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# Pulsed Control of Nonlinear Mechanical Systems with Rate Constrained Velocities. *

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## Abstract

A pulsed feedback control strategy is proposed for the robust stabilization of a class of multivariable mechanical systems in which rate constrained angular, or linear, velocity variables are regarded as control inputs. The trapezoidal character of the pulsed-width regulation policy complies with the physical limitation of having corresponding acceleration variables, i.e., applied torques, or forces, of bounded magnitude. The proposed approach is used in the approximate feedback control regulation of a perturbed multivariable differentially flat system

**Keywords :** Trapezoidal Pulse Width Modulation, Differentially flat systems

## 1 Introduction

Pulse-Width-Modulation (PWM) control of dynamical systems has been the subject of sustained theoretical and practical developments due to its inherent simplicity, robustness, and widespread possibilities for inexpensive hardware implementation. Early work, in connection with the regulation of linear systems, is due, among many other authors, to Skoog and Blankenship [11]. Developments casting PWM as a robust feedback control technique for nonlinear systems may be found in the work of Kuntsevich and Cherkhovoi [5], Sira-Ramírez and coworkers [7], [8], [9] and Taylor [12]. The prevailing characteristic of PWM strategies is the discontinuity of the applied feedback control input signal, constituted by rectangular pulses of varying width. As a consequence, the time derivatives of such train of width-varying pulses, exhibit infinte magnitudes.

For a large class of mechanical systems, such as nonholonomically velocity constrained systems, velocity variables (whether angular or linear velocities) are sometimes considered as control inputs (see Bloch *et al* [1] and Murray and Sastry [6]). Computation of the required torques, or forces, as ultimate control variables is then carried out, if necessary, by means of straightforward differentiation and simple algebraic manipulations. This procedure, however, is, evidently, not suitable for discontinuous feedback control techniques, such as sliding mode control and pulse-width-modulation (PWM) since infinite applied forces or torques would be required as feedback contol actions. To circumvent this difficulty, one resorts to a procedure which still regards the velocity variables as control inputs but it also considers magnitude constraints on the acceleration variables. This alternative results only in stable convergence to the regulation objectives, rather than asymptotically stable behavior. From a practical viewpoint the performance is, nevertheless, surprisingly satisfactory in spite of the presence of unmodelled high-frequency stochastic perturbation signals.

In this article a robust *trapezoidal* pulse width modulation scheme (TPWM) is proposed for the stabilization of a class of mechanical systems in which velocity variables are taken as control inputs for controller design purposes.

Section 2 presents a fundamental stability result regarding a simple integrator system feedback regulated by means of a TPWM strategy. This development is later shown, by means of an example, to be essential for the decoupled stabilization of multivariable nonlinear systems. The design example considers an application to TPWM regulation of a nonholonomically constrained system constituted by a "hopping robot". This system has been shown to be *differentially flat* (see Fliess *et al* [2] ) i.e. it is linearizable by means of *dynamical endogenous feedback*. This fact is shown to greatly facilitate the TPWM controller design task. The performances of the proposed multivariable TPWM feedback controllers is evaluated when the system is subject to unmodelled bounded stochastic disturbances. Section 3 contains the con-

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clusions of the article.

# 2 Main Result and an Application Example

## 2.1 Trapezoidal pulse width modulation control of a simple scalar system

Consider the following scalar closed loop system characterized by a state variable $s$,

$$\begin{aligned} \dot{s} &= v \\ v &= -W\, TPWM\ (s) \end{aligned} \qquad (1)$$

$$TPWM\ (s) = \begin{cases} \frac{1}{p\,\tau[s(t_k)]\,T}\,(t - t_k)\ \text{sign}\ s(t_k)\ \text{for}\ t_k \le t < t_k + p\,\tau[s(t_k)]\,T \\ \text{sign}\ s(t_k) \quad \text{for} \quad t_k + p\,\tau[s(t_k)]\,T \le t < t_k + \tau[s(t_k)]\,T\,[1-p] \\ -\frac{1}{p\,\tau[s(t_k)]\,T}\,(t - t_k - \tau[s(t_k)]\,T)\ \text{sign}\ s(t_k) \quad \text{for} \\ t_k + \tau[s(t_k)]\,T\,[1-p] \le t < t_k + \tau[s(t_k)]\,T \\ 0 \quad \text{for} \quad t_k + \tau[s(t_k)]\,T \le t < t_k + T \end{cases}$$

