Sliding Mode Observer-Controller Design

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Abstract

Motivated by an appropriate matched generalized state-space model, a robust sliding mode control is described which uses plant output information in conjunction with a particular sliding mode observer. The need for the usual structural matching constraints relating the input and output spaces, is circumvented. The proposed methodology is illustrated by considering a tutorial design example.

1. Introduction

Sliding mode observation and control schemes for both linear and nonlinear systems have caused considerable interest in recent times. Discontinuous nonlinear control and observation schemes, based on sliding modes, exhibit fundamental robustness and insensitivity properties of great practical value [11], [1]. A fundamental limitation found in the sliding mode control of linear perturbed systems and in sliding mode feedforward regulation of observers for linear perturbed systems, involves the necessity to satisfy some structural conditions of the "matching" type. These conditions have been widely recognized [11], [12], [2]. Such structural constraints on the system and the observer have also been linked to *strictly positive real* conditions [12], [13]. More recently, a complete Lyapunov stability approach for the design of sliding observers, where the above-mentioned limitations are also apparent, has been presented [3].

Here a different approach to the problem of output feedback control for any controllable and observable, perturbed linear system is taken. For the sake of simplicity, single-input single-output perturbed plants are considered, but the results can be easily generalized to multivarable linear systems. As is inherent in any sliding mode approach, the system must be relative degree one with respect to the measured output, and must also be minimum phase with respect to that output. However, this work does not impose any additional assumptions on the class of systems to which it may be applied.

Using a *Generalized Matched Observer Canonical Form* (GMOCF), similar to those developed in [5], it is found that, for the sliding mode state observation problem in observable systems, the structural conditions of the matching type are largely irrelevant. This statement is justified by the fact that a perturbation input "rechannelling" procedure *always* allows one to obtain a matched realization for the given system. Such rechannelling is never carried out in practice and its only possible purpose may be to obtain a reasonable estimate (bound) of the influence of the perturbation inputs on the state equations of the proposed canonical form. It is shown that the chosen matched output reconstruction error feedforward map, which is a design quantity, uniquely determines the stability features of the reduced order sliding state estimation error dynamics. The state vector of the proposed realization is, hence, robustly asymptotically estimated, independently of whether or not the matching conditions are satisfied by the original system.

The sliding mode output regulation problem for controllable and observable minimum phase systems, using a combination of a sliding mode observer and a sliding mode controller is then addressed. For this, a suitable modification of the GMOCF is proposed. The matched canonical form turns out to be, quite surprisingly, in traditional Kalman state space representation form. The resulting Matched Output Regulator Canonical Form (MORCF) is constructed in such a way that it is always matched with respect to the "rechannelled" perturbation inputs. The output signal of the system, expressed now in canonical form, is shown to be controlled by a suitable dynamical "precompensator" input, which is physically realizable. For the class of systems treated, the combined state estimation and control problem (i.e. output regulation problem) is therefore always robustly solvable by means of a sliding mode scheme, independently of any matching conditions.

In Section 2, a matched Generalized Observer Canonical Form, based on the input-output description of the given system, is proposed. In Section 3 a sliding mode control policy is presented for the matched Generalised Observer Canonical Form. It is shown that the chosen system realisation permits matching of the input with the disturbance. It is demonstrated in Section 4 that an observer which robustly estimates the required states may also be constructed. A robust closed-loop controller/observer pair is thus defined. A tutorial design example demonstrates the results of this paper in Section 5. Section 6 contains conclusions and suggestions for further research.

2. A Generalised Canonical Form for Robust Sliding Mode Control using State Reconstruction

Consider the following input-output representation of a linear time-invariant perturbed system

$$\begin{aligned} y^{(n)} + k_n y^{(n-1)} + \cdots + k_2 \dot{y} + k_1 y = \beta_0 u \\ + \beta_1 \dot{u} + \cdots + \beta_m u^{(m)} \\ + \gamma_0 \xi + \gamma_1 \dot{\xi} + \cdots + \gamma_q \xi^{(q)} \end{aligned} \tag{1}$$

where ξ represents the bounded external perturbation signal and the integer q is assumed to satisfy $q \leq n-1$.

