On the Adaptive Dynamical PWM Feedback Regulation of Switch–mode Power Supplies [1]

Hebertt Sira–Ramírez, Departamento Sistemas de Control. Escuela de Ingeniería de Sistemas Universidad de Los Andes, Mérida 5101, Venezuela, e-mail: isira@ing.ula.ve,

Mauricio García–Esteban, [2] Centro de Instrumentos, Universidad Nacional Autónoma de México, México, D.F., México, e-mail: mauricio@ing.ula.ve

Abstract

An adaptive dynamical feedback controller is proposed for the synthesis of duty ratio functions in uncertain Pulse–Width–Modulation (PWM) controlled dc–to–dc power supplies. The input–output dynamic linearizing, overparametrized, adaptive scheme is shown to provide indirect output capacitor voltage stabilization by means of suitable direct regulation of the minimum-phase input inductor current. A design example is included dealing with the boost converter circuit in continuous conduction mode. The robustness of the approach is tested via computer simulations which incorporate large, unmatched, external stochastic input signals of bounded amplitude.

1. Introduction

The results of adaptive *dynamical* input–output linearization, presented in Sira-Ramírez [1], are applied to the case of indirect stabilization of non-minimum phase output capacitor voltages of PWM regulated dc–to–dc power converters, such as the boost converter. The adaptive versions of *static*, partially linearizing, feedback controllers have already been treated by Sira-Ramírez *et al* in [2] and by Sira-Ramírez and Llanes-Santiago in [3]. General background results on exact state feedback linearization and on dynamical input-output linearization of dc–to–dc power supplies can be obtained, respectively, from the articles by Sira-Ramírez and Ilic–Spong, [4], and by Sira-Ramírez and Lischinsky-Arenas [5].

A feasible PWM feedback compensation method is developed for the adaptive PWM regulation of dc–to–dc power supplies with observable average dynamics. The method is based on indirect regulation of the output capacitor voltage trajectory in the presence of unknown, but constant, circuit parameter values. The approach is shown to also produce a substantially smoothed feedback duty ratio function trajectory thanks to the dynamic nature of the adaptive linearizing duty ratio generator.

The proposed approach suitably combines three important features : 1) Linearizing compensation, based on full, and not partial, exact input–output linearization, 2) The overall scheme is adaptive and, hence, assumes no knowledge of the converter parameters, 3) The proposed adaptive duty ratio synthesizer (controller) is dynamical in nature. Hence, the oscillatory nature of the duty ratio function trajectory, arising from the actual closed

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[2] On temporary leave from the Instituto de Cibernética, Universidad Politécnica de Catalunya, Barcelona, España

loop PWM operation, is further smoothed out by the natural integration involved in the dynamical compensation scheme.
Section 2 is devoted to apply the results of nonlinear adaptive input–output linearization, as developed in [1], to the adaptive dynamical feeedback regulation of dc–to–dc power converters. We specify an adaptive dynamic PWM regulation scheme for the indirect stabilization of non–minimum phase output capacitor voltages in dc–to–dc power converters. The boost converter is presented along with extensive computer simulations. The simulations include testing of the robust performance of the proposed adaptive scheme under severe, unmodelled, external stochastic perturbation inputs. Section 3 contains some conclusions and suggestions for further research.

2. Dynamical Adaptive Stabilization of Minimum Phase DC–to–DC Power Converters

2.1 The boost converter

Consider the boost converter model shown in figure 1. This circuit is described by the following state equation model,

$$\begin{aligned} \dot{x}_1 &= -(1-u)\, x_2/L + E/L \\ \dot{x}_2 &= (1-u)\, x_1/C - x_2/RC \\ y &= x_1 \end{aligned} \tag{2.1}$$

where x_1 and x_2 represent, respectively, the input inductor current and the output capacitor voltage variables. The positive quantity E is the external input voltage. The variable u denotes the switch position function, acting as a control input. Such a control input takes values in the discrete set $\{0, 1\}$. The output, y, of the system is represented by the output capacitor voltage x_2.
We define

$$\theta_1 = 1/L \quad ; \quad \theta_2 = 1/C \quad ; \quad \theta_3 = 1/RC \quad ; \quad \theta_4 = E/L \tag{2.2}$$

