On the Nonlinear Regulation of Derived Power Supplies via Discrete Models *

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November 1994

Abstract

Feedback controllers, based on Pulse–Width–Modulation (PWM), are derived for the average input current stabilization and tracking problems in derived dc–to–dc power supplies of the buck, boost and buck-boost types. The stabilization problems are solved on the basis of steady state considerations about the "ripple" and exactly discretized nonlinear models describing the sampled PWM regulated input current trajectories. In all but the buck–derived converter, the stabilization problems require *implicit* static nonlinear feedback controllers, or duty ratio synthesizers, which require on–line solutions of transcendental equations. The signal tracking problems are solved on the basis of discrete-time models describing the average PWM regulated input current. These models naturally result in generalized, i.e. non–kalmanian, state representations from which *explicit* dynamical, rather than static, feedback regulators are readily derived. Computer simulations, including unmodelled external stochastic perturbation inputs are presented which test the robustness of the proposed PWM controller performances

Keywords : DC–to–DC power converters, Choppers, PWM feedback control, Nonlinear discrete-time control.

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*This research was supported by the Consejo de Desarrollo Científico, Humanístico and Tecnológico of the Universidad de Los Andes under Research Grant I–456–94, and by the Consejo Nacional de Ciencia y Tecnología (CONACYT), México

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1 Introduction

Derived dc–to–dc power converters constitute simplified versions of traditional dc power converters, such as the buck, the boost, and buck–boost converters (see Severns and Bloom [1]). The derived converters, also known, respectively, as "choppers", step–up, and step up–down converters are obtained from the traditional converter structures by removing the storing capacitors from the output circuits (see Rashid [2] for details).

The feedback regulation of dc–to–dc power supplies is accomplished, through either Pulse–width–modulation (PWM) feedback strategies or by inducing appropriate stabilizing sliding regimes (see Venkataramanan *et al* [3], Sira-Ramírez and co–workers [4]–[5]). Typically, control objectives include input current stabilization, around a given constant value, or time–varying reference input signal tracking. In this context, ***infinite frequency average models*** (see Middlebrook and Cúk [6]) or ***equivalent control models*** are frequently invoked, at the controller design stage, in order to obtain a smooth feedback specification of the computed duty ratio function (see also [5], Sira-Ramírez [7], and Sira-Ramírez and coworkers [8]–[10]). The performance features of the actual PWM controlled circuit responses, with respect to those predicted by the average PWM model, depend on the magnitude of the sampling frequency associated with the pulse width modulator. For low sampling frequencies, the closed–loop precision deteriorates allowing substantial errors in the stabilization and tracking tasks.

The use of average models, however, may not be entirely justified for derived dc–to–dc power supplies. First, the appealing simplicity of the dynamic models does not seem to require further simplification through a questionable smooth approximation and, secondly, the possibilities for exact discretization, which is certainly a much more involved process in the traditional version of the converters, make it reasonable to attempt a direct PWM controller design based on an exact discrete–time model of the derived converter (see Kassakian *et al* [11]). The exact discretization circumvents all problems related to the approximation involved in the finite magnitude of the sampling frequency used for the pulse width modulator.

It is the purpose of this paper to explore, in detail, the feasibility of PWM stabilizng controller designs for derived dc–to–dc power converters based on exact discretization of the sampled input current. Discrete–time regulation policies based on approximate discretization and approximate linearization were explored by Ehsani *et al* [12]. The outline of an approximate discretization approach for the stabilization of more complex dc–to–dc power supplies can also be found in [11]. Related developments, from a viewpoint different to that of feedback control, are found in [2]. Useful information is also available from the recently published collection of articles edited by Bose [13].

In this article we present the fundamentals of an exact discretization approach for the input current stabilization in the derived versions of the buck, boost and buck–boost dc–to–dc power supplies. The linearity in the input, associated with the traditional infinite frequency average models of the converters, is effectively destroyed by the exact discretization procedure, although the resulting models still remain linear in the state. For all converters, except the buck–derived converter, the resulting non-linear discrete–time duty ratio synthesizers (controllers) are of the *implicit* type, i.e., at each sampled instant, the feedback duty ratio function is given by the solution of a transcendental equation. Similar transcendental equations allow for the off–line computation of the desired steady state average input current in terms of the steady state sampled input current. Only for the buck–derived converter it is possible to obtain an explicit expression relating steady–state average input current values to steady–state sampled input current values.

Section 2 contains an exact discretization approach for PWM feedback regulator designs solving stabilization problems defined on derived dc–to–dc power supplies of the buck, boost and buck–boost types. In this section we also present simulations of the proposed control schemes for the various derived converters. Section 3 is devoted to the conclusions and suggestions for further research in this area.

2 Regulation of Derived DC–to–DC Power Converters via an Exact Discretization Scheme

In this section we consider derived dc–to–dc converters and their exact discretization models, naturally associated with PWM feedback regulation strategies. The explicit off–line calculation of the steady–state value of the input current "ripple" effectively allows for the sampled state feedback solution of the average input current stabilization problem.

2.1 The buck–derived converter

Consider the buck–derived converter circuit shown in Figure (see 1) [2]. The switch regulated model describing the behaviour of the input current, denoted by x, is given by

$$\begin{aligned} \dot{x} &= -\frac{R}{L}x + \frac{E}{L}u \\ y &= Rx \end{aligned} \tag{2.1}$$

where y is the output load voltage and the parameters R, L and E stand, respectively, by the load resistance, the inductance of the input circuit, and the constant input source voltage. The variable u denotes the *switch position function* taking values on the discrete set $\{0,1\}$.

