# Adaptive Switching Control for the Stabilisation of a Power System<sup>1</sup>

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#### Abstract

An adaptive switching control strategy is proposed for the stabilisation of a power system. The switching control strategy consists of two components: an adaptive sliding control which enables a local sliding mode with fast transient response, and a switching control which enables reachability of the local sliding mode. Simulations are presented.

#### 1. Introduction

Generators for an AC power system must rotate at exactly the same speed, there is no average power flow if the generators are not in synchronism. Any disturbance will cause the angles between generators to oscillate and, for sufficiently small disturbances, the oscillations eventually decay. Large disturbances can cause loss of synchronism and require the breaking of the connection between the generators with consequent loss of supply to customers. Analysis tools have been developed to predict the oscillations between generators following disturbances such as lightning strikes to transmission lines [1]. The original form of suppression of oscillations was based on control of the field

current of the generators which then influenced terminal voltage and hence power flow [2]. In contrast to the slow speed of response and the localisation of excitation control to generator sites, the fast response and flexible location of Static Var Shunt Compensators (SVCs) makes them a desirable asset in the suppression of oscillations of groups of generators [3]. These SVCs can be based on phase control of inductors in parallel with capacitors [4]. The system effectively becomes a variable capacitor from the line to ground and influences power flow by changing effective series line impedance and by changing the voltage at load busses which tends to influence the load drawn. A more recent development has been the controlled series compensation of transmission lines. This has been shown to be very effective in the control of power flow and hence in stability enhancement of power systems [5]. The control for excitation, shunt and series compensators has traditionally been by linear control laws with a limited control action, restricted by the rating

An adaptive switching control design is examined in this paper with the expectation of improved performance for a wide range of operating conditions. The model is of a single machine [14] with constant voltage behind machine reactance, and we focus on the design of a switching controller for supression of oscillations in the post fault system. The controller structure is same as in [14], which consists of a sliding controller

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and a switching controller which enables reachability of the local sliding mode. However in this article, instead of using a constant switching line, the sliding controller employes an adaptively adjusted switching line that enables fast transient response. The control is represented by the pulsed addition of inductance in parallel with the compensating capacitor.

This article is organised as follows. Section 2 presents the main results of the article about the adaptive switching control strategy referred to the pulsed feedback stabilization of the power system. Section 3 contains the simulation results of the proposed control scheme. Conclusion is drawn in Section 4.

## 2. An Switching Control Solution to the Power System Stabilization Problem

#### 2.1. Problem formulation and existing results

The model of the single machine system is based on the difference between the electrical output power Pelec and the shaft power Ps from the turbine resulting in acceleration of the combined rotating inertia (see Figure 1). The electrical power flow between two ac voltages  $E_1$  and  $E_2$  at the same frequency separated by angle  $\delta$  over an effective reactance  $X_e$ . The effective reactance at mains frequency of the line plus compensator  $X_e$ , varies then between two corresponding possble values X and  $X + \epsilon$  where X is addressed as the fully compensated reactance and  $X + \epsilon$  is referred to as the uncompensated reactance. The power flow can thus be modelled by the introduction of a discrete-valued control input variable u, representing the thyristor switching action, as

$$P_{elec} = \frac{E_1 E_2}{X} \left( 1 - u \frac{\epsilon}{X + \epsilon} \right) \sin \delta \tag{1}$$

The control unput u, thus, takes values in the set  $\{0,1\}.$ 

The angle  $\delta$  is the angle of the rotor of the machine so the acceleration of the machine from the normal synchronous frequency is  $\ddot{\delta}$ . For a rotational inertia J expressed in per unit form and normalised for synchronous speed J, and provided we ignore the small damping component provided by machine damper bars, the dynamics of the angle  $\delta$  is given by

$$J\ddot{\delta} = P_s - P_{elec} = P_s - \frac{E_1 E_2}{X} \left( 1 - u \frac{\epsilon}{X + \epsilon} \right) \sin \delta$$
 (2)

Equation (2) is referred as the swing equation. This equation can also be written as

$$\begin{aligned}
\dot{\delta}_1 &= \delta_2 \\
\dot{\delta}_2 &= \frac{P_s}{J} - \frac{E_1 E_2}{JX} \left( 1 - u \frac{\epsilon}{X + \epsilon} \right) \sin \delta_1 \quad (3)
\end{aligned}$$

For the sake of simplicity, we denote  $\delta = (\delta_1, \delta_2)^T$  as the state of the swing equation (3).