$$t_k + T = t_{k+1} \quad \text{for} \quad k = 0, 1, 2, \ldots$$

where the function $\tau(s)$ represents the *duty ratio* function. Its sampled values, at every instant of time $t_k$, determines the width of the trapezoidal pulse for the current inter-sampling interval (see Figure 1). The trapezoidal pulse width is determined at each sampling period as $\tau[s(t_k)]\ T$, where $T$ denotes the sampling interval, or *duty cycle*, considered here to be constant. The duty ratio function is necessarily bounded by the closed interval $[0,1]$. However, in order to avoid infinite slopes in the signal $v$, we need to hypothesize a minimum positive value, or constant lower bound, for the duty ratio function. Such a value will be denoted by the constant, $\tau_{min}$. We also let the maximum control input rate to be specified by the constant, $A_{\max}$. The scalar $p$ is then a positive real number defining the fraction of the pulse width $\tau[s(t_k)]\ T$ on which the signal $v$ is allowed to either grow from zero to $W$, or to decrease from $W$ to zero. This number $p$, evidently, must also be bounded away from 1. The minimum allowable value of the duty ratio function, $\tau_{\min}$, is evidetly related to $p$, to the sampling interval $T$ and to the gain $W$ by the relation:

$$\frac{W}{p\,\tau_{\min}\,T} \le A_{\max} \qquad (2)$$

The positive constant gain $W$ is a design parameter representing the maximum amplitude, in absolute value, of the control input signal $v$.

The duty ratio function, $\tau(s)$, is synthesized as a feedback function as follows (see Figure 2):

$$\tau(s) = \begin{cases} 1 & \text{for} \quad |\,s\,| \ge \frac{1}{\beta} \\ \beta\,|\,s\,| & \text{for} \quad \frac{\tau_{\min}}{\beta} < |\,s\,| < \frac{1}{\beta} \\ \tau_{\min} & \text{for} \quad |\,s\,| \le \frac{\tau_{\min}}{\beta} \end{cases} \qquad (3)$$

The next paragraphs describes the stable features of the closed loop system (1),(3), along with an amplitude estimate of the underlying *limit cycle* behavior exhibited by $s$.

**Propostion 2.1** *The closed loop system (1) is stable. Moreover, the trajectories of $s$ are ultimately bounded by a vicinity of the origin given by*

$$|s(t)| \le \frac{\tau_{\min}}{\beta}$$

*provided*

$$(1-p)\,\beta\,W\,T < 1 \qquad (4)$$

**Proof**

The proof of stability is quite straightforward by simply adopting the function

$$V(s) = \frac{1}{2}\,s^2$$

as a Lyapunov function candidate. According to (1), one has, along its solutions,

$$\dot{V} = \dot{s}\,s \le 0.$$

Moreover, as long as $s \neq 0$, the sets where $\dot{V} = 0$ do not constitute trajectories of the system for an indefinite period of time. The system is therefore stable. Ultimate boundedness of the controlled trajectories to a small vicinity of zero easily follows from exact discretization of the scalar differential equation at the sampling instants and straightforward stability considerations under the prescribed sufficient conditon (the interested reader may find the details in Sira-Ramirez and Llanes-Santiago [10]).

## 2.2 The hopping robot

Consider the dynamics of a hopping robot in flight phase (see [6]) Let the state variables be defined as the length $l$ of the leg and the angular position coordinates $\psi$ and $\theta$, respectively, of the unit mass body and of the leg with respect to the horizontal axis. The leg of the robot, of mass $m_l$, can rotate with respect to its attachment to the body. The total angular momentum is, however, conserved during such motions ( such is the nature of the nonholonomic constraint). During the flight phase, the leg is also capable of extending and contracting within a given range, taken here, for simplicity, between 1 and $1 + L$ (see Figure 3). The control inputs $v_1$ and $v_2$ are constituted, respectively, by the angular velocity of the leg's rotation and the rate of change of the length of the leg. The equations describing the dynamics are given by:

$$\begin{aligned} \dot{\psi} &= v_1 \\ \dot{l} &= v_2 \\ \dot{\theta} &= -\frac{m_l\,(1+l)^2}{1+m_l\,(1+l)^2}\,v_1 \end{aligned} \tag{5}$$