A regulation strategy, based on a PWM specification of the switch position function, may be, generally speaking, specified by (see [8]–[10]):

$$u(t) = \begin{cases} 1 & \text{for} \quad t_k \leq t < t_k + \mu(t_k)T \\ 0 & \text{for} \quad t_k + \mu(t_k)T \leq t < t_k + T \end{cases} \tag{2.2}$$
$$t_{k+1} = t_k + T \; ; \; k = 0,1,2,\ldots$$

where $\mu(\cdot)$ is known as the *actual duty ratio* function, taking values in the interval $[0,1]$ of the real line. T is the sampling period and t_k is the sampling instant. A typical example of a PWM commanded switch position function trajectory is depicted in Figure 2.

Since the duty ratio μ is specified on–line in a feedback manner, i.e. computed as a function explicitly depending on the sampled value of the input current $x(t_k)$ at each instant t_k, one may obtain values of μ which lie outside the closed interval $[0,1]$. We must, therefore, make a distinction between the *computed duty ratio function*, denoted by $\mu_c(\cdot)$ and the *actual duty ratio function*, denoted by $\mu(\cdot)$. The relation between these variables is simply given by:

$$\mu(t) = \begin{cases} 1 & \text{for } \mu_c > 1 \\ \mu_c(t) & \text{for } 0 \leq \mu_c(t) \leq 1 \\ 0 & \text{for } \mu_c(t) < 0 \end{cases} \tag{2.3}$$

The actual duty ratio function is thus the forceful limitation of the computed duty ratio function to the closed interval $[0,1]$.

It is widely known that the PWM controlled trajectory $x(t)$ invariably exhibits a natural "chattering", or "zig–zag" motion due to the discontinuities associated with the "input" variable $u(t)$. The following paragraphs compute a discrete time model yielding the value of the sampled state at each instant t_k. The values of the "ripple" are not taken into account by such a model alone but they can still be computed from additional considerations.

The buck–derived converter owes its popular name as "chopper" from the fact that the input current is limited to take values on the interval $[0, E/R]$, as can be easily verified from the circuit equations. The corresponding (positive) output load voltages delieverd by the converter cannot, therefore, exceed the value E of the external source voltage.

2.2 An Exact Discretization of the PWM Regulated Buck–derived Converter

The linear nature of the two possible topologies of the converter circuit facilitate the derivation of an *exact* discrete–time model for the evolution of the sampled values of the input current in the buck–derived converter (2.1), when subject to a switching policy of the form (2.2). Indeed, given the value of x at time t_k, denoted by $x(t_k)$, the value of the input current at the end of the "pulse", of width $\mu(t_k)T$, is obtained after use of the *variations of constants formula* as

$$x(t_k + \mu(t_k)T) = e^{-\theta_1 \mu(t_k)T} x(t_k) + \frac{\theta_2}{\theta_1}\left[1 - e^{-\theta_1 \mu(t_k)T}\right] \tag{2.4}$$

where we have let the parameter θ_1 denote the quotient R/L and θ_2 denote E/L.

The sampled value of the input current at the end of the sampling interval is obtained, after some further computations, as

$$x(t_k + T) = e^{-\theta_1 T} x(t_k) + \frac{\theta_2}{\theta_1} e^{-\theta_1 T}\left[e^{\theta_1 \mu(t_k)T} - 1\right] \tag{2.5}$$

If we denote $\Psi_1 = e^{-\theta_1 T}$ and $\Psi_2 = \theta_2/\theta_1$, the discrete–time model for the evolution of the input current, depicted at the sampling instants, is given by the following model

$$x(t_{k+1}) = \Psi_1 x(t_k) + \Psi_1 \Psi_2 \left[\Psi_1^{-\mu(t_k)} - 1\right] \tag{2.6}$$

where the value of the duty ratio function at time t_k, $\mu(t_k)$, must now be effectively regarded as the "control input" variable, to be specified at the begining of each sampling period. The discrete time model for the sampled input current is, therefore, nonlinear in the new control input, $\mu(t_k)$.

The only eigenvalue associated with the linear sampled state dynamics, given by Ψ_1, is evidently positive and strictly smaller than unity. The steady state value of the sampled input current, denoted by x_∞^-, corresponding to constant duty ratio function of value μ_∞, is then readily obtained from (2.6) as

$$x_\infty^- = \frac{\Psi_1 \Psi_2}{1 - \Psi_1}\left(\Psi_1^{-\mu_\infty} - 1\right) \tag{2.7}$$

The supper–index "$-$" in (2.7) refers to the "lower" portion of the actual zig-zagged trajectory partially described by (2.6) (see Figure 3). Evidently, a feedback regulation policy which specifies the duty ratio function $\mu(t_k)$ solely on the basis of the sampled state $x(t_k)$ is, by no means, satisfactory. The reason for such a statement stems from the fact that the "ripple", unavoidably associated with the switch–regulated evolution of $x(t_k)$, is not taken into account by the model (2.6) alone. One must also take into account the values of $x(t)$ at the end of each width–modulated control input pulse occurring within the sampling period of length T. In other words, one must take into account the values of $x(t)$ at the instants $t = t_k + \mu(t_k)T$; $k = 0, 1, 2, \ldots$ (see Figure 3).

We now relate the values of x at times $t_{k+1} + \mu(t_{k+1})T$ and $t_k + \mu(t_k)T$ so as to obtain the values of the "upper" corners of the zig-zagged input current trajectory.