Notice that the equilibrium points corresponding to the uncontrolled system (i.e. with u = 0), are given by  $(\delta_{\bullet}^{n}, 0)^{T}$ , where

$$\delta_s^n = n\pi + (-1)^n \sin^{-1} \left( \frac{P_s X}{E_1 E_2} \right), \quad n = 0, \pm 1, \pm 2, \cdots$$
(4)

The equilibrium point for n = 0,  $(\delta_s^0, 0)^T$ , with

$$\delta_s^0 = \sin^{-1}\left(\frac{P_s X}{E_1 E_2}\right) \tag{5}$$

is deemed as desirable for the steady state operation of the system. without loss of generality, we assume that  $0 \le \delta_{\star}^{0} \le \pi/2$ . Notice also that, necessarily, such an equilibrium actually exists whenever  $P_{\bullet}X < E_1E_2$ . Similarly for the uncompensated reactance case whenever  $P_s(X + \epsilon) < E_1 E_2$ , an equilibrium exists. We also assume that  $0 < \sin^{-1}(P_s(X+\epsilon)/E_1E_2) < \pi/2$ .

Note that the equilibrium points with  $\delta_s^{-1}$  ( $\delta_s^{-1}$  =  $-\pi - \delta_s^0$ ) and  $\delta_s^1$  ( $\delta_s^1 = \pi - \delta_s^1$ ) are unstable, which are actually saddle points. There is a separatrix through  $\delta_{\cdot}^{1}$ which separates the closed orbits representing periodic solutions, from unbounded orbits [12]. Since the saddle point  $\delta_s^1$  is connected with itself by the separatrix, the orbit is also called homoclinic orbit. Therefore the stability region we are concerned, denoted as  $\Omega_{sr}$ , is bounded within the range  $(\delta_{sp}, \delta_s^1)$ , where  $\delta_{sp}$  and  $\delta_s^1$ are the two intersection points of the homoclinic orbit with the axis  $x_2 = 0$ . Within this region, the orbits are bounded.

For the design of sliding mode control, we introduce some desired dynamics

$$s = \delta_2 + \lambda(\delta_1 - \delta_s^0), \ \lambda > 0 \tag{6}$$

and we have the following result [14]:

Propostion 2.1 If the sliding mode control is

$$u_{ss} = \begin{cases} 0 & \text{for } \sin \delta_1 s > 0 \\ 1 & \text{for } \sin \delta_1 s < 0 \end{cases}$$
 (7)

then the sliding mode s = 0 defined by (6) exists only in the existence region

$$\Omega_{ss} = \{ (\delta_1, \delta_2) \mid l_1(\delta) \le 0, \ l_2(\delta) \ge 0 \}$$
 (8)

where the two delimiting lines are

$$l_1(\delta) = \frac{P_s}{J} - \frac{E_1 E_2}{JX} \sin \delta_1 + \lambda \delta_2 = 0$$
 (9)  
$$l_2(\delta) = \frac{P_s}{J} - \frac{E_1 E_2}{J(X + \epsilon)} \sin \delta_1 + \lambda \delta_2 = 0$$
 (10)

$$l_2(\delta) = \frac{P_s}{J} - \frac{E_1 E_2}{J(X + \epsilon)} \sin \delta_1 + \lambda \delta_2 = 0 \quad (10)$$

Figure 2 shows the construction of  $\Omega_{ss}$  and  $\Omega_{sr}$ .