The *Generalized Matched Observer Canonical Form* (GMOCF) of the above system is given by the following generalized state representation model [5]

$$\begin{aligned} \dot{\chi}_1 &= -k_1 \chi_n + \beta_0 u + \beta_1 \dot{u} + \cdots + \beta_m u^{(m)} + \lambda_1 \eta \\ \dot{\chi}_2 &= \chi_1 - k_2 \chi_n + \lambda_2 \eta \\ &\vdots \\ \dot{\chi}_{n-1} &= \chi_{n-2} - k_{n-1} \chi_n + \lambda_{n-1} \eta \\ \dot{\chi}_n &= \chi_{n-1} - k_n \chi_n + \eta \\ y &= \chi_n \end{aligned} \tag{2}$$

where η is an "auxiliary" perturbation signal, modelling the influence of the external signal ξ on every equation of the proposed system realization.

The relation existing between the signal η and its generating signal ξ, is obtained by computing the input-output description of system (2) in terms of the perturbation input η. The input-output description of the hypothesized model (2) is then compared with that obtained for the original system (1). This procedure results in a scalar linear, time-invariant, differential equation for η which accepts as an input the signal ξ.

The model presented below constitutes a realization of such an input-output description.

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_{n-1} &= -\lambda_1 z_1 - \lambda_2 z_2 - \cdots - \lambda_{n-1} z_{n-1} + \xi \\ \eta &= (\gamma_0 - \gamma_{n-1}\lambda_1) z_1 + (\gamma_1 - \gamma_{n-1}\lambda_2) z_2 + \cdots \\ &\quad + (\gamma_{n-2} - \gamma_{n-1}\lambda_{n-1}) z_{n-1} + \gamma_{n-1}\xi \end{aligned} \tag{3}$$

where $\gamma_{n-1} = 0$ for $q < n-1$.

Assumption 2.1 Suppose the components of the auxiliary perturbation distribution channel map $\lambda_1, ..., \lambda_{n-1}$, in equation (2), are such that the following polynomial in the complex variable s is Hurwitz

$$p_r(s) = s^{n-1} + \lambda_{n-1} s^{n-2} + \cdots + \lambda_2 s + \lambda_1 \tag{4}$$

Equivalently, Assumption 2.1 implies that the output of system (3) which generates the auxiliary perturbation η is a *bounded signal* for every bounded external perturbation signal ξ. If, for instance, ξ satisfies $||\xi|| < N$. Then, given N, the signal η satisfies $||\eta|| \leq M$ for some positive constant M. An easy to compute, although conservative, estimate for M is given by $M = \sup_{\omega \in [0,\infty)} ||G(j\omega)||\, N$ where $G(s)$ is the Laplace transfer function relating η to ξ in the complex frequency domain.

Remark It should be stressed that the purpose of presenting the state space model for the auxiliary perturbation signal η, which accepts as a forcing input the signal ξ, is to show how an estimate, through η, of the influence of ξ on the proposed state realization (2) of the original system (1) may be obtained. It should be noted that in terms of controller implementation, the model (3) plays no part.

The development of a sliding mode control strategy for the system realisation (2) will now be explored.

3. A Robust Sliding Mode Control Policy

In order to formulate the closed-loop control procedure, further consideration must be given to the system representation (1). Here it is assumed that the control objective is to force the system output y to zero in finite time. This objective will be realised by inducing a sliding mode upon $y = 0$. The objective can only be achieved if $m = n - 1$ with $\beta_{n-1} \neq 0$; the system must therefore have relative degree 1. In addition, the polynomial

$$q(s) = \beta_{n-1}s^{n-1} + \cdots + \beta_1 s + \beta_0 \tag{5}$$

must be Hurwitz so that the system is strictly minimum phase. It should be noted that the relative degree 1 and minimum phase assumptions are inherent in the sliding mode approach and are not particular to this discussion. Referring to the GMOCF presented in equation (2) it is necessary to consider the selection of a control such that the disturbance η acts wholly within channels which are transparently implicit in the input. This is necessary to ensure that the performance in the sliding mode will be independent of the external disturbance. Consider the introduction of a precompensator

$$\frac{\tilde{u}(s)}{\tilde{\nu}(s)} = \frac{s^{n-1} + \lambda_{n-1}s^{n-2} + \cdots + \lambda_2 s + \lambda_1}{\beta_{n-1}s^{n-1} + \cdots + \beta_1 s + \beta_0} \tag{6}$$

where $\lambda_1, ..., \lambda_{n-1}$ are as defined in (4) and ν is the auxiliary input to the system obtained from the precompensator. The matched realisation for control becomes, from (2),

$$\begin{aligned} \dot{\chi}_1 &= -k_1\chi_n + \lambda_1(\eta + \nu) \\ \dot{\chi}_2 &= \chi_1 - k_2\chi_n + \lambda_2(\eta + \nu) \\ &\vdots \\ \dot{\chi}_{n-1} &= \chi_{n-2} - k_{n-1}\chi_n + \lambda_{n-1}(\eta + \nu) \\ \dot{\chi}_n &= \chi_{n-1} - k_n\chi_n + \eta + \nu \\ y &= \chi_n \end{aligned} \tag{7}$$

