developed for the Boost converter, can be used to achieve stabilization of the dynamically generated control input trajectory of u around a small neighborhood of the required controller equilibrium point $\mu = U$.

2.4 Simulation Results

Simulations were performed for a dynamically feedback controlled Buck–Boost converter with a controller resetting policy. The same parameter values, used for the Boost converter example, were used in this simulation example. The desired unstable equilibrium value for the duty ration function, generated by the dynamical controller, was set to be $\mu = U = 0.556$. The corresponding equilibrium points for the normalized average input inductor current, and output capacitor voltage, was found to be $Z_1(0.556) = 0.2$, $Z_2(0.556) = -0.084$.

The simulations shown in figure 6 depict the behavior of the average normalized controlled state trajectories, as well as the resetting strategy performed on the controller output $\mu(t)$. The average normalized state variables z_1 and z_2 are shown to converge, in an average sense, towards the required equilibrium point.

3 Conclusions

A discontinuous feedback strategy, based on suitable resettings of an unstable dynamical linearizing feedback controller, has been proposed for the robust stabilization of output voltages, in nonlinear average models of dc–to–dc power converters, around given non–minimum phase equilibrium points. The resetting procedure simply entitles the creation of a quasi–sliding regime for the dynamical controller output (or, plant input variable) around the required controller's constant, but unstable, equilibrium value which corresponds to the desired output plant equilibrium point.

Many other classes of systems, and control objectives, may benefit from the possibilities of the proposed class of discontinuous (feedback) control actions. Extensions to trajectory tracking problems in non–minimum phase systems seems to be straightforward with enhanced possibilities and potential for further applications.

References

[1] V.I., Utkin, *Sliding Modes and Their Applications in Variable Structure Systems*, MIR Publishers, Moscow 1978.

[2] Y. Z., Tsypkin, *Relay Control Systems*, Cambridge University Press, Cambridge, 1984.

[3] R. A., Skoog and G., Blankenship, "Generalized pulse–modulated feedback systems: Norms, gains,

Lipschitz constants and stability," *IEEE Transactions on Automatic Control*, Vol. AC-15, pp. 300–315, 1970.

[4] H., Sira-Ramírez, "Dynamical pulse-width-modulation control of nonlinear systems," *Systems and Control Letters*, Vol. 18, No. 2, pp. 223–231, 1992.

[5] H., Sira-Ramírez, P., Lischinsky-Arenas and O., Llanes-Santiago., "Dynamic compensator design in nonlinear aerospace systems," *IEEE Transactions on Aerospace and Electronics Systems*, Vol. 29, No. 2, pp. 364–379, 1993.

[6] D. G., Taylor, "Pulse-Width Modulated Control of Electromechanical Systems" *IEEE Transactions on Automatic Control*, Vol. AC-37, No. 4, pp. 524–528, 1992.

[7] S., Abu el Ata-Doss and M., Fliess., "Nonlinear Predictive Control by Inversion," *IFAC Symposium on Nonlinear Control Systems Design*, Preprints, pp. 70-75, Capri, June 14-16, 1989.

[8] S., Abu el Ata-Doss, A., Coïc and M., Fliess, "Nonlinear Predictive Control by Inversion: Discontinuities for Critical Behavior," *International Journal of Control*, Vol. 55, No. 6, pp. 1521-1533, 1992.

[9] L. Benvenuti, M. D. Di Benedetto, J. W. Grizzle, "Approximate output tracking for nonlinear non-minimum phase systems with applications to flight control," Report CGR-92-20, Michigan Control Group Reports. University of Michigan, Ann Arbor Michigan, U.S.A. 1992.

[10] M. Fliess and H. Sira-Ramírez, "Regimes glissants, structures variables linéaires et modules, *C. R. de la Acad. de Sci. Paris*. Serie I, Automatique. pp. 703–706, 1993

[11] M., Fliess, "Nonlinear Control Theory and Differential Algebra," in *Modeling and Adaptive Control*, Ch. I. Byrnes and A. Khurzhansky (Eds.), Lecture Notes in Control and Information Sciences, Vol. 105. Springer-Velag, 1989.

[12] J. G., Kassakian, M. F., Schlecht, and G. C., Verghese, *Principles of Power Electronics*, Addison Wes-'.ر Publishing Co., Reading, Massachusetts, 1991.

[13] H. Sira-Ramírez and P. Lischinsky-Arenas, "Differential algebraic approach in non-linear dynamical compensator design for dc-to-dc power converters", *International Journal of Control*, Vol. 54, No. 1, pp. 111–133, 1991.