Because of the localisation of the sliding mode, the following switching controller was designed to ensure the asymptotical stability of the system in  $\Omega_{sr}$  [14]:

Propostion 2.2 If the switching control strategy is

$$u = \begin{cases} u_{sr} & \text{if } \delta \in \Omega_{sr} \backslash \Omega_{ss} \\ u_{ss} & \text{if } \delta \in \Omega_{ss} \end{cases}$$
 (11)

where

$$u_{ss} = \begin{cases} 0 & for \sin \delta_1 \ \delta_2 > 0 \\ 1 & for \sin \delta_1 \ \delta_2 < 0 \end{cases}$$
 (12)

then the system (3) is asymptotically stable and exhibits the sliding mode s=0 in  $\Omega_{ss}$ .

## 2.2. The adaptive switching controller

The problem with the local sliding controller in Section 2.1 is that the switching line exhibits an exponential decay with time because of its constant line slope. It has been proved [13] that for second-order linear systems an adaptive relay control scheme enables a near time optimal response. It is desirable that the similar adaptation mechanism can be introduced to the power system stabilisation problem to achieve a fast transient response without oscillations.

Similar to the adaption scheme [13], we employ the following: Initially set  $\lambda = \lambda_0 > 0$ . If the sliding motion occurs, the current switching line

$$s_i = \lambda_i (\delta_1 - \delta_s^0) + \delta_2 = 0 \tag{13}$$

is then rotated to be just ahead of the state point, i.e.

$$s_{i+1} = \lambda_{i+1}(\delta_1 - \delta_s^0) + \delta_2 = 0 \tag{14}$$

where

$$\lambda_{i+1} = \lambda_i (1+\mu) \tag{15}$$

with a very small positive value  $\mu$ . If the system is not sliding, the switching line is kept fixed. Note that when  $\lambda$  is changing, the existence region of sliding mode, denoted as  $\Omega_{ss}^{\lambda}$ , is changing as well because the two delimiting lines are changing.

To determine the so called *sliding boundary locus* [13] we use the switching line

$$s = \lambda(\delta_1 - \delta_s^0) + \delta_2 = 0 \tag{16}$$

and the delimiting line

$$\lambda = -\frac{P_s}{J\delta_2} + \frac{E_1 E_2}{J(X + \epsilon)\delta_2} \sin \delta_1 \tag{17}$$

which passes the point  $(\sin^{-1}(P_s(X+\epsilon)/(E_1E_2)), 0)$ . The other delimiting line is of little use because the state near the line in  $\Omega_{ss}^{\lambda}$  always tends to leave  $\Omega_{ss}^{\lambda}$ .

Assume that the system state satisfies (16) and (17), and consider that  $\lambda$  varies, eliminiting  $\lambda$  in (16) and (17) yields the sliding boundary locus

$$\delta_2^2 + (\delta_1 - \delta_s^0)(-\frac{P_s}{J} + \frac{E_1 E_2}{J(X + \epsilon)} \sin \delta_1) = 0$$
 (18)

Differentiating  $\lambda$  with respect to time yields

$$\dot{\lambda} = \frac{\delta_2^2 - \dot{\delta_2}(\delta_1 - \delta_s^0)}{(\delta_1 - \delta_s^0)^2} = \lambda(\lambda + \frac{\dot{\delta_2}}{\delta_2})$$
 (19)

Differentiating (18) and solving  $\delta_2$  yields

$$\dot{\delta}_2 = -0.5\lambda - 0.5(\delta_1 - \delta_s^0) \frac{E_1 E_2}{J(X + \epsilon)} \cos \delta_1 \qquad (20)$$

