A Sliding Mode Control Approach to Predictive Regulation [1]

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Abstract

A sliding mode feedback control scheme, of dynamic nature, is proposed as an efficient alternative to robustly deal with the intrinsic tracking problem associated with every Model Based Predictive Control strategy. The results apply to nonlinear single-input single-output perturbed systems for which the tracking of a prespecified desirable output reference signal is required. The scheme is shown to handle efficiently large modelling errors and unmatched perturbation inputs

1. Introduction

The Model Based Predictive Control (MBPC) technique has received sustained attention, from both a theoretical as well as an applied viewpoint, ever since it was introduced by Richalet *et al* [1], 15 years ago. The technique has been developed over the years by many authors, specially by Clark *et al* [2]–[3], Richalet [4], Richalet *et al* [5], Bitmead *et al* [6]. Predictive control has received fundamental impetus towards its applicability in the chemical process industry by the research efforts of Morari and his coworkers [7]– [9] and García and Morari [10]. On the theoretical side, extensions to the nonlinear case, in fruitful combination with the concept of system inversion, have been presented by Abu el Ata and Fliess [11] and Abu el Ata *et al* [12]. A recent book on the subject is that by Soeterboek [13]. More recently, interesting developments, related to nonlinear optimal control theory, have been presented in a series of works by Mayne and Michalska [14]–[16].

In this article we develop an approach that uses an advantageous combination of dynamical sliding mode control (see Sira-Ramirez [17]) and input-output system inversion (Fliess [18]) in MBPC schemes. These techniques naturally blend together to yield a robust solution to the nonlinear output tracking problem associated with any predictive control scheme and defined within a prespecified prediction interval. Initial steps in this direction have also been taken by Sira–Ramírez and Fliess [19].

In section 2 a general description of the Predictive Functional Control problem using a dynamical, i.e., chattering–free, sliding mode control approach is presented. Section 3 contains an illustrative example along with digital computer simulations. Section 4 presents conclusions and suggestions for further work in this area.

2. Robust Predictive Control via a Dynamical Sliding-Mode Strategy

2.1. A sliding mode control result for scalar perturbed systems

Propostion 2.1 *Let W and N represent strictly positive quantities and let "sign" stand for the* signum *function. Suppose ν is a scalar bounded perturbation signal such that $|\nu| \leq N$. Then, the perturbed scalar discontinuous system :*

$$\dot{w} = \nu - W \text{ sign } w \tag{1}$$

globally exhibits a sliding regime *([20]) on $w = 0$, provided $W > N$. Furthermore, any trajectory starting on the initial value $w = w(0)$, at time $t = 0$, reaches the condition $w = 0$ in finite time T_r. An estimate of the reaching time T_r is given by :*

$$T_r \leq \frac{|w(0)|}{W - N} \tag{2}$$

Proof Immediate upon checking that globally : $w \, dw/dt < 0$ whenever $w \neq 0$, and $W > N$. This is a well known condition for the existence of a sliding regime [20]. The estimation of the reaching time in (2) is immediate upon integration of (1) and consideration of the most disfavorable perturbation case.

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2.2. Predictive Functional Control via Dynamical Sliding Modes

Consider a nonlinear n-dimensional single-input single-output dynamical system, expressed in GOCF [18]

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \eta_3 \\ &\cdots \\ \dot{\eta}_n &= c(\eta, u, \dot{u}, \ldots, u^{(\alpha)}) + \nu \\ y &= \eta_1 \end{aligned} \qquad (3)$$

where the scalar signal function ν comprises all known information about external bounded perturbation signals and an assessment of possible modeling errors. Note that a particular advantage of the GOCF is that perturbation signals are always *matched* with respect to the highest derivative of the control input, $u^{(\alpha)}$, which is taken as the effective control input signal in any dynamical feedback regulation scheme. This fact avoids the need for complying with the well known *matching conditions*, set almost 25 years ago in the work of Drazenovic [21].