Having obtained an appropriate matched realisation, it is now necessary to consider appropriate reachability conditions to ensure that the sliding mode, where $y = 0$, is both attained and maintained. The usual expectation of a sliding mode approach is the ability to specify the dynamic performance during sliding. In other words, the designer specifies a desirable reduced order dynamics for the system when sliding. Consider first ν defined by

$$\nu = -W\text{sign } y = -W\text{sign } \chi_n \tag{8}$$

where $W > M$ and M is an upper bound on the magnitude of the perturbation signal η. It may be shown that a sliding mode exists in the region

$$\chi_n = 0 \; ; \; \|\chi_{n-1}\| \leq \quad W - M \tag{9}$$

This control signal attains a sliding mode using only output information. However, although $y = 0$, the system dynamics are not prescribed in a desirable manner. In effect, a sliding patch results. Consider now ν defined by

$$\nu = -W\text{sign } \chi_n - \chi_{n-1} + k_n\chi_n \tag{10}$$

Here a global sliding mode exists on $\chi_n = y = 0$ for $W > M$. Furthermore the ideal sliding dynamics using this control configuration is determined by the characteristic polynomial $p_r(s) = 0$ where $p_r(s)$ as given in (4), is by definition Hurwitz. The designer thus has the ability to specify $p_r(s)$ to yield appropriate desirable performance. The equivalent feedforward signal, ν_{eq}, is obtained from the *invariance conditions* [1]

$$\chi_n = 0 \; , \qquad \dot{\chi}_n = 0 \tag{11}$$

One obtains from (11) and (7)

$$\nu_{eq} = -\chi_{n-1} - \eta \tag{12}$$

This is a *virtual* feedforward action that is not synthesized in practice, but which helps to establish the salient features of the *average* behaviour of the sliding mode regulated system. The resulting dynamics governing the state evolution on the sliding region are then ideally described by

$$\begin{aligned} \dot{\chi}_1 &= -\lambda_1\chi_{n-1} \\ \dot{\chi}_2 &= \chi_1 - \lambda_2\chi_{n-1} \\ &\vdots \\ \dot{\chi}_{n-1} &= \chi_{n-2} - \lambda_{n-1}\chi_{n-1} \\ y &= \chi_n = 0 \end{aligned} \tag{13}$$

The roots of the Hurwitz characteristic polynomial (4) are seen to determine the behaviour of the reduced order system (13), and an asymptotically stable behaviour to zero of the state components $\chi_1, \ldots, \chi_{n-1}$ is therefore achievable since the state χ_n undergoes a sliding regime on the relevant portion of the "sliding surface" $\chi_n = 0$.

However, the control strategy (10) requires knowledge of the state χ_{n-1} which is unmeasurable. The possibilities of reconstructing, in a robust manner, an estimate of this state will now be explored.

4. Robust State Observation

An observer for the system realization (7) is proposed as follows

$$\begin{aligned}
\dot{\hat{\chi}}_1 &= -k_1\hat{\chi}_n + h_1(y-\hat{y}) + \lambda_1(v+\nu) \\
\dot{\hat{\chi}}_2 &= -k_2\hat{\chi}_n + \hat{\chi}_1 + h_2(y-\hat{y}) + \lambda_2(v+\nu) \\
&\vdots \\
\dot{\hat{\chi}}_{n-1} &= -k_{n-1}\hat{\chi}_n + \hat{\chi}_{n-2} + h_{n-1}(y-\hat{y}) \\
&\quad + \lambda_{n-1}(v+\nu) \qquad (14)\\
\dot{\hat{\chi}}_n &= -k_n\hat{\chi}_n + \hat{\chi}_{n-1} + h_n(y-\hat{y}) + (v+\nu) \\
\hat{y} &= \hat{\chi}_n
\end{aligned}$$

