MODELING AND SIMULATION OF SWITCH REGULATED DC-TO-DC POWER CONVERTERS OF THE BOOST TYPE*

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Abstract

This paper presents the bond graph modeling and simulation of a switched dc-to-dc power converter of the boost type, regulated by means of a pulse-width-modulation (PWM) strategy. Two models are presented, the average model, which represents the switch as an ideal linear transformer element, and the actual (discontinuous) model, which represents it as a non-linear transformer element whose function is a non-periodic pulse train taking values in the discrete set {0,1}. Both transformes are modulated by the pulse width. The bond graphs were simulated and both the average and the actual discontinuous PWM regulated responses of the dc-to-dc power converter for a given feedback duty ratio policy were obtained.

1 Introduction

Switched electrical networks play an important role in a variety of devices, most notably in power converters such as dc-to-dc switchmode power converters. These power converters are, in turn, of great value in many growing application areas such as communication and data handling systems, portable battery-operated equipment, and uninterruptible power sources. New requirements for higher performance, smaller volume and lighter weight power converters makes new demands on technology and warrant a fresh look at associated modeling, simulaton and feedback control strategies for such devices.

DC-to-DC power converters have been extensively studied by researchers from the areas of Power Electronics and of Automatic Control Theory (see the book recently edited by Bose [1]). These devices are frequently regulated by means of PWM feedback control policies, whose complexity varies from traditional linear feedback schemes (see Severns and Bloom, [2], Kassakian *et al* [3] and Rashid [4]) to rather complex non-linear and adaptive feedback control options (see among many other authors, the work of Sira-Ramírez and colleages [5]-[6]-[7]).

Feedback regulation design of dc-to-dc converters, governed by PWM feedback laws, frequently resorts to *average models* (see Ćuk [8], Middlebrook and Ćuk [9] and Sira-Ramírez, [10]). Generally speaking, such average models are not of discontinuous nature and exhibit the duty ratio function as an effective control input parameter that can be synthesized by means of linear or nonlinear, dynamic or static, continuous or sampled, feedback policies.

Bond graph models of physical systems were introduced by Professor H. Paynter as a systematic graphical means of tracing the energy flow in the several interconnections, or *bonds*, between the elements or subsystems comprising a particular system under study (see the book by Karnopp *et al* [11]). The technique has undergone great development and enjoys well deserved popularity in many engineering and pure science disciplines.

In this article new and different models from the ones proposed in the literature are presented. These models are based on the bond graph technique, and are essentially computer models capable of predicting in a precise manner the behaviour of the boost converter.

This article is organized as follows. Section 2 contains a brief description of the dc-to-dc power boost converter. Section 3 and Section 4 contain the development of the average and actual models of the converter. Section 5 presents the simulation results obtained using a modeling and simulation tool for bond graph models. The conclusions and suggestions for further work are collected in Section 6.

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2 The Boost Converter Circuit

Consider the switch-regulated boost converter circuit of Figure 1, where x_1 and x_2 represent, respectively, the input inductor current and the output capacitor voltage variables. The positive quantity E represents the constant voltage value of the external voltage source. The variable u denotes the switch position function, acting as a control input. Such a control input takes values in the discrete set $\{0, 1\}$.

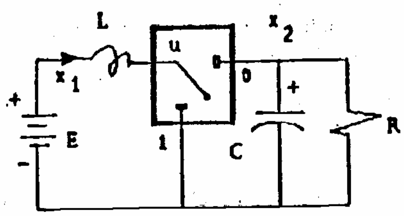

Figure 1: Boost Converter Circuit.

A Pulse Frequency Modulation (PWM) regulation policy for the switch position function may be specified as follows,

$$u = \begin{cases} 1 \text{ for } & t_k \le t < t_k + \mu(t_k)T \\ 0 \text{ for } & t_k + \mu(t_k)T \le t < t_k + T \end{cases}$$
$$t_{k-1} = t_k + T \quad ; \quad k = 0, 1, \ldots \qquad (2.1)$$

where t_k represents a sampling instant; the parameter T is the fixed sampling period, also called the *duty cycle*; the sampled values of the state vector $x(t)$ of the converter are denoted by $x(t_k)$ and the function $\mu(\cdot)$ is the *duty ratio function*. The value of the duty ratio function, $\mu(t_k)$, determines, at every sampling instant, t_k, the width of the upcoming "pulse" (switch at the position $u = 1$) as $\mu(t_k)T$.