The integer α in (3) is considered to be a strictly positive integer. For systems which are *exactly input-output linearizable*, i.e. where $\alpha = 0$, (see Isidori [22]), the same developments presented here are still applicable, except that, in order to obtain our proposed chattering-free responses, a first, or higher, order *dynamical extension* of the system becomes necessary (the concept of dynamical extension can be found in the book by Nijmeijer and van der Schaft [23])

The signal ν is assumed to satisfy :

$$\sup |\nu| \leq N \qquad (4)$$

Let $y_R(t)$ be a prescribed reference output function, assumed to be sufficiently smooth and defined over a given prediction interval $[0, Tp]$. Such an interval is determined below, in section 2.3.

Define a tracking error function, $e(t)$, as the difference between the actual system output, $y(t)$, and the output reference signal, $y_R(t)$:

$$e(t) = y(t) - y_R(t) \qquad (5)$$

We then have :

$$\begin{aligned} e^{(i)}(t) &= n_{i+1} - y_R^{(i)}(t) \;\; ; \;\; 0 \leq i \leq n-1 \qquad (6) \\ e^{(n)}(t) &= \dot{\eta}_n - y_R^{(n)}(t) = c(\eta, u, \ldots, u^{(\alpha)}) - y_R^{(n)} + \nu \end{aligned}$$

Defining $e_i = e^{(i-1)}$, $(i = 1, 2, \ldots, n)$, as components of an error vector $\mathbf{e}$, we may also express the tracking error system (5)–(6) in GOCF as :

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ &\cdots \\ \dot{e}_n &= c(\mathbf{e} + \xi_R(t), u, \dot{u}, \ldots, u^{(\alpha)}) - y_R^{(n)}(t) + \nu \\ e &= e_1 \end{aligned} \qquad (7)$$

with :

$$\begin{aligned} \xi_R(t) &= \mathrm{col}\left(y_R(t), y_R^{(1)}(t), \ldots, y_R^{(n-1)}(t)\right) \\ \mathbf{e} &= \mathrm{col}\,(e_1, e_2, \ldots, e_n) \end{aligned} \qquad (8)$$

The model-based predictive controller synthesis entitles the unambiguous specification of the (desired) system output tracking error, $e(t)$, within the specified prediction horizon $[0, Tp]$. This task is easily accomplished by prescribing a reduced order linear dynamical tracking error behaviour, which is known to asymptotically converge to zero, i.e. we specify the desirable tracking error dynamics as

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ &\cdots \\ \dot{e}_{n-1} &= -m_{n-1}e_{n-1} - \cdots - m_1 e_1 \\ e &= e_1 \end{aligned} \qquad (9)$$

where the set of real coefficients $\{m_1, \ldots, m_{n-1}\}$ is such that the following (characteristic) polynomial, in the complex variable "s ", is Hurwitz:

$$p(s) = s^{n-1} + m_{n-1}s^{n-2} + \cdots + m_2 s + m_1 \qquad (10)$$

We denote by μ the smallest real part, in absolute value, of all the complex stable roots of the polynomial equation $p(s) = 0$, associated with (10). The parameter μ actually represents the smallest time constant associated with the asymptotical exponentially stable decay of the controlled tracking errror response, under *ideal* sliding mode conditions (see [20]). Such a design parameter is used in the computation of the current prediction interval $[0, Tp]$ and, evidently, it may be specified *a priori* during the design stage (see also section 2.3).

The prescription of the desired linear tracking error dynamics (9), in turn, uniquely specifies a corresponding *sliding surface coordinate function* on the output tracking error phase space of the adopted model. In order to achieve such a desirable tracking error dynamics the coordinate e_n must satisfy, acording to (7) and (9),

$$e_n = -m_{n-1}e_{n-1} - \cdots - m_1 e_1 \qquad (11)$$

Motivated by this requirement, we next define an auxiliary scalar output variable w, in terms of the output tracking error coordinates e_i ; $(i = 1, \ldots, n)$ as,

$$w = e_n + m_{n-1}e_{n-1} + \cdots + m_1 e_1 \qquad (12)$$

Note that if the auxiliary output function w is driven to zero by means of a suitable control action, say, in finite time, then the desired error dynamics, specified in (9), is accomplished, and asymptotic exponential stability of the tracking error towards zero is obtained.