The following state coordinate transformation

$$x_1 = \psi \;\; ; \;\; x_2 = -\frac{m_l\,(1+l)^2}{1+m_l\,(1+l)^2} \;\; ; \;\; x_3 = \theta \tag{6}$$

and the redefinition of the input variables

$$\begin{aligned} u_1 &= v_1 \\ u_2 &= -\frac{2\,m_l\,(1+l)}{\left(1+m_l\,(1+l)^2\right)^2}\,v_2 \end{aligned} \tag{7}$$

takes the system (5) into a *2-input, 1-chain, single generator chained system* form (see [6]),

$$\begin{aligned} \dot{x}_1 = u_1 \quad & \dot{x}_2 = u_2 \\ & \dot{x}_3 = x_2\,u_1 \end{aligned} \tag{8}$$

The system is evidently differentially flat since all variables in the transformed system can be expressed as a differential function of the linearizing outputs $y_1 = x_1$ and $y_2 = x_3$ (i.e., as a function of the leg and the body angular position coordinates and a finite number of its time derivatives). Indeed,

$$\begin{aligned} x_1 &= y_1 \\ x_2 &= \frac{\dot{y}_2}{\dot{y}_1} \\ x_3 &= y_2 \\ u_1 &= \dot{y}_1 \\ u_2 &= \frac{\ddot{y}_2\dot{y}_1 - \dot{y}_2\ddot{y}_1}{\dot{y}_1^2} \end{aligned} \tag{9}$$

It is easy to see that the *row relative degrees* (see Isidori [4]) of $y_1$ and $y_2$ are both equal to 1, while the *essential orders* (see Glumineau and Moog [3]) are both equal to 2. The system is not decouplable by means of static state feedback and a dynamic extension of order 1 is required on the transformed control input $u_1$ in order to make the structure at infinity of the extended system coincide with the essential structure. This extension is thus necessary for the appropriate definition of a dynamical feedback law which achieves decoupling of the system.

The dynamically extended version of the transformed system, which is now suitable for static linearly decoupling feedback, is given by

$$\begin{aligned} \dot{x}_1 = u_1 \quad & \dot{x}_2 = u_2 \\ \dot{u}_1 = \nu_1 \quad & \dot{x}_3 = x_2\,u_1 \end{aligned} \tag{10}$$

where $\nu_1 = \dot{u}_1$ is a new control input to the system and the variable $u_1$ is just an additional state variable for the extended system. The extended control inputs as differential functions of the linearizing outputs are simply given by

$$\nu_1 = \ddot{y}_1 \;\; ; \;\; u_2 = \frac{\ddot{y}_2\dot{y}_1 - \dot{y}_2\ddot{y}_1}{\dot{y}_1^2} \tag{11}$$

Suppose it is desired to have the linearizing coordinate $y_1 = \theta$, adopt the constant value $\Theta$, at the end of the flying phase, while the lenght $l$ is driven to a constant value, say $L/3$. Appropriate error functions $s_1$ and $s_2$ may be defined as

$$\begin{aligned} s_1 &= x_3 - \Theta = y_2 - \Theta \\ s_2 &= x_2 + \frac{m_l\,(1+L/3)^2}{1+m_l\,(1+L/3)^2} \\ &= \frac{\dot{y}_2}{\dot{y}_1} + \frac{m_l\,(1+L/3)^2}{1+m_l\,(1+L/3)^2} \end{aligned} \tag{12}$$

While the first choice is clear, it is also easy to realize, from equation (12), that the only phisically meaningful solution for $l$, from the condition $s_2 = 0$, is given by $l = L/3$. Note that the first regulated output $z_1 = y_2 = \theta$ coincides with a linearizing, or flat, output $y_2$ of the sytem, while the second regulated output, $z_2 = x_2$, is a differential function of the flat outputs. In the extended system, the regulated output $z_1$ has relative degree two, while the regulated output $z_2$ has only relative degree 1. Thus, an imposed second order dynamics on $z_1$ and an imposed first order dynamics on $z_2$ already contain expressions involving the highest order time derivatives of the linearizing outputs $y_1$ and $y_2$. As it will be seen, such time derivatives of the flat outputs are solvable from these imposed relations.

In accordance with the particular form of the proposed regulated error functions, a second order closed loop TPWM dynamics will be proposed for $s_1$. This is achieved by imposing a first order TPWM dynamics on a (Hurwitz) linear combination of $s_1$ and $\dot{s}_1$. A first order closed loop TPWM controlled dynamics suffices for the error coordinate $s_2$.