Using the variation of constants formula one obtains,

$$x(t_{k+1} + \mu(t_{k+1})T) = \Psi_1^{\mu(t_{k+1})} \Psi_1^{1-\mu(t_k)} x(t_k + \mu(t_k)T) + \Psi_2(1 - \Psi_1^{\mu(t_{k+1})}) \tag{2.8}$$

The eigenvalue associated with the above linear state dynamics is clearly given by the product $\Psi_1^{\mu(t_{k+1})} \Psi_1^{1-\mu(t_k)}$ This quantity is strictly positive and samller than unity for values of μ bounded by the unit interval $[0, 1]$. The steady state value of the "upper" corners of the state trajectory, described by (2.8), corresponding to a constant value μ_∞ of the duty ratio function, is given by (see Figure 3),

$$x^+_\infty = \frac{\Psi_2}{1-\Psi_1}\left(1-\Psi_1^{\mu_\infty}\right) \tag{2.9}$$

The relation between the steady state values x^+_∞ and x^-_∞ can be obtained from (2.7) and (2.9) as,

$$x^-_\infty = x^+_\infty \Psi_1^{1-\mu_\infty} \tag{2.10}$$

Since Ψ_1 is a positive number which is strictly less than 1, one can conclude that $x^-_\infty < x^+_\infty$ for $\mu_\infty \in [0,1]$.

The steady state "ripple", denoted by r_∞, may then be described as the following difference

$$r_\infty = x^+_\infty - x^-_\infty = \frac{\Psi_2}{1-\Psi_1}\left(1-\Psi_1^{\mu_\infty}\right)\left(1-\Psi_1^{1-\mu_\infty}\right) \tag{2.11}$$

We define a *steady state average value* for the input current trajectory as

$$x_{av}(\infty) = x^-_\infty + r_\infty = \frac{1}{2}\left(x^-_\infty + x^+_\infty\right) \tag{2.12}$$

Using the expressions (2.7) and (2.9) in (2.12) one obtains

$$x_{av}(\infty) = \frac{1}{2}\left(\frac{\Psi_2}{1-\Psi_1}\right)\left(1-\Psi_1^{\mu_\infty}\right)\left(1+\Psi_1^{1-\mu_\infty}\right) \tag{2.13}$$

We proceed to express the steady state value of the sampled input current trajectory x^-_∞, in terms of the average steady state input current $x_{av}(\infty)$. This relation allows us to define a suitable stabilizing feedback duty ratio (control) policy on the basis of the sampled states of the discrete–time model (2.6). The feedback policy properly takes into account the ripple associated to the controller trajectory and asymptotically achieves a pre-specified desired steady state value for the average input current. To achieve this goal one simply eliminates the steady state value of the duty ratio, μ_∞ from the expressions (2.7) and (2.13). One then obtains,

$$x^-_\infty = -\Psi_2\left[\left(\frac{1}{2}(1-\frac{2x_{av}(\infty)}{\Psi_2}) + \frac{\Psi_1}{1-\Psi_1}\right) - \sqrt{\frac{1}{4}\left(1-\frac{2x_{av}(\infty)}{\Psi_2}\right)^2 + \frac{\Psi_1}{(1-\Psi_1)^2}}\,\right] \tag{2.14}$$

2.3 A Stabilizing PWM Control Policy for the Buck–derived Converter

The stabilization problem for the buck–derived converter consists in specifying a PWM feedback regulation policy of the form (2.2) such that the steady state average value of the controlled input current trajectory $x(t)$ reaches a desired constant value $x_{av}(\infty) = X$.

A stabilizing feedback regulation policy $\mu(t_k)$ can then be explicitly obtained on the basis of the sampled states of the discrete–time model (2.6) by forcing $x(t_k)$ to asymptotically stabilize around the value x^-_∞, corresponding to X, which we denote by $x^-_\infty(X)$ and rewrite as,

$$x^-_\infty(X) = -\Psi_2\left[\left(\frac{1}{2}(1-\frac{2X}{\Psi_2}) + \frac{\Psi_1}{1-\Psi_1}\right) - \sqrt{\frac{1}{4}\left(1-\frac{2X}{\Psi_2}\right)^2 + \frac{\Psi_1}{(1-\Psi_1)^2}}\,\right] \tag{2.15}$$

We impose on the sampled controlled system the following linear asymptotically stable closed–loop behaviour

$$x(t_{k+1}) = \alpha\left(x(t_k) - x^-_\infty(X)\right) + x^-_\infty(X) \ \ ; \ \ |\alpha| < 1 \tag{2.16}$$

Substituting the right hand side of expression (2.6) on (2.16) and solving for the duty ratio function $\mu(t_k)$ one obtains the following non–linear computed duty ratio feedback control policy,

$$\mu_c(t_k) = -\frac{1}{\log \Psi_1} \log \left[1 + \frac{(\alpha - \Psi_1)\, x(t_k) + (1-\alpha)\, x_\infty^-(X)}{\Psi_1 \Psi_2} \right] \quad ; \quad k = 0, 1, 2, \ldots \tag{2.17}$$

The actual duty ratio function $\mu(t_k)$ may be readily obtained from the expression (2.3). Figure 4 depicts the PWM feedback regulation scheme based on the exact discrete time dynamics model of the sampled input current.