A dynamical discontinuous controller inducing a robust sliding motion on the zero level set of the proposed sliding surface, $w = 0$, may be found by standard *system inversion* performed on the unperturbed version of system (7) (i.e. by setting $\nu = 0$). Consider then the following dynamical feedback controller in terms of an implicit ordinary differential equation with discontinuous right hand side,

$$c(\xi_R + \mathrm{e}, u, \dot{u}, \ldots, u^{(\alpha)}) - y_R^{(n)} + \sum_{i=1}^{n-1} m_i e_{i+1} = -W \text{ sign } \left(\sum_{i=1}^{n} m_i e_i \right) \quad (13)$$

It easily follows, by taking the time derivative in (12), and using (7) that the controller (13) determines the following evolution of the auxiliary output function w:

$$\dot{w} = \nu - W \text{ sign } \left(\sum_{i=1}^{n} m_i e_i \right) = \nu - W \text{ sign } w \quad (14)$$

According to the result of Proposition 2.1, the controlled values of w go to zero in finite time and a sliding regime can be indefinitely sustained on the condition $w = 0$, provided $W > N$.

A truly *variable structure controller* is obtained from (13) since on each one of the regions: $w > 0$, and $w < 0$, a different *dynamic* feedback controller "structure" acts on the regulated system. The corresponding implicit differential equation (13) is to be independently solved for the controller u, on the basis of knowledge of the predicted error vector e and the vector of future desired output time derivative functions $\xi_R(t)$, computed, in turn, from knowledge of the future output reference trajectory $y_R(t)$. Under the additional assumption that, locally, $\partial c / \partial u^{(\alpha)}$ is non zero in (13), then no singularities, of the *impasse* points type need be locally considered ([12]). If singularities do arise they may be handled by the introduction of appropriate discontinuities on the dynamical controller output u (see Example 3.1).

Note that for $\alpha \geq 1$, the obtained sliding mode controller output u is actually *continuous*, rather than bang-bang. This result is nontypical in sliding mode control where, traditionally, bang-bang inputs, and its associated *chattering* output responses are usually obtained (See [20]).

After convergence to zero of the output tracking error, the dynamical controller exhibits the following *remaining dynamics* :

$$c(\xi_R, u, \dot{u}, \ldots, u^{(\alpha)}) = y_R^{(n)} \quad (15)$$

It is assumed that the nonlinear time-varying dynamics (15) is globally stable for the given desired output reference function $y_R(t)$. The dynamics (15) is, evidently, coincident with the *zero dynamics* (See Fliess [24] and also [22]) for those cases in which the desired value of the output function $y_R(t)$ is identically zero, or a given constant. In such cases, our previous assumption implies that the given system is locally, or globally, *minimum phase* [22] (see also [23]). In this last class of systems, an asymptotically stable response is obtained, as a solution of (15) for the control input u, towards a stable equilibrium value.

2.3. The prediction interval

The above procedure is evidently based on the validity of the available mathematical model for the system. Such a mathematical model, as usual, may be at variance with respect to the actual system behaviour. In using the predictive dynamical discontinuous controller on the actual system, one may generally obtain, at the end of the prediction horizon, a nonzero tracking error, or a nonzero sliding surface coordinate function value. These nonzero values are unknown functions of the model mismatch. The predictive control technique proposes then a number of procedures for obtaining an improvement, in the actual closed loop system behavior, for the next prediction interval, $[T_p, T'_p]$ (see [1], [4], [5]).

A reasonable choice for the setting of the new prediction horizon $[Tp, T'p]$ may be devised as:

$$T'_p = \frac{|w(T_p)|}{W - N} + \frac{2}{\mu} \quad (16)$$

i.e. the new prediction interval is comprised of the reaching time to the sliding surface, $w = 0$, computed from the sliding surface value based on the new reset tracking error initial conditions (see equation (12)), plus *twice* the slower time constant of the impossed linear error dynamics (this choice, roughly speaking, guarantees at the end of the new prediction horizon a theoretical decrease, in absolute value, of the slowest tracking error mode to about 13% of its initial value at the hitting of the proposed sliding surface).

