On the order of observable monovariable systems obtained by sliding mode control strategies

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Abstract

A systematic procedure is proposed for the determination of the system relative degree using a growing family of sliding surface candidates in the phase space of the system. A sliding regime exists on the zero level set on a candidate surface if, and only if, it is relative degree one. The procedure yields the order if, and only if, the system is differentially flat.

1. Introduction:

An important problem in the automatic regulation of systems is the determination of the orders of the associated system model (system dimension n, relative degree n* and dimension of zero dynamics α=n-n*) whose performance is to be improved by means of state feedback. This usually considers two possible operating modes: stabilisation and tracking, for repetitive or non repetitive tasks. In this article, some new concepts introduced by Fliess [1] [2] [3], Sira-Ramirez [4] and Messager [5] (Generalized Controller Canonical Form, linearizing dynamical state feedback and generalized variable structure system dynamics) will be used to propose a discontinuous feedback approaches for the determination of system orders. In particular *sliding modes* will be used [7].

A *sliding surface* may be defined in terms of state or phase variables depending on observability properties of the system. Nonlinear sliding surface may be proposed in general. However the sliding manifold is usually defined as an hyperplane due to simplicity of synthesis and the possibility of inducing closed loop linear dynamics.

The objective of this paper is to show that, in the case of a free system fedback with a discontinuous control, the analysis of the sliding surface coordinate allows to validate the local dimensions of the presumed model associated to the system.

2. Mathematic formulation of the problem:

Consider an n-dimensional monovariable dynamic system of the form:

$$\begin{cases} \dfrac{dx}{dt} = f(x,u) \\ y = h(x) \end{cases} \tag{1}$$

where $u \in R$, $x \in R^n$, $y \in R$ are respectively the input, the state and the output. Under very mild conditions ([12] Conte, Moog & Perdon) there exists an input-dependent state coordinate transformation:

$$z = \Phi(x, u, \dot{u}, \ldots, u^{(\alpha-1)}) \tag{2}$$

that places system (1) in generalized observability canonical form (GOCF)

$$\begin{cases} \dot{z}_i = z_{i+1} \quad i = 1,\dots,n-1 \\ \dot{z}_n = C(z,u,\dot{u},\dots,u^{(\alpha)}) \\ y = z_1 \end{cases} \tag{3}$$

where α=n-r is the dimension of the *zero dynamics* ($C(0,u,\dot{u},\dots,u^{(\alpha)}) = 0$). In case α=0, the system is said to be *differentially flat* [13] with linearizing output given by y=h(x).

The relations of the GOCF with Isidori [14] normal canonical form are clear from the fact that only state dependent coordinate transformations are allowed $(\eta,\xi) = \psi(x)$. Such that $\frac{\partial}{\partial u}\left[\dot{\xi}\right] = 0$, which take the system into:

$$\begin{cases} \dot{\eta}_i = \eta_{i+1} \quad i = 1,\dots,r-1 \\ \dot{\eta}_r = \Theta(\eta,\xi,u) \\ \dot{\xi} = \sigma(\xi,\eta) \\ y = \eta_1 \end{cases} \tag{4}$$

The autonomous differential equation $\dot{\xi} = \sigma(\xi,0)$ is also the zero dynamics of dimension α=n-r.

A stabilizing sliding surface can be readily proposed for the above system as:

$$s = \lambda_1\eta_1 + \lambda_2\eta_2 + \dots + \lambda_{r-1}\eta_{r-1} + \eta_r \tag{5}$$

such that the set $\{\lambda_1,\lambda_2,\dots,\lambda_{r-1},1\}$ constitute coefficients of an r-th order Hurwitz polynomial.

If s is forced to zero in finite time, the closed loop dynamics is given by:

$$\begin{cases} \dot{\eta}_i = \eta_{i+1} \quad i = 1,\dots,r-1 \\ \dot{\eta}_{r-1} = -\lambda_1\eta_1 - \lambda_2\eta_2 - \dots - \lambda_{r-1}\eta_{r-1} \\ \dot{\xi} = \sigma(\xi,\eta_1,\eta_2,\dots,\eta_{r-1},-\lambda_1\eta_1 - \lambda_2\eta_2 - \dots - \lambda_{r-1}\eta_{r-1}) \\ y = \eta_1 \end{cases} \tag{6}$$

it is assumed that the system is *minimum phase* i.e. the dynamics $\dot{\xi} = \sigma(\xi,0)$ is asymptotically stable to an equilibrium point.

The creation of a sliding motion on s entitles forcing the coordinate s to satisfy the reaching condition: $s\dot{s} < 0$. This, may be guaranteed by adopting the following dynamics with discontinuous right hand side for s:

$$\dot{s} = -W\,\mathrm{sign}(s) \tag{7}$$

Evidently a sliding regime can always be locally created on an open set of s=0 if and only if $\dot{s}$ depends explicitly on u i.e. if the s is relative degree one with respect to u.