A PWM feedback control strategy for the regulation of the boost converter circuit is typically given by the following prescription of the switch position function.

$$u = \begin{cases} 1 \text{ for } & t_k \leq t < t_k + \mu[x(t_k)]T \\ 0 \text{ for } & t_k + \mu[x(t_k)]T \leq t < t_k + T \end{cases} \qquad k = 0, 1, \dots \tag{2.3}$$

where t_k represents a sampling instant, the parameter T is the fixed sampling period, also called the ***duty cycle***, the sampled values of the state vector $x(t)$ of the converter are denoted by $x(t_k)$. The function, $\mu(\cdot)$, is the ***duty ratio function*** acting as a truly feedback policy. The value of the duty ratio function, $\mu[x(t_k)]$, determines, at every sampling instant, t_k, the width of the upcomming "pulse" as $\mu[x(t_k)]T$. The actual duty ratio function, $\mu(\cdot)$, is, evidently, a function limited to the closed interval, $[0, 1]$, of the real line. The control problem, associated to the stabilization of the discontinuously controlled system, (2.1),(2.3), towards some (feasible) prespecified constant equilibrium point, consists in specifiying the duty ratio function, μ, *as a feedback policy*, i.e., as a function of the state vector x. As formulated, the problem of synthesizing a suitable duty ratio function $\mu(x)$ is quite involved, due to the difficulty in performing an exact discretization of the nonlinear PWM controlled circuit model (2.1),(2.3) (see Kassakian *et al* [6]).

A conceptually useful and rather practical alternative consists in resorting to the *Infinite Frequency Average PWM model*, of the switch regulated system. Such an average model is also known as *State Space Average Model* (see Middlebrook and Ćuk [7]). The assumption of an infinite sampling frequency results in a smooth nonlinear average system model of (2.1) in which the duty ratio function, μ, is readily interpreted as a control input to the average system. Such an interpretation results in the formal replacement of the switch position function u by the duty ratio function μ in the averaging process. In fact, the duty ratio function becomes the *equivalent control* input in the corresponding *Sliding Mode* (see Utkin [8]) interpretation of the obtained idealization (see also Sira-Ramírez [9]). An extensive use of the described infinite frequency average models, with successful results, has been made in a series of articles dealing with nonlinear feedback regulation policies for dc-to-dc power supplies (see references [10]-[12])
The average PWM model of system (2.1),(2.3) is then described by,

$$\begin{aligned} \dot{\zeta}_1 &= -\theta_1(1-\mu)\,\zeta_2 + \theta_4 \\ \dot{\zeta}_2 &= \theta_2(1-\mu)\,\zeta_1 - \theta_3\zeta_2 \\ y &= \zeta_1 \end{aligned} \tag{2.4}$$

where the averaged state variables are denoted by ζ_1 and ζ_2. Note that we still, abusively, denote the average output voltage by the same variable y.
The constant equilibrium point of the average system dynamics (2.4), corresponding to a constant value U, $(0 < U < 1)$, of the duty ratio function μ, is given by

$$\mu = U \;\;;\;\; \zeta_1 = Z_1(U) = \frac{\theta_3\theta_4}{\theta_2\theta_1(1-U)^2} \;\;;\;\; \zeta_2 = Z_2(U) = \frac{\theta_4}{\theta_1(1-U)} \tag{2.5}$$

The equilibrium values $Z_1(U)$ and $Z_2(U)$ are, both, positive quantitities.
The observability matrix (see Conte *et al* [13]) associated to the output, $y = \zeta_1$, of the average system (2.4) is given by

$$\mathcal{O} = \begin{bmatrix} 1 & 0 \\ 0 & -\theta_1(1-\mu) \end{bmatrix} \tag{2.6}$$