The process described next is systematically repeated at the end of each prediction interval. Such process entitles,

1. Assessment of the actual values of the tracking error, or of the proposed stabilizing sliding surface,
2. Re–initialization of the desirable error dynamics in accordance with the obtained actual tracking error performance (this step may include a redesign of the parameters defining the desired tracking errors)
3. Calculation of the new prediction interval and, by direct system inversion techniques, calculation of the required sliding mode control policy.

4. Implementation of the reassessed control policy, and monitoring of the obtained response during the adopted planning horizon.

3. An Illustrative Example

Example 3.1 *A d.c. motor example*

Consider the following nonlinear model of a stator voltage controlled d.c. motor (see [22])

$$\begin{aligned} \dot{x}_1 &= -\frac{R_r}{L_r}x_1 + \frac{V_r}{L_r} - \frac{K}{L_r}x_2\,u \\ \dot{x}_2 &= -\frac{F}{J}x_2 + \frac{K}{J}x_1\,u \\ y &= x_1 \end{aligned} \tag{17}$$

where x_1 represents the armature circuit current and x_2 is the angular velocity of the rotating axis. V_r is the fixed voltage applied to the armature circuit, while u is the field winding input voltage, acting as a control variable. The constants R_r, L_r and K represent, respectively, the resistance, the inductance in the armature circuit and the torque constant. The parameters F and J are the viscous damping coefficient and the moment of inertia associated with the rotor.

An input–output representation of the system is obtained by elimination of the state vector (see Diop [25] for general results)

$$\begin{aligned} \ddot{y} &= -\frac{R_rF}{L_rJ}\,y - \left(\frac{F}{J}+\frac{R_r}{L_r}\right)\dot{y} + \frac{KV_r}{L_rJ}\,u \\ &\quad -\frac{K^2}{L_rJ}\,y\,u^2 + \frac{\dot{u}}{u}\left[\dot{y}+\frac{F}{J}\,y\right] \end{aligned} \tag{18}$$

Suppose it is desired to track a known angular velocity profile, or reference trajectory, $y_R(t)$.

The zero dynamics for $y=0$ degenerates into the algebraic condition $u=0$. Since a common objective in velocity control is to track reference trajectories that eventually include constant angular velocities, we consider the zero dynamics of the system associated with such constant values of y. Such a zero dynamics is readily obtained after setting the outut y to a constant equilibrium value, say Ω, and setting to zero the output derivatives $\dot{y}$; $\ddot{y}$. Hence, one obtains

$$\dot{u} - \frac{R_a}{L_a}\,u + \frac{KV_r}{\Omega L_rF}\,u^2 - \frac{K^2}{L_rJ}\,u^3 = 0 \tag{19}$$

Aside from the trivial equilibrium point $u=0$, there exists, for every constant angular velocity Ω, two other phisically meaningful equilibrium points for the zero dynamics, provided $V_a^2 > 4\,R_rF\Omega^2$. We denote here such equilibria by $u=U$. The minimum or non-minimum phase nature of a particular equilibrium point $u=U$ depends, respectively, on whether the quantity: $R_rF - K^2U^2$ exhibits a positive or negative value.

A GOCF representation for the tracking error dynamics, with state components defined by $e_1 = e = x_2 - y_R(t)$ and $e_2 = \dot{x}_2 - \dot{y}_R(t)$, is readily obtained as

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= -\frac{R_rF}{L_rJ}(e_1+y_R(t)) - \left(\frac{F}{J}+\frac{R_r}{L_r}\right)(e_2+\dot{y}_R(t)) \\ &\quad +\frac{KV_r}{L_rJ}\,u - \frac{K^2}{L_rJ}(e_1+y_R(t))\;u^2 \\ &\quad +\frac{\dot{u}}{u}\left[e_2+\dot{y}_R(t)+\frac{F}{J}(e_1+y_R(t))\right] - \ddot{y}_R(t) \end{aligned} \tag{20}$$

A predictive dynamical sliding mode controller is next designed by considering the sliding surface w as,

$$w = e_2 + m_1\,e_1 \tag{21}$$

Note that such a sliding surface is a nonlinear, time-varying, input-dependent sliding manifold of the form

$$w = -\frac{F}{J}\,x_2 + \frac{K}{J}\,x_1\,u - \dot{y}_R(t) + m_1\;(x_2 - y_R(t)) \tag{22}$$