Define then the *auxiliary error coordinates* $\sigma_1$ and $\sigma_2$, as

$$\begin{aligned} \sigma_1 &= \dot{s}_1 + \lambda\,s_1 \;\; ; \;\; \lambda > 0 \\ \sigma_2 &= s_2 \end{aligned} \tag{13}$$

A decoupled set of closed loop TPWM dynamics guarantee desirable stability features, for $s_1$ and $s_2$ after the auxiliary error functions $\sigma_1$ and $\sigma_2$ are approximately driven to zero, in finite time. Such imposed TPWM dynamics are given by,

$$\begin{aligned} \dot{\sigma}_1 &= -W_1 \,\mathrm{TPWM}\, \sigma_1 \;\; ; \;\; W_1 > 0 \\ \dot{\sigma}_2 &= -W_2 \,\mathrm{TPWM}\, \sigma_2 \;\; ; W_2 > 0 \end{aligned} \qquad (14)$$

The preceeding closed loop dynamics lead to error dynamics governed by

$$\begin{aligned} \ddot{s}_1 &= -\lambda\, \dot{s}_1 - W_1 \,\mathrm{TPWM}\,(\dot{s}_1 + \lambda\, s_1) \\ \dot{s}_2 &= -W_2 \,\mathrm{TPWM}\, s_2 \end{aligned} \qquad (15)$$

After small amplitude stable oscillations occur around the zero level set of the auxiliary error functions $\sigma_1 = 0$ and $\sigma_2 = 0$, the $s_1$ and $s_2$ coordinates will approximately satisfy the following equations,

$$\begin{aligned} \dot{s}_1 &= -\lambda\, s_1 \\ s_2 &= 0 \end{aligned} \qquad (16)$$

One may conclude that the proposed scheme guarantees a stable convergence of $s_1$ and $s_2$ to a small vicinity of zero. This accomplishes, in an approximate, but efficient manner, the proposed control objectives. The leg's angular coordinate $\theta$ is seen to converge towards the vicinity of the prescribed value $\Theta$, while also closely achieving the required length $l = L/3$ for the rotating leg.

By virtue of the differential flatness of the system, the regulated dynamics (14) can be immediately translated into requied autonomous dynamics for the linearizing outputs $y_1 = x_1$ and $y_2 = x_3$. Indeed, in terms of $y_1$ and $y_2$ the equations (14) result in the following nonlinear set of differential equations with right hand sides specified by TPWM feedback policies.

$$\begin{aligned} \ddot{y}_1 &= \\ &\frac{1}{\dot{y}_2}\Big\{ \Big[ -\lambda\, \dot{y}_2 - W_1 \,\mathrm{TPWM}\, \left(\dot{y}_2 + \lambda (y_2 - \Theta)\right) \Big]\, \dot{y}_1 \\ &\quad + \dot{y}_1^2\, W_2 \,\mathrm{TPWM} \left( \frac{\dot{y}_2}{\dot{y}_1} + \frac{m_l (1 + l/3)^2}{1 + m_l (1 + L/3)^2} \right) \Big\} \\ \ddot{y}_2 &= -\lambda\, \dot{y}_2 - W_1 \,\mathrm{TPWM}\, [\dot{y}_2 + \lambda\, (y_2 - \Theta)] \end{aligned} \qquad (17)$$

Using the highest order derivatives $\ddot{y}_1$ and $\ddot{y}_2$, obtained from the previous set of differential equations, on the expressions for the (extended) control inputs given in equation (11), one immediately obtains, in transformed coordinates, the required extended control input $\nu_1$ as $\nu_1 = \dot{u}_1 = \ddot{y}_1$ and the (static) control input $u_2$ as,

$$\begin{aligned} \dot{u}_1 &= -\frac{1}{x_2} \{ [\lambda\, x_2 \\ &\quad -W_2 \,\mathrm{TPWM} \left( x_2 + \frac{m_l\, (1 + L/3)^2}{1 + m_l\, (1 + L/3)^2} \right) \Big]\, u_1 \\ &\quad + W_1 \,\mathrm{TPWM}\, [x_2 u_1 + \lambda\, (x_3 - \Theta)]\, \Big\} \\ u_2 &= -W_2 \,\mathrm{TPWM} \left( x_2 + \frac{m_l\, (1 + L/3)^2}{1 + m_l\, (1 + L/3)^2} \right) \end{aligned} \qquad (18)$$