The rank of the matrix is everywhere equals to 2, except when μ is identically 1. This condition physically corresponds to a permanently *saturated* duty ratio function, thus entitling a fixed switch position ($u \equiv 1$) and, hence, no possibilities exist of feedback regulation.
Consider now the input-output description of the average PWM controlled boost converter (see [5]),

$$\ddot{y} = -\theta_1\theta_2(1-\mu)^2 y - \frac{\dot{y}-\theta_4}{1-\mu}\left(\theta_3(1-\mu) + \dot{\mu}\right) \tag{2.7}$$

The *zero dynamics*, associated to the constant equilibrium point of the system, $y = Z_1(U)$, $\dot{y} = 0$, is given by

$$\dot{\mu} = \frac{\theta_3}{(1-U)^2}(1-\mu)(\mu-U)\;(\mu+U-2) \tag{2.8}$$

The equilibrium points of the zero dynamics are clearly given by $\mu = U$, $\mu = 1$ and $\mu = 2-U$. As the phase diagram of system (2.8) reveals (see Figure 2), the equilibrium point $\mu = 1$ is unstable while the other two are asymptotically stable. The only physically meaningful equilibrium point is given by $\mu = U$, since $\mu = 2-U$ corresponds to a duty ratio larger than 1.

Suppose it is desired to regulate the input inductor current x_1 towards a desired constant value $X_1(U) = Y$, modulo small chattering due to the pulsed regulatory actions of the actual PWM feedback controller. It has been rigorously shown in [9] that the actual PWM regulated state trajectories can be made to converge, in an arbitrarily close fashion, towards the average closed loop PWM controlled state responses, provided a sufficiently high value of the sampling frequency $1/T$, associated to every PWM regulation scheme, is used. Our PWM stabilization problem is, therefore, primarily solved on the basis of the average PWM model (2.4), as if such a model accurately represented the actual PWM controlled system (2.1),(2.3).
Let the desired asymptotically stable closed loop linear dynamics for the average PWM controlled converter be of the form:

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2(y - Y) = 0 \tag{2.9}$$

The corresponding dynamical feedback regulator which synthesizes the required *computed* duty ratio function μ, is written, in transformed variables ζ_1 and ζ_2, as

$$\dot{\mu} = \frac{1}{\theta_1\zeta_2}\left[-\omega_n^2(\zeta_1 - Y) + 2\xi\omega_n\theta_1(1-\mu)\zeta_2 - 2\xi\omega_n\theta_4 + \theta_1\theta_2(1-\mu)^2\zeta_1 - \theta_1\theta_3(1-\mu)\zeta_2\right] \tag{2.10}$$

If the circuit parameters $\theta_1, \ldots, \theta_4$ are unknown then the adaptive version of the dynamic linearizing controller (2.10) is specified as,

$$\dot{\mu} = \frac{1}{\hat{\theta}_1\zeta_2}\left[-\omega_n^2(\zeta_1 - Y) + 2\xi\omega_n\hat{\theta}_1(1-\mu)\zeta_2 - 2\xi\omega_n\hat{\theta}_4 + \widehat{\theta_1\theta_2}(1-\mu)^2\zeta_1 - \widehat{\theta_1\theta_3}(1-\mu)\zeta_2\right] \tag{2.11}$$

In terms of the *overparametrization* vector $\hat{\Theta}$ (see Campion and Bastin [14]), defined as,

$$\Theta = [\Theta_1, \ldots, \Theta_{14}] = [\theta_1, \ldots, \theta_4, \theta_1^2, \theta_1\theta_2, \ldots, \theta_2^2, \ldots, \ldots, \theta_3\theta_4, \theta_4^2]^T \tag{2.12}$$

the dynamical feedback controller (2.11) is rewritten as,

$$\dot{\mu} = \frac{1}{\hat{\Theta}_1\zeta_2}\left[-\omega_n^2(\zeta_1 - Y) + 2\xi\omega_n\hat{\Theta}_1(1-\mu)\zeta_2 - 2\xi\omega_n\hat{\Theta}_4 + \hat{\Theta}_6(1-\mu)^2\zeta_1 - \hat{\Theta}_7(1-\mu)\zeta_2\right] \tag{2.13}$$