Since  $0 \leq \delta_s^0 \leq \pi/2$ ,  $\lambda > 0$ , and for  $\Omega_{ss}^\lambda$   $\delta_2 < 0$ , and also  $0 < \delta_1 < \sin^{-1} P_s(X+\epsilon)/E_1E_2 < \pi/2$ , therefore  $\dot{\lambda} > 0$ . This means that the adaptive adjustment of  $\lambda$  is always towards increasing the switching slope  $\lambda$ . Denote  $\Sigma_\lambda = \{\lambda_0, \lambda_1, \cdots\}$  the set of the sequence of the switching slopes, then the existence region of sliding mode is

$$\Omega_{ss} = \bigcap_{\lambda \in \Sigma_{\lambda}} \Omega_{ss}^{\lambda} \tag{21}$$

which is depicted in Figure 3. Ideally when  $\mu \to 0$  and the sliding motion can be detected instantaneously so that the adaptive adjustment can be carried out continuously in time, the path in  $\Omega_{ss}$  before reaching the sliding boundary locus is actually the path of the system with u=0 (see Figure 3). The fast transient response is realised because with the constantly increasing  $\lambda$ , the time taking a state to the equilibrium point becomes shorter and shorter.

The above analysis is for those states which are accidentally above an initial switching line. The problem is that the system state may enter the existence region  $\Omega_{ss}$  at a point on the sliding boundary locus (for example C'' in Figure 3) that may be below an initial prescribed switching line. This may slow down the convergence speed towards the equilibrium point. To overcome this problem, the switching line should be initialised as follows: suppose the system trajectory at some instant reaches the sliding boundary locus at  $(\delta_1^c, \delta_2^c)^T$ , then the initial switching line should be set up as

$$s = \delta_2 + \lambda(\delta_1 - \delta_s^0) \tag{22}$$

$$\lambda = -\frac{\delta_2^c}{\delta_1^c - \delta_3^0} \tag{23}$$

The above analysis is summarised in the following proposition.

Propostion 2.3 If the variable structure control is given by

$$u_{ss} = \begin{cases} 0 & \text{for } \sin \delta_1 \ s > 0 \\ 1 & \text{for } \sin \delta_1 \ s < 0 \end{cases}$$
 (24)

where the switching line is adaptively adjusted based on the adaptation mechanism described above, then when the system state enters  $\Omega_{ss}$ , it will eventually converge to the equilibrium pont  $(\delta_s^0, 0)^T$  along the sliding boundary locus (18).

The reachability of the existence region of sliding mode can also be proved using the similar reasoning for Proposition 2.2 [14]. It is omitted here.

#### 3. Simulation Results

For the simulation studies of the adaptive switching control strategy, the following values of parameters were chosen in p.u.

$$P_{s} = 1$$
,  $J = 0.06$ ,  $X = 0.5$ ,  $E_{1} = 1$ ,  $E_{2} = 1$ ,  $\epsilon = 0.2$ 

The sampling rate was set to 10ms to model the fact that the thyristor firing can only occur once every 10ms for a 50Hz power system. The values of X=0.5 and  $\epsilon=0.2$  correspond to effective line reactances varying in the range of 0.5 to 0.7 p.u.. The steady state operating point u=0 is chosen to give the minimum effective line reactance and hence the minimum prefault angle thus minimizing the first swing angle for a given disturbance.

Figure 4 gives the simulation results showing the effectiveness of the adaptive switching control strategy. The initial state was chosen as  $\delta_1(0) = 0.3$ ,  $\delta_2(0) = 0$ . Comparing with the simulation with a fixed  $\lambda = 1$  where the system state  $\delta_2$  reaches zero at about t = 3 seconds, here  $\delta_2$  reaches zero at about t = 1.35 seconds, a much fast transient response.

Figure 5 illustrates the simulation results with a different initial state  $\delta_1(0) = 0.8$ ,  $\delta_2(0) = -0.3$ . The system state approaches the sliding boundary locus from below. Again, a fast response is shown.

#### 4. Conclusion

In this article an adaptive switching control stragegy has been proposed for the stabilization of a simple power system consisting of an a.c. generator and a single transmission line. The feedback strategy is constituted by the pulsed addition of inductive impedance to the transmission line through a suitable arrangement of thyristors. It has been shown that an adaptive local sliding controller enables fast transient response.

Discontinuous feedback control techniques enjoy great robustness advantages and simplicity of implementation when compared with more traditional feedback controller design schemes. In forthcoming publications we shall extend the methodology developed to multimachine system stabilization problem.