The multivariable TPWM controller thus includes a first order dynamical TPWM compensator for the control input $u_1$ controlling the leg's angle and a static feedback TPWM regulator for the control input $u_2$ regulating the leg's length. The dynamical and statical controllers can also be expressed in the original system coordinates as

$$\begin{aligned} \dot{v}_1 &= -\frac{1 + m_l\, (1 + l)^2}{m_l\, (1 + l)^2} \left[ \left( \lambda\, \frac{m_l\, (1 + l)^2}{1 + m_l\, (1 + l)^2} \right. \right. \\ &\quad \left. + W_2 \,\mathrm{TPWM}\, \sigma_2 \,\right)\, v_1 - W_1 \,\mathrm{TPWM}\, \sigma_1 \,] \\ v_2 &= -W_2\, \frac{\left(1 + m_l\, (1 + l)^2\,\right)^2}{2\, m_l\, (1 + l)} \,\mathrm{TPWM}\, \sigma_2 \end{aligned} \qquad (19)$$

with

$$\begin{aligned} \sigma_1 &= -\left( \frac{m_l (1 + l)^2}{1 + m_l\, (1 + l)^2} \right) v_1 + \lambda\, (\theta - \Theta) \\ \sigma_2 &= -\frac{m_l\, (1 + l)^2}{1 + m_l\, (1 + l)^2} + \frac{m_l\, (1 + L/3)^2}{1 + m_l\, (1 + L/3)^2} \end{aligned} \qquad (20)$$

## 2.3 Simulation Results

Computer simulations were carried out for the system and the designed dynamical TPWM control policy. In order to test the robustness of the proposed controller an (unmodelled) computer generated stochastic perturbation signal $\eta$ was added to the hopping robot plant model. The perturbed model used for the simulations was then take to be:

$$\begin{aligned} \dot{\psi} &= v_1 \\ \dot{l} &= v_2 + \eta \\ \dot{\theta} &= -\frac{m_l\, (1 + l)^2}{1 + m_l\, (1 + l)^2}\, v_1 + \eta \end{aligned}$$

where $\eta$ is the hypothesized zero-mean pseudo white noise.

The required leg's angular position was set to be $\Theta = -2\pi/3[\mathrm{rad}] \approx -2.095[\mathrm{rad}]$. The leg's length was taken as $L = 1[\mathrm{mt}]$, so that the desired final length was $L/3 = 0.333[\mathrm{mt}]$. The controller constants were chosen as

$$\lambda = 5\, [\mathrm{s}^{-1}] \;\; ; \;\; W_1 = 8\, [\mathrm{rad/s}^2] \;\; ; \;\; W_2 = 0.2$$

$$\beta_1 = 1.4 \text{ [s/rad]} \;\; ; \;\; \beta_2 = 60$$

$$\tau_{\min 1} = 0.05 \;\; ; \;\; \tau_{\min 2} = 0.05 \;\; ; \;\; T = 0.1 \text{ [s]}$$

The value of $p$ was set to be $p = \frac{1}{6}$ for both TPWM signals. Figure 4 shows the TPWM controlled trajectory of the perturbed evolution of the angle $\theta$ and the leg's length $l$. These variables are seen to approach the prescribed equilibrium values, respectively, in a stable manner and achieving the desired performance in finite time. The original control input signals $v_1$ and $v_2$ are also shown in this figure. The control $v_1$ is the output of a dynamical TPWM compensator and therefore exhibits a "smoother" behavior than the control signal $v_2$, which is the output of a static feedback controller. The signal $v_2$ thus exhibit a characteristic pulsed behavior.

The body angle $\psi$ is seen to exhibit a stable response towards an arbitrary equilibrium point. Figure 4 also shows a sample of the perturbation signal $\eta$. The peak–to–peak amplitude bound for this signal was allowed to be 4.

## 3 Conclusions

In this article a new class of pulsed feedback control strategies, without steplike discontinuities, has been proposed for the regualtion of nonlinear multivariable systems. The feedback technique, addressed as "trapezoidal pulse width modulation" control, has been shown to be suitable for the regulation of a large class of nonlinear multivariable systems with limited control input rates.

A fundamental result on the stability features of the trapezoidal pulsed regulation, of a single integrator scalar system, provides the basis for suitable error stabilization in more complex systems, such as multivariable nonlinear systems.