Note that the controller (2.13) exhibits a singularity at $\zeta_2 = 0$, which may be considered to be far away from the desired equilibrium point (2.5). Initial conditions for the system, and the controller state, can always be appropriately set such that this singularity is conveniently avoided.
Following the developments in [1] (see also Sastry and Isidori [15]), we obtain the following expression for the closed-loop behavior of the output variable y,

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2(y - Y) = [\tilde{\Theta}_1, \ldots, \tilde{\Theta}_{14}]\begin{bmatrix} W_1(\zeta, \hat{\Theta}, \mu) \\ \vdots \\ W_{14}(\zeta, \hat{\Theta}, \mu) \end{bmatrix} \tag{2.14}$$

The equations constituting the parameter estimation error update law are summarized as follows:

$$\dot{\tilde{\Theta}}_i = -\dot{\hat{\Theta}}_i = -\gamma_i\frac{e_1\vartheta_i}{1 + \vartheta^T\vartheta} \quad ; \quad i = 1, \ldots, 14 \tag{2.15}$$

where,

$$\begin{aligned} W_i(\zeta,\hat{\Theta},\mu) &= 0 \ ; \ i=2,3,5,8,\ldots,14 \\ W_1(\zeta,\hat{\Theta},\mu) &= -2\xi\omega_n(1-\mu)\zeta_2+\zeta_2\dot{\mu} \\ W_4(\zeta,\hat{\Theta},\mu) &= 2\xi\omega_n \\ W_6(\zeta,\hat{\Theta},\mu) &= -\zeta_1(1-\mu)^2 \\ W_7(\zeta,\hat{\Theta},\mu) &= -\zeta_2(1-\mu) \end{aligned} \tag{2.16}$$

and,

$$e_1=\sum_{i=1}^{14}\tilde{\Theta}_i\vartheta_i = y - Y - \sum_{i=1}^{14}\hat{\Theta}_i\vartheta_i + \frac{1}{s^2+2\xi\omega_n s+\omega_n^2} * \left[\sum_{i=1}^{14}\hat{\Theta}_i W_i(\zeta,\hat{\Theta},\mu)\right] \tag{2.17}$$

The filtered regressor vector components ϑ_i are obtained as solutions of the following differential equations, with zero initial conditions,

$$\begin{aligned} \ddot{\vartheta}_i &= -2\xi\omega_n\dot{\vartheta}_i-\omega_n^2\vartheta_i+W_i(\zeta,\hat{\Theta},\mu) \\ & \quad i=1,4,6,7 \end{aligned} \tag{2.18}$$

Evidently,

$$\vartheta_i = 0 \quad \text{for} \ \ i=2,3,5,8,\ldots,14 \tag{2.19}$$

The adaptive dynamically synthesized duty ratio function, (2.13),(2.15)-(2.19), is now easily translated into an acutal PWM regulation scheme of the form (2.3), by means of high frequency sampling of the on-line genarated (computed) duty ratio function trajectory $\mu(t)$. Figure 3 depicts the proposed adaptive dynamical PWM feedback regulation scheme. Note that in such a feedback scheme, rather than using the average values of the state vector ζ_1, ζ_2, as demanded by the expression (2.13), the actual PWM regulated values of the state vector components, x_1, x_2, are used, for feedback purposes. The above PWM feedback control technique has been fully justified in [9]- [12] for a variety of nonlinear feedback regulation strategies.