The actual duty ratio function, $\mu(\cdot)$, is evidently a function limited to the closed interval $[0, 1]$.

3 Average Model of the Boost Converter Circuit

We consider separately the bond graphs of the two circuits associated with each one of the two possible positions of the regulating switch. The aim in carrying out such representations is to gain some insight on the physical effects of the switching action in terms of a suitable corresponding "average bond" describing an intermediate "effort-flow" connection. In particular, we would like to determine the nature and possible physical interpretation of such a suitable average physical "bond" which should also be capable of approximately representing the PWM behaviour of the switch regulated circuit.

Consider then, $u = 1$. The resulting circuit is as shown in Figure 2. In this case two separate, or decoupled, circuits are clearly obtained and the corresponding bond graph representation of the circuits is presented in Figure 3. Note that the effort variable v_1, acting on the inductor element, and the flow variable i_2, corresponding to the resistor element, satisfy

$$v_1 = E \quad ; \quad i_R = -i_2 \qquad (3.1)$$

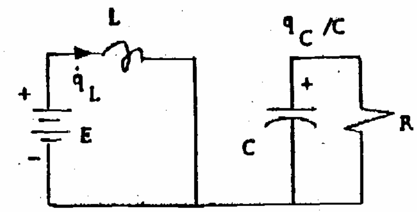

Figure 2: Boost Converter Circuit ($u = 1$).

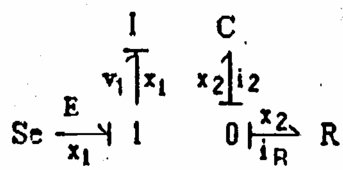

Figure 3: Bond Graph Boost Converter Circuit ($u = 1$).

Consider now the case $u = 0$. The resulting circuit is as shown in Figure 4.

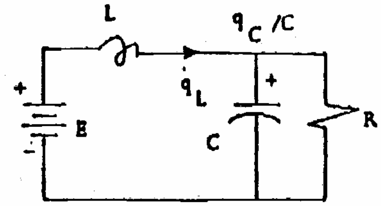

Figure 4: Boost Converter Circuit ($u = 0$).

The corresponding bond graph of the resulting circuit is shown in Figure 5. Note that the effort variable v_1, acting on the inductor element, and the flow variable i_2 corresponding to the resistor element now satisfy

$$v_1 = E - x_2 \quad ; \quad i_R = x_1 - i_2 \qquad (3.2)$$

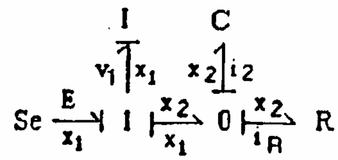

Figure 5: Bond Graph of Boost Converter Circuit ($u = 0$).

According to the PWM switching policy (2.1), at every sampling interval of period T, the decoupled bond graph of Figure 3 is valid only $\mu(t_k)$ percent of the time while the fully coupled bond graph of Figure 5 is valid $(1-\mu(t_k))$ percent of the time. Thus, the equivalent bond joining the two decoupled bond graphs of Figure 3 should be a bond "modulated" by the quantity $(1-\mu)$ in a fashion that neglects the underlying discrete time sampling process. However, such a smooth "bond modulation" should be *consistent* with the fact that when μ adopts the extreme saturation values, 1 or 0, the decoupled and the fully coupled bond graphs of Figures 3 and 5 should be exactly recovered, respectively.

We propose to use as a "complementary duty ratio-modulated bond" one representing an ideal linear transformer element with *turns ratio*, or transformer modulus, equal to the *complementary duty ratio* function, $(1-\mu)$. Such an average PWM bond graph is shown in Figure 6.

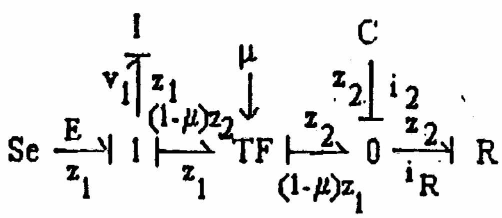

Figure 6: Bond Graph Model for the Average PWM Switched Regulated Boost Converter.