Expression (2.17) allows for the determination of the region of non–saturation of the actual duty ratio function. Indeed, the double inequality: $0 < \mu_c < 1$, yields the following corresponding region for the sampled state,

$$0 < (\alpha - \Psi_1) x(t_k) + (1-\alpha) x_\infty(X) < \Psi_2 (1 - \Psi_1) \tag{2.18}$$

2.4 Simulation Results

In order to test the robustness of the previously proposed PWM feedback regulation policy we carried out simulations on the following noise perturbed model of the buck–derived converter,

$$\dot{x} = -\frac{R}{L} x + \left(\frac{E + \nu(t)}{L} \right) u \tag{2.19}$$

where $\nu(t)$ is a (computer generated) stochastic perturbation signal representing an unmodelled additive noisy voltage source afecting the behaviour of the circuit. The values for the parameters defining the converter were taken to be

$$R = 2.8 \times 10^{-2} \ \ \Omega \ \ ; \ \ L = 1.0 \times 10^{-2} \ \ \text{mH} \ \ ; \ \ E = 126 \ \ \text{Volts}$$

The sampling period was chosen to be $T = 0.125$ ms ($1/T = 8$ KHz) and the desired steady state value of the average dynamics was set to be $X = 1237$ amp. The eigenvalue for the closed loop linear dynamics, α, was set to be 0.3. The corresponding value of the steady state input current was found to be $x_\infty^-(1237) = 1080.7$ A. The required steady state average value of the input current as well as the steady state values x_∞^+ and x_∞^- are well within the allowable range which guarantees non–saturation of the actual duty ratio function.

Figure 5 depicts a typical simulated PWM feedback controlled trajectory for the input current arising from the perturbed model (2.19). This figure also shows the actual duty ratio function $\mu(t)$ and the corresponding switch position function $u(t)$. At the end of the figure we show the perturbation signal $\nu(t)$. As shown, in spite of the unmodelled perturbation signal the derived nonlinear discrete–time duty ratio controller performs remarkably well.

2.5 The boost–derived converter

Consider the boost–derived converter circuit shown in Figure 6 (see [2]). The switch regulated model describing the behaviour of the input current, denoted by x, is given by

$$\begin{aligned} \dot{x} &= -\frac{R}{L} x (1 - u) + \frac{E}{L} \\ y &= R x \end{aligned} \tag{2.20}$$

where y is the output load voltage and the parameters R, L and E stand, respectively, by the load resistance, the inductance of the input circuit, and the constant input source voltage. The variable u denotes the *switch position function* taking values on the discrete set $\{0, 1\}$.

A regulation strategy, based on a PWM specification of the switch position function, may be specified exactly as in (2.2):

The boost–derived converter is popularly known as the "step–up" converter. This is due to the fact that the average value of the input current is, theoretically, capable of achieving all values in the semi–open interval $[E/R, \infty]$, as can be verified from the circuit equations. The corresponding (positive) output voltage delieverd by the converter cannot be lower than the source voltage value E.

2.6 An Exact Discretization of the PWM Regulated Boost–derived Converter

Given the value of x at time t_k, denoted by $x(t_k)$, the value of the input current at the end of the "pulse", of width $\mu(t_k)T$, is obtained after use of the *variations of constants formula* as

$$x(t_k + \mu(t_k)T) = \theta_2\mu(t_k)T + x(t_k) \tag{2.21}$$

where we have, again, let the parameter θ_1 denote the quotient R/L and θ_2 denote E/L.

The sampled value of the input current at the end of the sampling interval is obtained, after some further computations, as

$$x(t_k + T) = e^{-\theta_1 T(1-\mu(t_k))}\left[\theta_2\mu(t_k)T + x(t_k)\right] + \frac{\theta_2}{\theta_1}\left[1 - e^{\theta_1 T(1-\mu(t_k))}\right] \tag{2.22}$$

If we denote $\Psi_1 = e^{-\theta_1 T}$, $\Psi_2 = \theta_2/\theta_1$ and $\Psi_3 = \theta_2 T$, the discrete–time dynamics describing the evolution of the input current, depicted at the sampling instants t_k, is given by the following expression

$$x(t_{k+1}) = \Psi_1^{(1-\mu(t_k))}x(t_k) + \Psi_1^{(1-\mu(t_k))}\left[\mu(t_k)\Psi_3 - \Psi_2\right] + \Psi_2 \tag{2.23}$$

The discrete time model for the sampled input current is, as in the Buck–derived case, nonlinear in the new control input, $\mu(t_k)$.

Note that the quantity $\Psi_3 = \theta_2 T$ represents, for all times t_k, an upper bound of the input current ripple $x(t_k + \mu(t_k)T) - x(t_k) = \theta_2\mu(t_k)T < \theta_2 T$.

The time–varying eigenvalue associated with the linear sampled state dynamics, given by $\Psi_1^{(1-\mu(t_k)}$, is evidently positive and not greater than unity for all values of μ restricted to the unit interval $[0,1]$. The steady state value of the sampled input current, denoted by x_∞^-, corresponding to constant duty ratio function of value μ_∞, is then readily obtained from (2.23) as

$$x_\infty^- = \frac{\Psi_1^{(1-\mu_\infty)}\left[\mu_\infty\Psi_3 - \Psi_2\right] + \Psi_2}{1 - \Psi_1^{(1-\mu_\infty)}} \tag{2.24}$$

We can now relate the values of x at times $t_{k+1} + \mu(t_{k+1})T$ and $t_k + \mu(t_k)T$ in order to obtain the "upper" corners of the zig-zagged input current trajectory.