2.2 Simulation Results

Simulations were carried out for a boost converter regulated by the proposed nonlinear adaptive dynamical feedback controller specified by (2.13),(2.15)-(2.19). The following parameter values were used in the simulation model, but otherwise they were assumed to be unknown: L = 0.020 H, C = 20 μF R = 30 Ω and E = 15V. The equilibrium value for the duty ration function, generated by the dynamical controller, was set to be $\mu = U = 0.6$. The corresponding "nominal" equilibrium point, for the average input inductor current and average output capacitor voltage, were taken to be , respectively, $\zeta_1 = Y = Z_1 = 3.125$Amp, $\zeta_2 = Z_2 = 37.5$Volts. The dynamically generated duty ratio function μ was used on a PWM controller of the form (2.3), with a sampling frequency of 5 KHz.

In order to test the robustness of the proposed adaptive feedback control scheme, the designed dynamical controller was used on a "perturbed" version of the boost converter. For this purpose a zero mean, computer generated, bounded stochastic signal η, was allowed to act as an unmodelled external perturbation input η to the converter in the following manner,

$$\begin{aligned} \dot{x}_1 &= -\theta_1(1-u)x_2+\theta_4+\eta \\ \dot{x}_2 &= \theta_2(1-u)\,x_1-\theta_3\,x_2 \\ y &= x_1 \end{aligned} \tag{2.20}$$

The peak to peak value of the "unmatched" perturbation signal, η, was set to be approximately 15% of the assigned simulation value used for the parameter E/L.
The desired second order asymptotically stable linear dynamics, (2.9), for the regulated input inductor current $y = x_1$ was characterized by the following parameters $\xi = 0.80$ $\omega_n = 500$.
The simulations, shown in Figure 4, depict the behavior of the nonlinear dynamical PWM regulated state trajectories $x_1(t)$, $x_2(t)$ for the parameter uncertain, externally perturbed, converter model (2.20). The smooth evolution of the dynamically generated duty ratio function, $\mu(t)$, is shown along with a small portion of the corresponding PWM regulated switch position trajectory $u(t)$.
Only the trajectories of the components of the vector of estimated parameters, $\hat{\Theta}(t)$, exhibiting significant variations from their arbitrarily assigned initial values, are shown in this figure. The adaptation gains γ_i ; $i = 4, 5, 6, 7$ were set to be $\gamma_1 = \gamma_4 = 9.0 \times 10^6$ and $\gamma_6 = \gamma_7 = 1$. Finally, a small portion of the, computer generated, external stochastic perturbation input signal η is depicted in the figure.
The PWM controlled state variables, x_1 and x_2, are seen to asymptotically converge towards the required equilibrium values, modulo the expected chattering associated to PWM regulation. In spite of the "unmatched" nature of the applied perturbation input, and its significant amplitude values, the system state variables are shown to exhibit remarkable robustness with respect to such an external perturbation signal. The control objective of indirectly driving the converter output capacitor voltage to a prespecifed equilibrium value with desirable stability features for the regulated variable is seen to be accomplished in a rather satisfactory manner.

3. Conclusions and Suggestions for Further Research

In this article a stable adaptive feedback control scheme has been proposed for the dynamical stabilization of minimum phase output signals in parameter uncertain dc–to–dc power supplies. The scheme constitutes an extension of the results given in [15], and by Sasty and Bodson in [16]. The developments, however, include the case in which the underlying linearizing feedback controller is dynamical.
The results were applied to the indirect regulation of non–minimum phase output capacitor voltages in a PWM controlled dc–to–dc power supply, such as the boost converter for which only static adaptive regulation schemes were available.
The proposed adaptive feedback regulation scheme is primarily based on the idealized infinte frequency sampling model, or state average model, of the PWM regulated converter. In spite of the idealization, the actual discontinuously controlled trajectories are seen to follow remarkably close the average closed loop trajectories for modestly high sampling frequencies. The scheme is also quite robust with respect to unmodelled external stochastic perturbation signals of bounded nature.
The problem of direct regulation of non–minimum phase output signals continues to be a challenging problem, specially for the case of parameter uncertain systems. In the switched mode dc–to–dc power converters case, the output capacitor voltage signal constitutes a non–minimum phase output on which the proposed regulation scheme is not applicable. In a sense, our approach of indirect output voltage regulation through stabilization of the input inductor current, already proposed in [5], also constitutes an application of the method recently proposed by Benvenuti *et al* in [17] (see also Fliess and Sira-Ramírez, [18] for a module theoretic justification of the method in linear systems).