Imposing the dynamics, $\dot{w} = -W$ sign w, on such a sliding surface coordinate w, one obtains the following dynamical predictive controller by inversion of the tracking error system equations,

$$\begin{aligned} \dot{u} &= \frac{u}{\left[e_2+\dot{y}_R(t)+\frac{F}{J}(e_1+y_R(t))\right]} \\ &\quad \left\{\frac{R_rF}{L_rJ}(e_1+y_R(t)) + \left(\frac{F}{J}+\frac{R_r}{L_r}\right)(e_2+\dot{y}_R(t))\right. \\ &\quad -\frac{KV_r}{L_rJ}\,u + \frac{K^2}{L_rJ}(e_1+y_R(t))\;u^2 + \ddot{y}_R(t) \\ &\quad \left. m_1\,e_2 - W\text{sign}\;w\right\} \end{aligned} \tag{23}$$

Simulations were performed for a d.c. motor with the following parameter values,

$$\begin{aligned} &R_r = 7.0\text{ Ohm} \;\; ; \;\; L_r = 120.0\text{ mH} \;\; ; \;\; V_r = 5.0\text{ V} \;\; ; \\ &F = 6.04\times10^{-6}\text{ N}-\text{m}-\text{s/rad} \; ; \\ &J = 1.06\times10^{-6}\text{ N}-\text{m}-\text{s}^2\text{/rad} \;\; ; \\ &K = 1.41\times10^{-2}\text{ N}-\text{m/A}. \end{aligned} \tag{24}$$

The following prescribed output trajectory, constituted by a picewise linear function, was proposed as the velocity profile to be followed by the motor's shaft angular velocity x_2

$$y_R(t) = \begin{cases} 300\text{ rad/sec for } t \le 0.5\text{s} \\ 300 - 100\,(t-0.5)\text{ rad/sec} \\ \qquad\qquad \text{for } 0.5\text{s} < t < 1.5\text{ s} \\ 200\text{ rad/sec for } t \ge 1.5\text{s} \end{cases} \tag{25}$$

In order to test the robustness of the proposed discontinuous predictive control scheme, we devised simulation trials on two unmodelled perturbation input cases, both of them corresponding to the *unmatched* perturbation type. Thus, the above controller was used in combination with the following (actual) nonadapted, perturbed, systems:

$$\begin{aligned} \dot{x}_1 &= -\frac{R_r}{L_r} x_1 + \frac{V_r}{L_r} - \frac{K}{L_r} x_2 u + \vartheta(t) \\ \dot{x}_2 &= -\frac{F}{J} x_2 + \frac{K}{J} x_1 u \\ y &= x_1 \end{aligned} \tag{26}$$

$$\begin{aligned} \dot{x}_1 &= -\frac{R_r}{L_r} x_1 + \frac{V_r}{L_r} - \frac{K}{L_r} x_2 u \\ \dot{x}_2 &= -\frac{F}{J} x_2 + \frac{K}{J} x_1 u + \vartheta(t) \\ y &= x_1 \end{aligned} \tag{27}$$

where the signal $\vartheta(t)$ was set to be a computer generated, normally distributed, white noise signal. The simulation results in both cases were highly encouraging and the performance obtained was remarkably robust. We only show the simulation results corresponding to the performance of the system (26), controlled by a predictive scheme that includes the dynamic sliding mode regulator (23).

Figure 1 shows the angular velocity response of the system in comparison with the desired trajectory (25) (shown in dashed lines). In spite of the unmatched nature of the perturbation signal ϑ, the controlled trajectory $y(t)$ follows, quite closely, the required angular velocity profile. Figure 2 depicts the corresponding armature circuit current x_1, while Figure 3 represents the bang-bang free control input signal u generated by the dynamical sliding mode predictive controller scheme. Figure 4) shows the applied perturbation input signal ϑ and, finally, Figure 5 depicts the corresespoding sliding surface evolution.