Using the variation of constants formula one obtains,

$$x(t_{k+1} + \mu(t_{k+1})T) = \Psi_1^{(1-\mu(t_k))}x(t_k + \mu(t_k)T) + \Psi_3\mu(t_{k+1}) + \Psi_2\left(1 - \Psi_1^{1-\mu(t_k)}\right) \tag{2.25}$$

The eigenvalue associated with the above linear state dynamics is clearly given by $\Psi_1^{(1-\mu(t_k))}$ This quantity is strictly positive and samller or equal than unity for values of $\mu(t_k)$ bounded by the unit interval $[0,1]$. The steady state value of the "upper" portions of the state trajectory, described by (2.25), corresponding to a constant value μ_∞ of the duty ratio function, is given by,

$$x_\infty^+ = \frac{\mu_\infty\Psi_3 + \Psi_2\left(1 - \Psi_1^{(1-\mu_\infty)}\right)}{1 - \Psi_1^{(1-\mu_\infty)}} \tag{2.26}$$

Contrary to the Buck–derived case, an expression, independent of μ_∞ relating x_∞^+ and x_∞^- cannot be obtained from the expressions (2.25) and (2.26). This fact has a direct influence on the impossibility to express explicitly the steady–state average value of the input current in terms of the steady state value of the sampled input current.

It follows from the expressions (2.24), (2.26) and the fact that $\Psi_1^{(1-\mu_\infty)} < 1$, that $x_\infty^- < x_\infty^+$ for $\mu_\infty \in [0,1]$.

The steady state "ripple", denoted by r_∞, may be described in a manner similar to (2.11), but in simpler terms,

$$r_\infty = x_\infty^+ - x_\infty^- = \Psi_3 \mu_\infty \tag{2.27}$$

The *steady state average value* for the input current trajectory is defined as before

$$x_{av}(\infty) = x_\infty^- + \frac{1}{2} r_\infty \tag{2.28}$$

Using the expressions (2.24) and (2.27) in (2.28) one obtains

$$x_{av}(\infty) = \frac{\left[\Psi_3 \mu_\infty \left(1 + \Psi_1^{(1-\mu_\infty)}\right) + 2\Psi_2 \left(1 - \Psi_1^{(1-\mu_\infty)}\right)\right]}{2\left(1 - \Psi_1^{(1-\mu_\infty)}\right)} \tag{2.29}$$

It is again impossible to express the steady state value of the sampled trajectory of the input current, x_∞^-, in terms of the average input current $x_{av}(\infty)$. One must, therefore, proceed to numerically find the value of μ_∞ corresponding to a desired steady–state value of the average input current $X = x_{av}(\infty)$. The required steady state duty ratio function, μ_∞, must then be substituted on the expression for the steady–state sampled input current (2.24). This procedure, which can certainly be performed off–line, yields the required steady state value of the sampled input current, $x_\infty^-(X)$ to which the corresponding exactly discretized controlled dynamics (2.23) must be driven, by means of an appropriate prescription of the feedback duty ratio function. The derived controlled policy forces the average input current to to reach the desired average steady state value.

We show that the required solution, $x_\infty^-(X)$, for the steady state sampled input current, in terms of the steady state average input current, $x_{av}(\infty) = X$, always exist for some μ_∞ in the interval $[0,1]$. Moreover, such a solution is unique. Indeed, manipulatiing expression (2.29) one obtains the following equivalent expression,

$$\Psi_1^{1-\mu_\infty} = \frac{2X - \Psi_3 \mu_\infty - 2\Psi_2}{2X + \Psi_3 \mu_\infty - 2\Psi_2} \tag{2.30}$$

We recall that both Ψ_1 and Ψ_3 are positive quantities, with $\Psi_1 < 1$. Consider the left and right hand sides of (2.30) as functions of μ_∞. The graph of the function of μ_∞ in the left hand side of (2.30) is seen to continuously increase with non–negative slope while taking values in the interval $[\Psi_1, 1]$ as μ varies in the interval $[0,1]$. The graph of the function of μ_∞ in the right hand side of (2.30), on the other hand, continuously decreases with strictly negative slope, as μ_∞ varies from 0 to 1, from the value 1 towards the quantity $M = (2X - \Psi_3 - 2\Psi_2)/(2X + \Psi_3 - 2\Psi_2)$ which is, certainly, less than unity. The two graphs, therefore, intersect at most once in the interval $[0,1]$ and a unique solution exists for μ_∞.

2.7 An Implicit Stabilizing PWM Control Policy for the Boost–derived Converter

The stabilization problem for the boost–derived converter consists in specifying a PWM feedback regulation policy of the form (2.2) such that the steady state average value of the controlled input current $x(t)$ trajectory equals a desired constant value $x_{av}(\infty) = X$.

A stabilizing feedback regulation policy $\mu(t_k)$ can then be (implicitly) obtained from the discrete–time model (2.23) by forcing $x(t_k)$ to asymptotically stabilize around the value x_∞^-, corresponding to X, which we denote by $x_\infty^-(X)$,

One imposes on the controlled system the following linear asymptotically stable closed loop behaviour

$$x(t_{k+1}) = \alpha\left(x(t_k) - x_\infty^-(X)\right) + x_\infty^-(X) \quad ; \quad |\alpha| < 1 \tag{2.31}$$

Substituting the right hand side of expression (2.23) in (2.31) one obtains the following transcendental expression, from which the computed duty ratio feedback control policy can be numerically obtained at each sampling instant,

$$\left[\Psi_1^{(1-\mu_c(t_k))} - \alpha\right] x(t_k) + \Psi_1^{(1-\mu_c(t_k))}\left[\mu_c(t_k)\Psi_3 - \Psi_2\right] + \Psi_2 - (1-\alpha)\, x_\infty^-(X) = 0 \tag{2.32}$$