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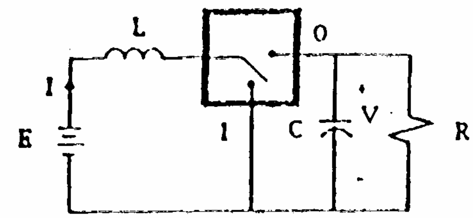

Figure 1: Boost Converter Circuit

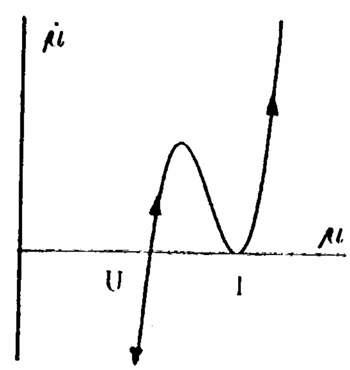

Figure 2: Phase diagram of Boost converter zero dynamics

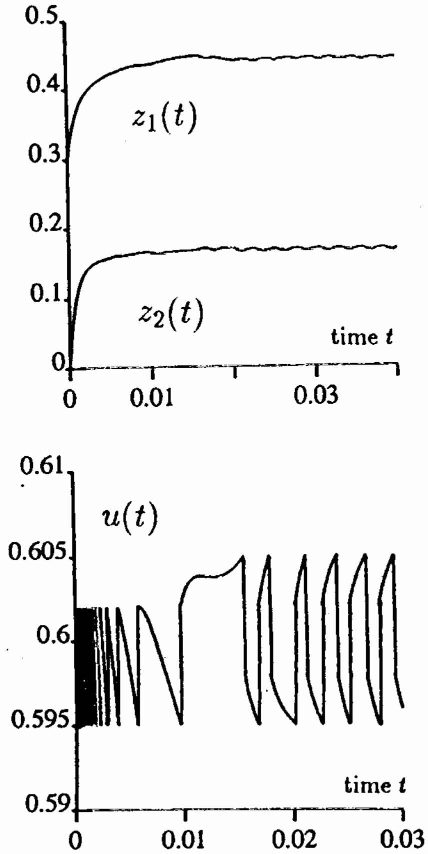

Figure 3: State trajectories and resettings of control input for output voltage stabilization in the Boost converter

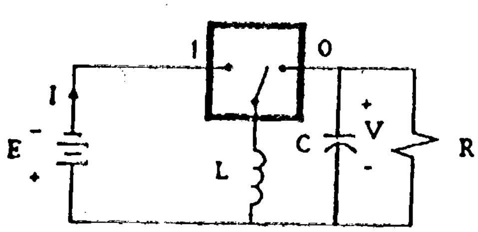

Figure 4: Buck-Boost Converter Circuit

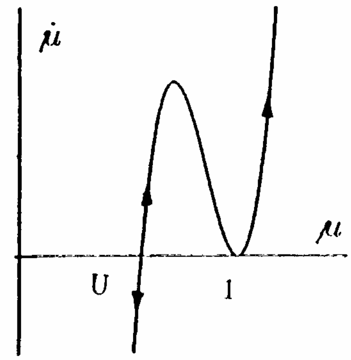

Figure 5: Phase diagram of Buck-Boost converter zero dynamics

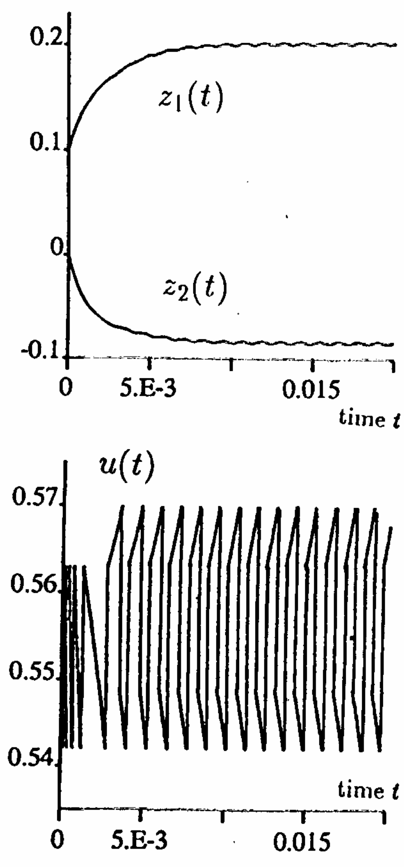

Figure 6: State trajectories and resettings of control input for output voltage stabilization in the Buck-Boost converter