Note that we have purposefully chosen exactly the same output error feedforward distribution map λ for the signal v, as that corresponding to the auxiliary perturbation input signal η and to the control input distribution map in (7). As a consequence, the matching conditions are satisfied by the proposed matched canonical realization (14). The observer has the following sliding mode feedforward regulated reconstruction error dynamics

$$\begin{aligned}
\dot{\epsilon}_1 &= -(k_1+h_1)\epsilon_n + \lambda_1(\eta - v) \\
\dot{\epsilon}_2 &= \epsilon_1 - (k_2+h_2)\epsilon_n + \lambda_2(\eta - v) \\
&\vdots \\
\dot{\epsilon}_{n-1} &= \epsilon_{n-2} - (k_{n-1}+h_{n-1})\epsilon_n \\
&\quad + \lambda_{n-1}(\eta - v) \qquad (15)\\
\dot{\epsilon}_n &= \epsilon_{n-1} - (k_n+h_n)\epsilon_n + (\eta - v) \\
\epsilon_y &= \epsilon_n
\end{aligned}$$

where ϵ_i represents the state estimation error components $\chi_i - \hat{\chi}_i$, for $i = 1, \ldots, n$.

In order to have a reconstruction error transient response associated with a preselected n th order characteristic polynomial, such as

$$p(s) = s^n + \alpha_n s^{n-1} + \cdots + \alpha_2 s + \alpha_1 \ , \qquad (16)$$

the gains h_i $(i = 1, \ldots, n)$ should be appropriately chosen as $h_i = \alpha_i - k_i$ $(i = 1, \ldots, n)$.

The feedforward output error injection signal v is chosen to be the discontinuous regulation policy

$$v = W \text{sign } \epsilon_y = W \text{sign } \epsilon_n \qquad (17)$$

where W is a positive constant. For a sufficiently large gain W, the proposed choice of the feedforward signal v results in a sliding regime on a region properly contained in the set expressed by

$$\epsilon_n = 0 \ , \qquad |\epsilon_{n-1}| \leq W - M \qquad (18)$$

The equivalent feedforward signal, v_{eq}, is again obtained from the invariance conditions

$$\epsilon_n = 0 \ , \qquad \dot{\epsilon}_n = 0 \qquad (19)$$

One obtains from (19) and the last of (15)

$$v_{eq} = \eta + \epsilon_{n-1} \qquad (20)$$

The equivalent feedforward signal is, generally speaking, dependent upon the perturbation signal η.

The resulting dynamics governing the evolution of the error system on the sliding region are then ideally described by

$$\begin{aligned}
\dot{\epsilon}_1 &= -\lambda_1\epsilon_{n-1} \\
\dot{\epsilon}_2 &= \epsilon_1 - \lambda_2\epsilon_{n-1} \\
&\vdots \\
\dot{\epsilon}_{n-1} &= \epsilon_{n-2} - \lambda_{n-1}\epsilon_{n-1} \qquad (21)\\
\epsilon_y &= \epsilon_n = 0
\end{aligned}$$

The resulting ideal sliding error dynamics exhibit, in a natural manner, a feedforward error injection structure of the "auxiliary output error" signal ϵ_{n-1}, through the design gains $\lambda_1, \ldots, \lambda_{n-1}$. As a result, the roots of the characteristic polynomial in (4) determining the behaviour of the homogeneous reduced order system (21), are completely determined by a suitable choice of the components of the feedforward vector, $\lambda_1, \ldots, \lambda_{n-1}$.

An asymptotically stable behaviour to zero of the estimation error components $\epsilon_1, \ldots, \epsilon_{n-1}$ is therefore achievable as the output observation error ϵ_n undergoes a sliding regime on the relevant portion of the "sliding surface" $\epsilon_n = 0$. The states of the estimator (14) are then seen to converge asymptotically towards the corresponding components of the state vector of the system realization (7).

The characteristic polynomial (4) of the reduced order observation error dynamics (21) coincides entirely with that of the transfer function relating the auxiliary perturbation model signal η to the actual perturbation input ξ. Hence, appropriate choice of the design parameters $\lambda_1, \ldots, \lambda_{n-1}$ not only guarantees asymptotic stability of the sliding error dynamics, but also ensures boundedness of the auxiliary perturbation input signal η, for any given bounded external perturbation ξ.