Rearranging expression (2.32) one obtains the following possible expression for the implicit controller:

$$\Psi_1^{(1-\mu_c(t_k))} = \frac{\alpha x(t_k) - \Psi_2 + (1-\alpha)x_\infty^-(X)}{x(t_k) - \Psi_2 + \mu_c(t_k)\Psi_3} \tag{2.33}$$

The left hand side of the expression varies, as a function of μ_c, from $\Psi_1 < 1$ to 1, as μ_c varies from zero to one. For a given value of $x(t_k)$, the right hand side of (2.33) is seen to take values, as a function of μ_c, on the interval

$$\left[\frac{\alpha x(t_k) - \Psi_2 + (1-\alpha)x_\infty^-(X)}{x(t_k) - \Psi_2}, \frac{\alpha x(t_k) - \Psi_2 + (1-\alpha)x_\infty^-(X)}{x(t_k) - \Psi_2 + \Psi_3}\right] \tag{2.34}$$

It is easy to see that a necessary and sufficient condition for the existence of an intersection point of the graphs of the two functions, on the interval $[0,1]$, is given by the condition,

$$\frac{\alpha x(t_k) - \Psi_2 + (1-\alpha)x_\infty^-(X)}{x(t_k) - \Psi_2 + \Psi_3} < 1 \tag{2.35}$$

which is equivalent to the following simplified condition,

$$x_\infty^-(X) < x(t_k) + \frac{\Psi_3}{1-\alpha} \tag{2.36}$$

Note that in steady state, when $x(t_k) = x_\infty^-(X)$ the condition (2.36) is trivially satisfied, from the positivity of Ψ_3 and the fact that $|\alpha| < 1$. The implicit controller (2.33) yields then a unique solution for the duty ratio μ_∞ lying in the interval $[0,1]$. During the transient period, however, the condition (2.36) simply says that the desired value of the steady state sampled input current, $x_\infty^-(X)$, should not exceed the sampled input current, at any time, plus the upper value of all input current "ripples" (given by $\Psi_3 = \theta_2 T$), multiplied by a factor of $1/(1-\alpha)$. As it can be seen this is not a very stringent condition, specially if α is chosen to be positive and not close to zero. Note, furthermore, that in the absence of condition (2.36), the implicit controller (2.33) *always* yields a unique solution for $\mu_c(t_k)$ but one which is not, necessarily, constrained to the meaningful interval $[0,1]$.

The actual duty ratio function $\mu(t_k)$ may be readily obtained from the computed duty ratio function by use of the expression (2.3). Figure 7 depicts the PWM feedback regulation scheme for the boost–derived converter based on the exact discrete time dynamics model of the sampled input current and the implicit duty ratio synthesizer.

2.8 Simulation Results

In order to test the robustness of the previously derived PWM feedback regulation policy, based on exact discretization, we used the following noise perturbed model of the boost–derived converter,

$$\dot{x} = -\frac{R}{L}(1-u)x + \left(\frac{E+\nu(t)}{L}\right) \tag{2.37}$$

where $\nu(t)$ represented an unmodelled computer generated stochastic perturbation signal representing a noisy voltage source afecting the behaviour of the circuit. The values for the parameters defining the converter were taken to be the same as in the buck–derived case,

$$R = 2.8 \times 10^{-2} \ \ \Omega \ \ ; \ \ L = 1.0 \times 10^{-2} \ \ \text{mH} \ \ ; \ \ E = 126 \ \ \text{Volts}$$

The sampling period was chosen to be $T = 0.125$ ms ($1/T = 8$ KHz) and the desired steady state value of the average dynamics was set to be $X = 6000$ A. The eigenvalue for the closed loop linear dynamics, α, was set to be 0.3. The corresponding value of the steady state input current was found to be $x_\infty^-(6000) = 5804$ A. The required steady state average value of the input current as well as the steady state values x_∞^+ and x_∞^- are well within the allowable range which guarantees non–saturation of the actual duty ratio function.

Figure 8 depicts a typical simulated PWM feedback controlled trajectory for the input current arising from the perturbed model (2.37). This figure also shows the actual duty ratio function $\mu(t)$ and the corresponding switch position function $u(t)$. At the end of the figure we show the perturbation signal $\nu(t)$. As shown, in spite of the unmodelled perturbation signal the derived nonlinear discrete–time duty ratio controller performs remarkably well.

2.9 The Buck–Boost–derived Converter

Consider the buck-boost-derived converter circuit shown in Figure 9 (see [2]). In the next paragraphs we summarize all of the relevant equations leading to the non–linear stabilizing PWM controller design for the buck-boost derived converter. The feedback loop synthesizing the required duty ratio function is based on a desired steady state average input current of value X.

The buck–boost–derived switch regulated model

$$\begin{aligned} \dot{x} &= -\frac{R}{L}(1-u)x - \frac{E}{L}\,u \\ &= \theta_1(1-u)x - \theta_2 u \end{aligned} \tag{2.38}$$

PWM feedback regulation strategy for the switch position

$$u(t) = \begin{cases} 1 & \text{for} \quad t_k \le t < t_k + \mu(t_k)T \\ 0 & \text{for} \quad t_k + \mu(t_k)T \le t < t_k + T \end{cases} \tag{2.39}$$
$$t_{k+1} = t_k + T \ ; \ k = 0, 1, 2, \ldots$$

The buck–boost–derived converter is popularly known as the "step–up–down" converter. This is due to the fact that the average value of the input current is, theoretically, capable of achieving all values between 0 and ∞.