The basic result to be used in the simulation and experimental procedure which is here proposed is based in the fact a sliding regime exists on a given representative of a family of sliding surfaces if, and only if, the chosen coordinate has relative degree equal to one with respect to u.

So, consider a growing sequence of sliding surfaces:

$$s_\beta = \eta_\beta + \sum_{i=1}^{\beta-1} \lambda_{i,\beta}\eta_i \qquad \beta=1,2,\dots$$

with $\{\lambda_{1,\beta},\lambda_{2,\beta},\dots,\lambda_{\beta-1,\beta},1\}$ being the coefficient of an β-th order Hurwitz polynomial, but otherwise arbitrary. It is clear that a sliding regime exists on $s_\beta = 0$ if and only if β=r.

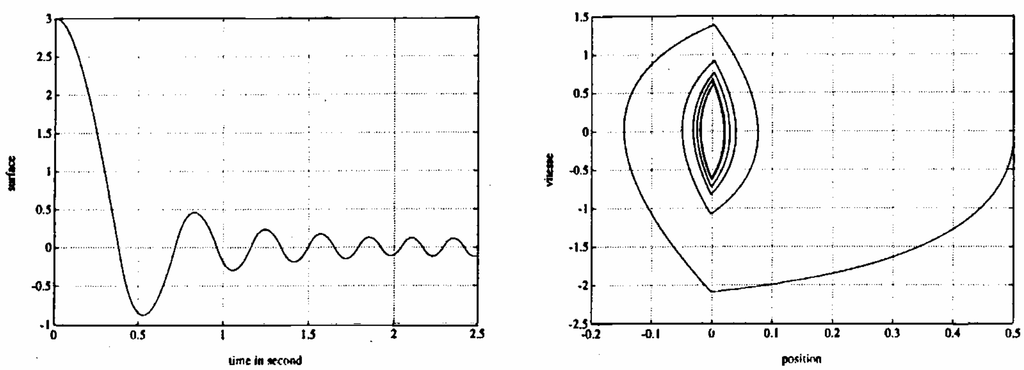
The experimental procedure is based on sequentially testing the sliding surface coordinate s_β for β=1,2,... until a sliding regime locally appears on $s_\beta = 0$. Note that if the system is differentially flat a sliding regime appears, for the first time, on $s_n = 0$ and there for the order of the systems is determined.

3. Simulation results:

Consider for simulation purposes a second order system already in normal form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -3x_1 - 4x_2 + u \\ y = x_1 \end{cases}$$

Choosing $s_0 = \lambda x_1$, for, say λ=6, the condition $s_0\dot{s}_0 < 0$ is only satisfied in the second and fourth quadrants of the phase space $\{y, \dot{y}\}$ and a sliding regime does not exist on $s_0 = 0$ as demonstrated on figure 1.



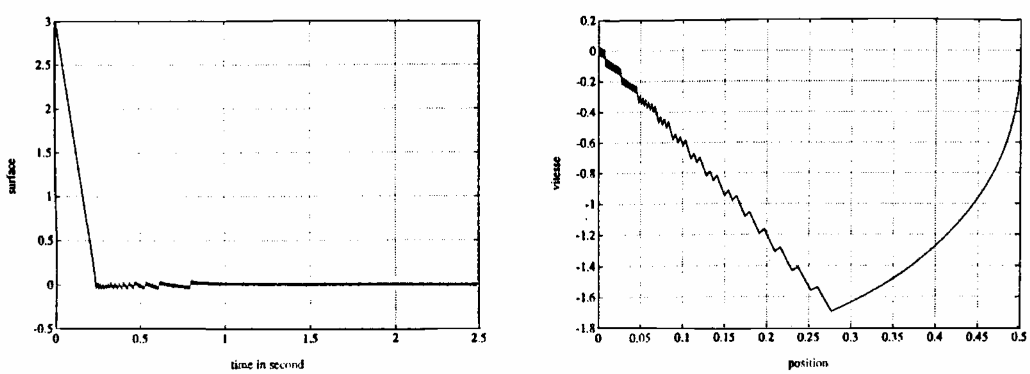
a) s(t) b) Plan de phase x_2=f(x_1)

Figure 1: Second order system - Surface: s_0=6 x_1

Choose now $s_1 = 6x_1 + x_2$ which is relative degree one. The condition

$$s_1\dot{s}_1 = s_1(-3x_1 + 2x_2 + u) < 0$$

is satisfied, for sufficient high k when the control u=-k sign(s_1) is used. The result, which drastically contrasts with that of figure 1, is shown in figure 2.



a) s(t) b) Plan de phase x_2=f(x_1)

Figure 2: Second order system - surface: $s_1 = 6x_1 + x_2$

The above procedure has been successfully used in an experimental set up for the validation of a third order model of a system consisting of a robotic manipulator with artificial muscles [16]. The computation of the required phase variables was made by numerical differentiation.

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