#### References

[1] Anderson, P.M., and A. A. Fouad, *Power System Control annd Stability*, Iowa State University Press Ames, IOWA, 1977.

- [2] DeMello, F.P., and C. Concordia, "Concepts of Synchronous machine stability as affected by excitation control", *IEEE Trans PAS*, **PAS-88**, 316-329(1969).
- [3] O'Brien, M., and G. Ledwich, "Static VAR Compensator controls for Improved System Stability", *IEE Proc C*, **134**, No. 1 (1987).
- [4] Torseng, S., "Shunt Connected Reactors and Capacitors Controlled by Thyristors", *IEE Proc C*, 128, No. 6(1981).
- [5] Johnson, R., D. Torgerson, D. Christl, P. Lutzelberger, K. Renz, K. Sadek, and S. Weiss, "Advanced Series Compensation provides Flexibility for AC Transmission Systems" CIGRE 1993 Region Meeting, Gold Coast Australia 242-250(1993).
- [6] Ghosh, A., G. Ledwich, O. P.Malik, and G.S. Hope, "Power system stabilizer based on adaptive control techniques", *IEEE Transactions on Power Apparatus and Systems*, **PAS-103**, 1983-1989(1984).
- [7] Erche, M., E. Lerch, D. Povh, and R. Mihalic, "Enhancement of Power system performance using power electronic equipmen", CIGRE (Conference International a Gros Haut Riseau Electric) 14/37/38-02 (1992).
- [8] DeCarlo, R.A., S.H. Zak, and G.P. Matthews, "Variable structure control of nonlinear multivariable systems: a tutorial", *Proceedings of IEEE*, 76, 212-232(1988).
- [9] Utkin, V.I., "Variable structure systems with sliding modes", *IEEE Transactions on Automatic Control*, AC-22, 212-222(1977).
- [10] Flugge-Lotz, I., Discontinuous control of nonlinear systems, Princeton University Press, 1953.
- [11] Zinober, A.S.I., Deterministic Control of Uncertain Systems, Peter Peregrinus Ltd, 1990.
- [12] Drazin, P.G., Nonlinear Systems, Cambridge University Press, 1992.
- [13] Zinober, A.S.I., "Adaptive relay control of second-order systems", *International Journal of Control*, 21, 81-98(1975).
- [14] Yu, X., H. Sira-Ramírez and G. Ledwich, "Switching control strategy for the power system stabilisation problem", submitted for publication.

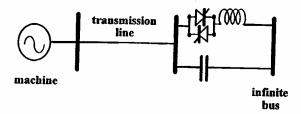


Figure 1

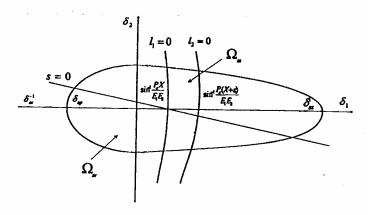


Figure 2

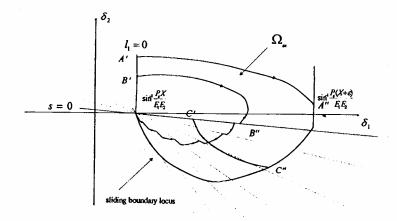


Figure 3

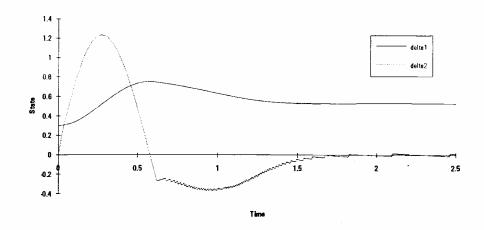


Figure 4

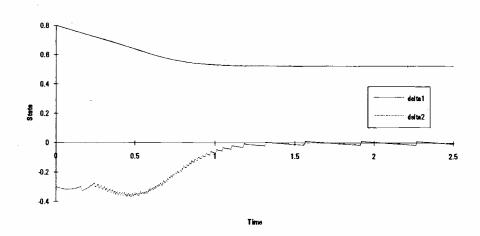


Figure 5