An exact discretization of the PWM regulated buck–boost–derived dynamics

$$x(t_k + \mu(t_k)T) = -\theta_2\mu(t_k)T + x(t_k) \tag{2.40}$$

$$x(t_k+T) = e^{-\theta_1 T(1-\mu(t_k))}\left[-\theta_2\mu(t_k)T + x(t_k)\right]$$
$$= \Psi_1^{(1-\mu(t_k))}x(t_k) - \Psi_1^{(1-\mu(t_k))}\Psi_3\mu(t_k) \tag{2.41}$$

Steady–state value of the sampled input current

$$x_\infty^- = -\frac{\Psi_1^{(1-\mu_\infty)}\mu_\infty\Psi_3}{1-\Psi_1^{(1-\mu_\infty)}} \tag{2.42}$$

Discrete–time dynamics of the "upper corners" of the PWM regulated input current trajectory

$$x(t_{k+1}+\mu(t_{k+1})T) = \Psi_1^{(1-\mu(t_k))}x(t_k+\mu(t_k)T) - \Psi_3\mu(t_{k+1}) \tag{2.43}$$

Steady state value of the "intersampling" peaks of the input current

$$x_\infty^+ = -\frac{\mu_\infty\Psi_3}{1-\Psi_1^{(1-\mu_\infty)}} \tag{2.44}$$

Steady state "ripple"

$$r_\infty = x_\infty^+ - x_\infty^- = -\Psi_3\mu_\infty \tag{2.45}$$

Steady state average value of the input current trajectory

$$x_{av}(\infty) = x_\infty^- + \frac{1}{2}r_\infty = -\frac{\left[\Psi_3\mu_\infty\left(1+\Psi_1^{(1-\mu_\infty)}\right)\right]}{2\left(1-\Psi_1^{(1-\mu_\infty)}\right)} \tag{2.46}$$

Note that $2X_{av} < -\Psi_3\mu_\infty$.

Existence of steady state duty ratio function for desired value of steady state average input current

$$\Psi_1^{(1-\mu_\infty)} = \frac{2X_{av}(\infty)+\Psi_3\mu_\infty}{2X_{av}(\infty)-\Psi_3\mu_\infty} \tag{2.47}$$

A solution for μ_∞ always exists in $[0,1]$ from the fact that $X_{av}(\infty) < 0$ and $\Psi_3 > 0$. Indeed, on the interval $[0,1]$, the left hand side varies from $\Psi_1 < 1$ to 1 while the right hand side varies from 1 to $(2X_{av}(\infty)+\Psi_3)/(2X_{av}(\infty)-\Psi_3)$, which is less than 1, and may even be negative, by virtue of the fact that $|2X_{av}(\infty)-\Psi_3| > |2X_{av}(\infty)+\Psi_3|$.

Desired linear asymptotically stable closed loop dynamics

$$x(t_{k+1}) = \alpha\left(x(t_k) - x_\infty^-(X)\right) + x_\infty^-(X) \quad ; \quad |\alpha| < 1 \tag{2.48}$$

Implicit nonlinear feedback duty ratio synthesizer

$$\Psi_1^{(1-\mu_c(t_k))} = \frac{\alpha x(t_k) + (1-\alpha)\,x_\infty^-(X)}{x(t_k) - \Psi_3\mu_c(t_k)} \tag{2.49}$$

Given $x(t_k)$ a solution exists for $\mu_c(t_k)$, in the interval $[0, 1]$ if and only if the following condition holds,

$$x_\infty^-(X) < x(t_k) - \frac{\Psi_3}{1-\alpha} \tag{2.50}$$

which has a similar interpretation to that of condition (2.36).

2.10 Simulation Results

The following noise-perturbed model of the buck-boost-derived converter was used in computer simulations for the proposed implicit feedback duty ratio synthesizer,

$$\dot{x} = -\frac{R}{L}(1-u)x - \left(\frac{E+\nu(t)}{L}\right) u \tag{2.51}$$

The values for the parameters defining the converter, the sampling period and the eigenvalue for the desired linear closed loop dynamics were taken to be the same as in the previous case. The deisred steady state value for the average input current was set to be -1500 A.

The corresponding value of the steady state input current was found to be $x_\infty^-(-1500) = -1304$ A.

Figure 10 depicts a typical simulated PWM feedback controlled trajectory for the input current arising from the perturbed model (2.51). The actual duty ratio function $\mu(t)$ and the corresponding switch position function $u(t)$, along with the perturbation signal $\nu(t)$, are also shown in this figure.

3 Conclusions

In this article an exact discretization scheme has been proposed for the regulation of perfectly known derived dc–to–dc power supplies of the buck, boost and buck–boost types. The complexities arising in the stabilization problem associated with such devices are related, fundamentally, to the highly nonlinear form of the derived duty ratio compensators. For the boost and the buck–boost converters, such controllers cannot be found explicitly. a transcendental equation must be solved on–line at each sampling instant on the basis of the current (sampled) state of the converter circuit.

Some of the difficulties encountered in the simple one–dimensional cases here treated become more striking when dealing with the traditional two dimensional converters, including output low pass filters based on RC arrangements. In this case, the symbolic manipulation task associated with the solution of the regulation problem becomes particularly intricate, even with the help of very efficient computer packages such as Maple, or Mathematica.