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FIGURES

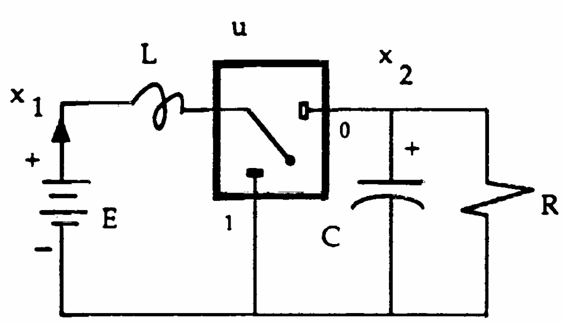

Figure 1. The boost converter.

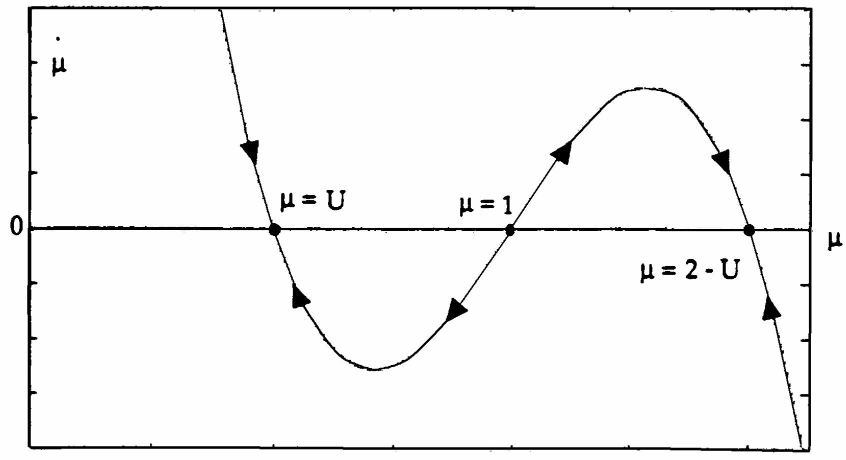

Figure 2. Zero dynamics for the boost converter.