If the state χ_{n-1} is not directly available for measurement, the proposed feedback control (10) can be modified to employ the estimated state obtained from the sliding observer (14) as

$$\hat{\nu} = k_n y - \hat{\chi}_{n-1} - W \text{ sign } y \qquad (22)$$

where we have used the fact that the output y is clearly available for measurement. This control policy still results in a finite time convergence of y to zero as can be seen from the closed loop output dynamical equation

$$\begin{aligned} \dot{y} &= (\chi_{n-1} - \hat{\chi}_{n-1}) + \eta - W \text{ sign } y \\ &= \epsilon_{n-1} + \eta - W \text{ sign } y \end{aligned} \quad (23)$$

Since ϵ_{n-1} is decreasing asymptotically to zero, the output y is seen to go to zero, in finite time, for sufficiently large values of $W > M$.

The output observation error signal e_y, and the output signal y itself, are seen to converge to zero in finite time. The combined reduced order ideal sliding/ideal observer dynamics is obtained from the same invariance conditions $\chi_n = 0$, $\dot{\chi}_n = 0$ as before. This results precisely in the same equivalent control input and the same equivalent feedforward signals. The resulting reduced order ideal sliding/ideal observation error dynamics is still given by (13) and (21). The overall scheme is, thus, asymptotically stable.

5. Design Example

Consider the average Boost converter model derived by Sira-Ramírez and Lischinsky-Arenas [10]:

$$\begin{aligned} \dot{z}_1 &= -\omega_0 z_2 + \mu\omega_0 z_2 + b \\ \dot{z}_2 &= \omega_0 z_1 - \omega_1 z_2 - \mu\omega_0 z_1 \end{aligned} \quad (24)$$

where z_i, $i = 1, 2$ denote the corresponding "averaged components" of the state vector x where $x_1 = I\sqrt{L}$, $x_2 = V\sqrt{C}$ represent the normalized input current and output voltage variables respectively. The quantity $b = E/\sqrt{L}$ is the normalised external input voltage. The LC (input) circuit natural oscillating frequency and the RC output circuit time constant are denoted by $\omega_0 = 1/\sqrt{LC}$ and $\omega_1 = 1/RC$ respectively. The variable μ is the control input. The equilibrium points of the average model (24) are obtained as

$$\mu = U; Z_1(U) = \frac{b\omega_1}{\omega_0{}^2(1-U)^2}; Z_2(U) = \frac{b}{\omega_0(1-U)} \quad (25)$$

where U denotes a particular constant value for the duty ratio function. The linearisation of the average PWM model (24) about the constant operating points (25) is given by

$$\begin{aligned} \dot{z}_{1\delta} &= -(1-U)\omega_0 z_{2\delta} + \frac{b}{1-U}\mu_\delta \\ \dot{z}_{2\delta} &= (1-U)\omega_0 z_{1\delta} - \omega_1 z_{2\delta} - \frac{b\omega_1}{(1-U)^2\omega_0}\mu_\delta \end{aligned} \quad (26)$$

with

$$\mu_\delta(t) = \mu(t) - U; z_{i\delta}(t) = z_i(t) - Z_i(U), i = 1, 2. \quad (27)$$

Taking the averaged normalised input inductor current z_1 as the system output in order to meet the relative degree 1, minimum phase assumption, the following input-output relationship is obtained

$$\frac{z_{1\delta}(s)}{\mu_\delta(s)} = \omega_0 Z_2(U)\frac{s + 2\omega_0}{s^2 + \omega_1 s + (1-U)^2\omega_0{}^2} \quad (28)$$

The observer-controller pair (22), (14) is now implemented on the average boost converter model. For simulation purposes nominal parameter values of $R = 30\Omega$, $C = 20\mu F$, $L = 20mH$ and $E = 15V$ are assumed. The desirable set point for the average normalized input inductor current is $z_1 = 0.4419$ which corresponds to a constant value $U = 0.6$. In order to demonstrate the robustness of the approach, the effects of noise in both the input current and output voltage dynamics, has been considered. The system representation thus becomes, from (28),

$$\begin{aligned} \dot{z}_{1\delta} &= -632.46z_{2\delta} + 265.17\mu_\delta + \alpha\xi \\ \dot{z}_{2\delta} &= 632.46z_{1\delta} - 1666.67z_{2\delta} - 698.77\mu_\delta + \beta\xi \end{aligned} \quad (29)$$