The average effort variable, denoted as $\bar{v}_1$, acting on the inductor element, and the average flow variable $\bar{i}_2$, corresponding to the resistor element, satisfy now the following relations

$$\bar{v}_1 = E - (1-\mu)z_2 \quad ; \quad \bar{i}_R = (1-\mu)z_1 - \bar{i}_2 \qquad (3.3)$$

where we denote by z_1, z_2, $\bar{v}_1$, $\bar{i}_2$ the *average* input inductor current, the average capacitor voltage, the average inductor voltage and the average capacitor current, respectively. This change in notation is made just to distinguish the average effort and flow variables from their actual PWM regulated values x_1, x_2, v_1, i_2. Note that when either $\mu = 0$, or $\mu = 1$, the average effort and average flow variables defined above should be made to coincide with the actual variables.

The duty ratio function μ is shown as an external input variable (shown with an activated bond) to the proposed bond graph converter model. The duty ratio function may be specified as a *feedback* function of the average states z_1, z_2 (see [5], [6], [7]). As such, the ideal transformer element is characterized by a *time-varying*, or modulated, turns ratio specified as a feedback function of the average state variables z_1 and z_2. If a constant value is specified for μ, one obtains a traditional, constant, turns ratio transformer element in the proposed bond graph. In this case, the bond graph is actually representing an average, *steady state*, PWM controlled boost converter.

Note that, as demanded above, the proposed average PWM bond graph of Figure 6 is consistent with the fact that in case μ takes one of the extreme values $\mu = 1$, or $\mu = 0$, one can recover the bond graphs corresponding to the circuits with the switch positions at the values $u = 1$ and $u = 0$, respectively. As a consequence of this property, the resulting average expressions (3.3) also generalize expressions (3.1) and (3.2) when μ is taken to be $\mu = 0$ and $\mu = 1$, respectively. This observation allows one to also use the proposed bond graph model for the adequate representation of the actual, rather than the average, PWM regulated boost converter circuit (see section 4).

The proposed average PWM bond graph model of Figure 6 allows one to readily obtain the differential equation model corresponding to the average PWM circuit model of the converter. These equations are given by

$$\begin{aligned} \dot{z}_1 &= -(1-\mu)\frac{1}{L}z_2 + \frac{E}{L} \\ \dot{z}_2 &= (1-\mu)\frac{1}{C}z_1 - \frac{1}{RC}z_2 \end{aligned} \qquad (3.4)$$

4 Actual Model of the Boost Converter Curcuit

It is well known that average PWM models, such as (3.4), of switchmode controlled dc-to-dc power supplies constitute a useful approximation of the actual PWM regulated circuit behavior provided the sampling frequency $1/T$, characterizing the feedback policy (2.1), is sufficiently high. The degree of approximation provided by the average PWM model (3.4) to the actual PWM circuit behaviour rapidly deteriorates as the sampling frequency is lowered. In order to obtain an actual simulation model of the PWM regulated dc-to-dc power converters one can still make a conceptual use of the average bond graph model of Figure 6 by letting the turns ratio of the linear ideal transformer $(1-\mu)$ adopt only two possible values: 0 or 1, corresponding to the actual switch position function values, $u = 1$ and $u = 0$, respectively. This effect is achieved by simply letting the duty ratio function μ appearing in the average PWM bond graph model to be specified as a possibly non–periodic, "pulse train" taking values in the discrete set $\{0, 1\}$. In other words, for representation purposes, we let μ be replaced by the switch position function u. This replacement is entirely permissible due to the consistency of the proposed average bond graph model with the intervening circuit topologies associated with each possible switch position function value.

Figure 7 thus depicts a general bond graph model that can be used for the simulation of actual PWM regulated behaviour of a boost converter or, alternatively, for the average PWM simulation of the same converter.In this bond graph the modulated transformer is a non-linear element whose function is a pulse train modulated by μ.