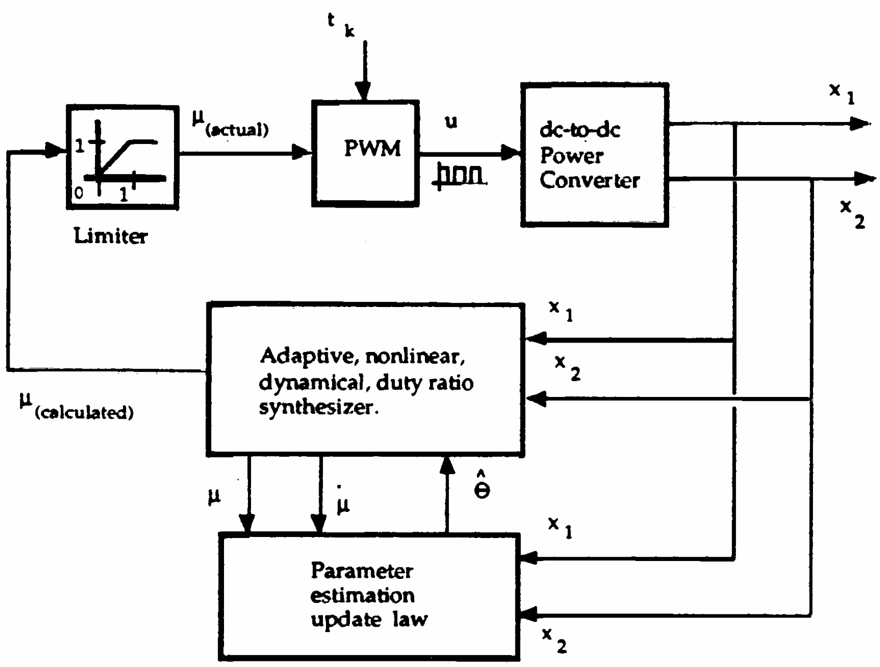

Figure 3. An adaptive dynamical PWM feedback control

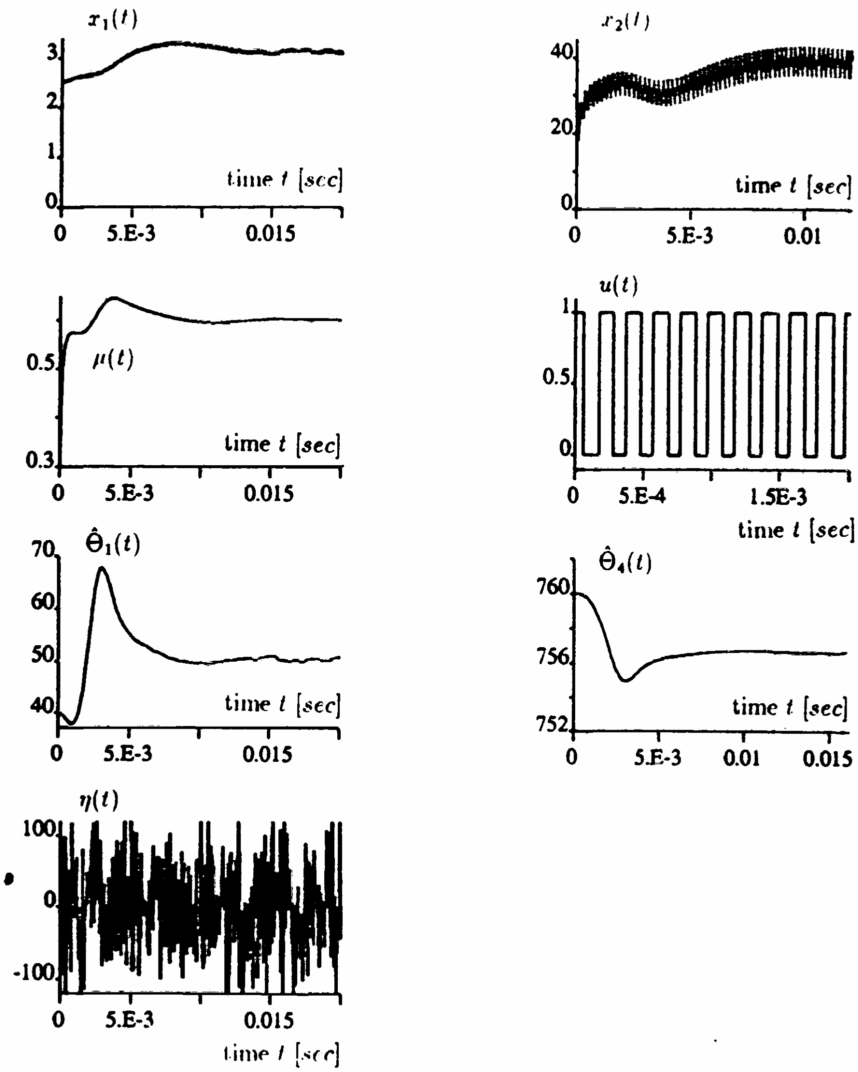

Fig. 4. Actual nonlinear adaptive PWM controlled state trajectories, duty ratio function trajectory, time evolution of the switch positions, trajectories of the estimated parameters and external perturbation signal for the boost converter example.