Here α and β define the noise distribution channel which is not necessarily matched. The polynomial (4) which defines the auxiliary perturbation distribution map is chosen to be

$$p_r(s) = s + 3000 \quad (30)$$

The rate of decay of the reconstruction error dynamics, (16), is determined by the roots of the following characteristic polynomial

$$p(s) = s^2 + 8500s + 18000000 \quad (31)$$

Using (30) and (31) an observer (14) for the system is given by

$$\begin{aligned} \dot{\hat{\chi}}_1 &= 400000\hat{\chi}_2 + 17600000(y - \hat{y}) \\ &\quad + 3000(v + \vartheta) \\ \dot{\hat{\chi}}_2 &= -1666.67\hat{\chi}_2 + \hat{\chi}_1 + 6833.33(y - \hat{y}) \\ &\quad + (v + \vartheta) \\ \hat{y} &= \hat{\chi}_2 \\ v &= W_{obs}\text{sign}(y - \hat{y}) \end{aligned} \quad (32)$$

The following state-space realisation may be used to determine the plant input μ_δ

$$\begin{aligned} \dot{w} &= -3333.33w + 0.0038\vartheta \\ \mu_\delta &= -333.33z + 0.0038\vartheta \\ \vartheta &= -W_{con}\text{sign } y - \hat{\chi}_1 + 1666.67y \end{aligned} \quad (33)$$

The magnitude of the discontinuous gain elements W_{con} and W_{obs} were chosen to be 120 and 220 respectively. These were tailored to provide the required speeds of response as well as appropriate disturbance rejection capabilities. Using a disturbance distribution map defined by $\alpha = 0.01$ and $\beta = -0.02$, which is clearly unmatched with respect to the input and output distributions of the system realisation (29), and a high frequency cosine representing the system noise, the following simulation results were obtained. Fig. 1 shows the convergence of the estimated inductor current to the actual inductor current. A sliding mode is reached whereby $z_1(t) - Z_i(t) = 0$. The required set point is thus attained and maintained despite the disturbance which is acting upon the system. Fig. 2 shows the control effort, μ. The discontinuous nature of this signal supports the assertion that a sliding mode has been attained.

6. Conclusions

In this article it has been shown that, when using a sliding mode approach, structural conditions of the *matching type*, are largely irrelevant for robust state reconstruction and regulation of linear perturbed systems. In other words, the class of linear systems for which robust sliding mode output feedback regulation can be obtained, independently of any *matching conditions*, comprises the entire class of controllable (stabilizable) and observable (reconstructible) linear systems which have relative degree one for the measured output and are minimum phase with respect to this measured output.

This result, first postulated by Sira-Ramírez and Spurgeon in [9], is of particular practical interest when the designer has freedom to propose a convenient state space representation for a given unmatched system. This is in total accord with corresponding results regarding, respectively, the robustness of the sliding mode control of perturbed controllable linear systems, expressed in the *Generalized Observability Canonical Form* [6], and the dual result for the sliding mode observation schemes based on the *Generalized Observer Canonical Form* [8].

Sliding mode output regulator theory (i.e. one considering an observer-controller combination) for linear systems may also be examined from an algebraic viewpoint using *Module Theory* [4]. The conceptual advantages of using a module theoretic approach to sliding mode control have also been recently addressed [7]. The module theoretic approach can also give further generalizations and insights related to the results presented.

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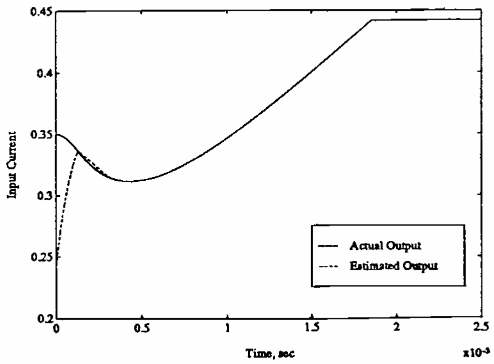

Figure 1: Response of the actual and estimated average normalized inductor current

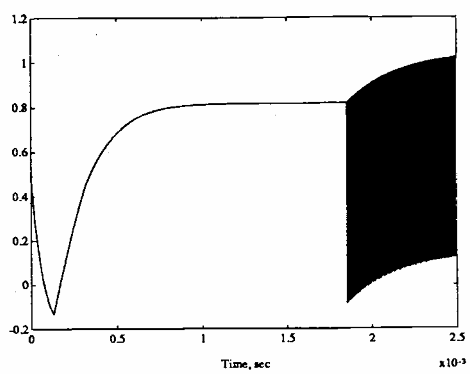

Figure 2: Response of the control effort μ