The proposed scheme is specially suitable for the solution of stabilization, and tracking, problems defined on nonlinear mechanical systems in which angular, or linear, velocity variables are natrually regarded as control input variables to the system. The limited slope assumption on the generated feedback input signals corresponds to magnitude acceleration constraints and, hence, it also naturally handles realistic torque, or force, magnitude limitations. The proposed pulsed feedback controller also represents a "smoothed" approximation strategy for traditional pulse width modulation feedback schemes of discontinuous nature.

Due to the lack of asymptotic stability features of the fundamental scheme, feedback TPWM regulation can only achieve stabilization to arbitrarily small neighborhoods of pre-specified constant equilibrium points. From a practical viewpoint, however, the precision features of the corresponding regulated position variables are, quite surprisingly, highly satisfactory. This is basically due to the averaging effects of the imposed integration of the induced small-amplitude velocity limit cycles.

One application example of physical flavor was presented, along with encouraging computer simulations. These included digitally generated stochastic perturbation signals.

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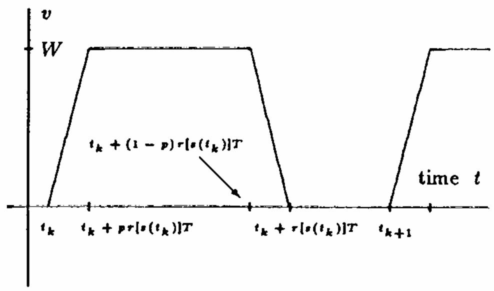


Figure 1: Trapezoidal Pulse Width Modulated Control Signal

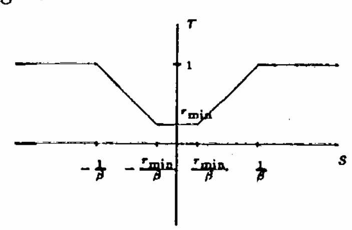


Figure 2: Duty Ratio Function for Scalar TPWM Feedback Strategy

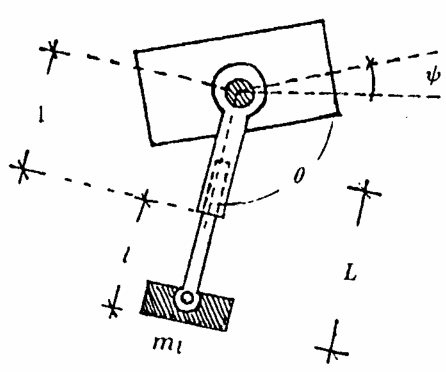


Figure 3: A Hopping Robot

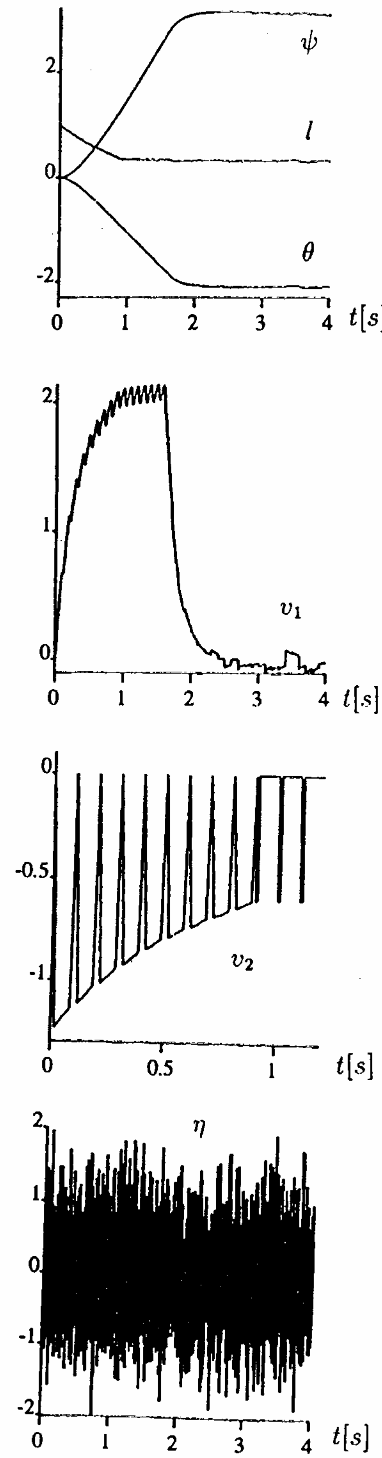


Figure 4: TPWM Regulated Trajectories, Control Input Signals and Perturbation Noises for the Hopping Robot