4. Conclusions

In this article a model based predictive control scheme has been proposed which combines the advantages of sliding mode control robustness, and its traditional high performance features, with the conceptual simplicity of nonlinear system inversion techniques. This association of these techniques was proven to be particularly suitable for conceptually dealing with the associated output tracking problem present in every predictive feedback control scheme.

The adopted framework of designing sliding controllers via a GOCF of the nonlinear system model, which is, fundamentally, an input-output design approach, results in the possibility of effectively compensating for bounded unmatched uncertainties.

The obtained results may be extended to the case of decouplable multivariable input-output systems. In this context research efforts are being directed to relate the approach proposed here with the theory of *differentially flat systems* (see the work of Fliess and his colleages in [26]-[28]). In forthcoming publications we will show that the combination of sliding mode control, model based predictive control and differentially flat systems, results in a most natural, and rather general, way to formulate, and design, robust predictive regulators for controllable nonlinear multivariable systems.

An illustrative single-input single-output example was presented in which a dynamical sliding mode control strategy is devised for robust error stabilization on non-adapted systems. The results show the insensitivity of the sliding mode controller to rather large (unmatched) external random perturbation signals.

The results also show that the advantageous combination of sliding modes, and perdictive control, results in an efficient feedback corrective scheme which accomplishes the desired control objectives.

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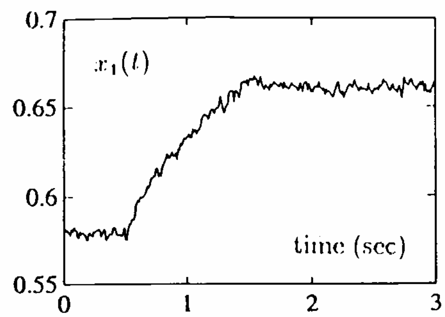

Figure 2: Armature circuit current response of predictive dynamical sliding mode controlled d.c. motor.

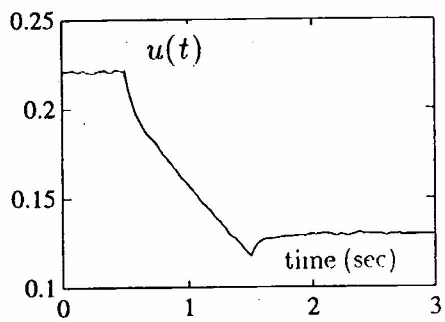

Figure 3: Dynamically generated bang–bang free control input signal for the d.c. motor example.

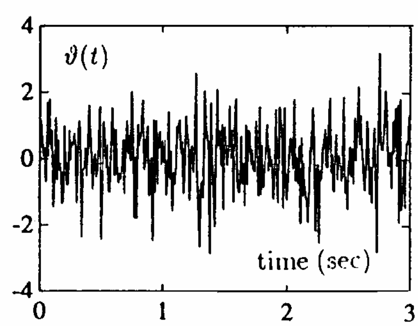

Figure 4: Unmatched perturbation input signal to the d.c. motor.

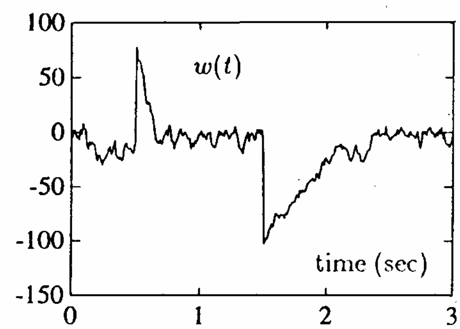

Figure 5: Evolution of the sliding surface coordinate for predictive dynamical sliding mode controlled d.c. motor.

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FIGURES

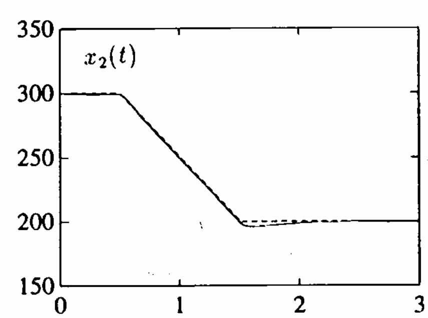

Figure 1: Angular velocity response of predictive dynamical sliding mode controlled d.c. motor.