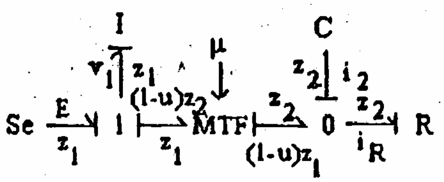

Figure 7: Bond Graph Model for the Actual PWM Switched Regulated Boost Converter

5 Simulation of the Average and Actual Models of the Boost Converter Circuit

Both models were simulated using DESIS [12]–[13] computer program for a given feedback duty ratio policy.

For the average model the transformer modulus was introduced as 0.4 corresponding to the complementary duty ratio function. The parameter values of the elements were taken from [14] as R=30 Ω, C=20 μF and L=20 mH. Since we wanted to observe the average input current and the average output capacitor voltage for a 15 volts step input, we specified the flow variable at bond 2, or f_2; and the effort variable at bond 5, or e_5, as output variables. The simulation results for the average model are shown in Figure 8.

To simulate the actual discontinuous model of the power converter, the transformer was not linear. In this case we used as non–linear function a pulse train taking values (0,1) with a 60% duty cycle and 10 Khz frequency. The simulation results for the actual model of the power converter are shown in Figure 9.

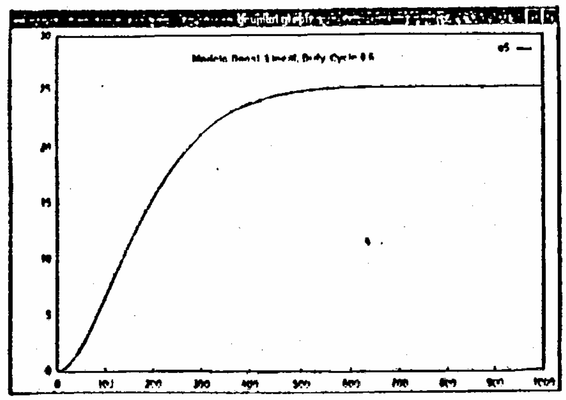

Figure 8: Simulation Results for Average Capacitor Voltage of the Boost Converter for a Constant Duty Ratio Function

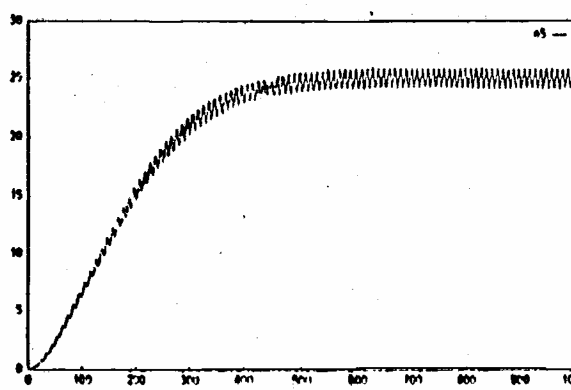

Figure 9: Simulation Results for Actual Capacitor Voltage of the Boost Converter for a Constant Duty Ratio Function

6 Conclusions

In this article we have used a bond graph approach to the modeling of switch–regulated dc–to–dc power converters of the boost type. The switch position function is assumed to undergo a PWM feedback strategy characterized by a given duty ratio function. An average PWM bond graph is proposed which replaces the "switching bond" by a linear transformer element with turns ratio coincident with the complementary duty ratio function associated with the PWM feedback strategy. The proposed average PWM bond graph model was shown to be entirely consistent with the individual bond graph models corresponding to each one of the possible circuit topologies occurring when the duty ratio function adopts one of the two possible extreme saturation values.

An actual PWM bond graph was also proposed which uses a non linear transformer element to represent the "switching bond". The non-linear function in this case is a pulse train taking values of the discrete set {0, 1} modulated by the duty ratio function.

We used DESIS program to simulate the average and actual PWM switch-regulated boost converter. For the simulations we used PWM policies characterized by constant duty ratio functions (i.e., steady-state average and actual PWM models). However, the methodology also allows for truly nonlinear feedback policies defining the duty ratio function.

The actual model proposed in this article has a closer behaviour to the real response of the system because it shows inherent oscillation of the operation of the commutation circuit absent in average models.

The bond graph modeling methodology developed in this article for dc-to-dc power converters can be readily extended to include some other representatives of the family of switchmode power supplies, such as the boost, the buck-boost and the Čuk converters.

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