As a topic for further research, the case of derived converters with uncertain parameters is of particular practical interest and one for which efficient nonlinear discrete–time adaptive control techniques must be developed.

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FIGURES

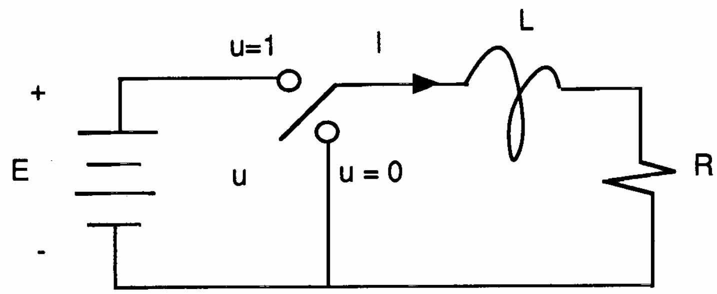

Figure 1: The Buck–derived Converter

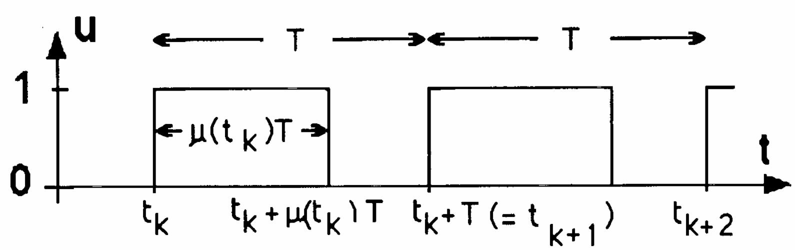

Figure 2: PWM Commanded Switch Position Function

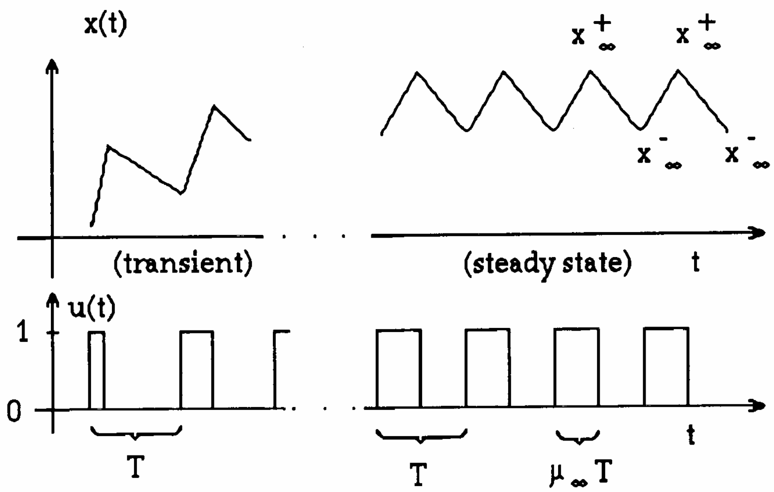

Figure 3: Transient and Steady State PWM Controlled State Trajectory

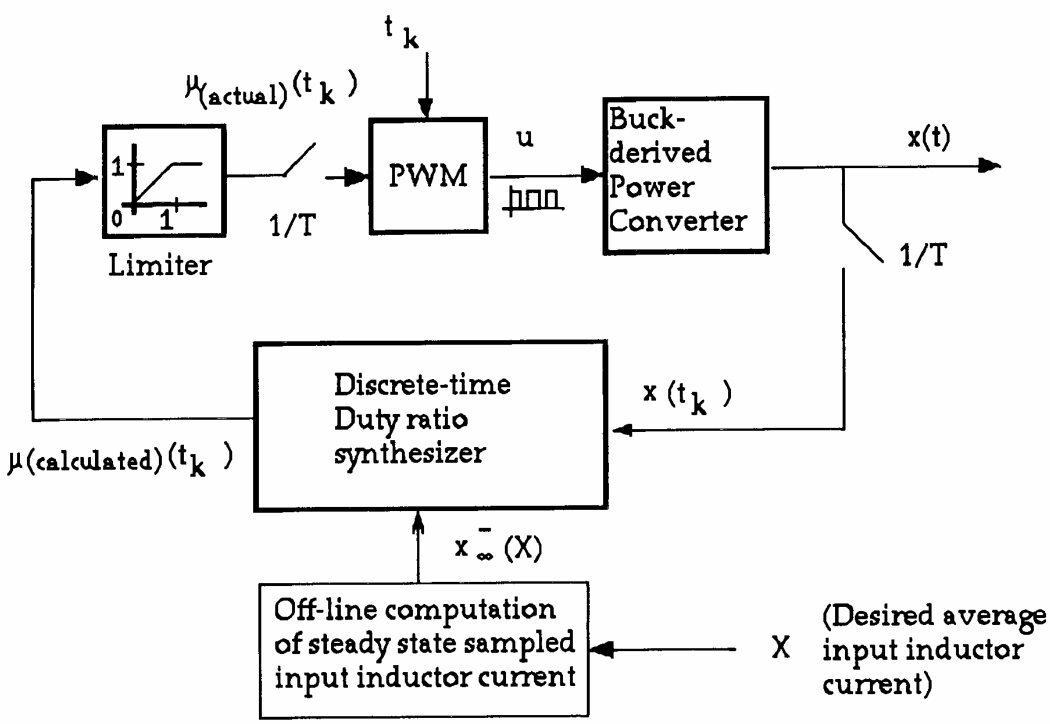

Figure 4: A PWM Feedback Regulation Scheme for the Buck–derived Converter Based on Exact Discretization

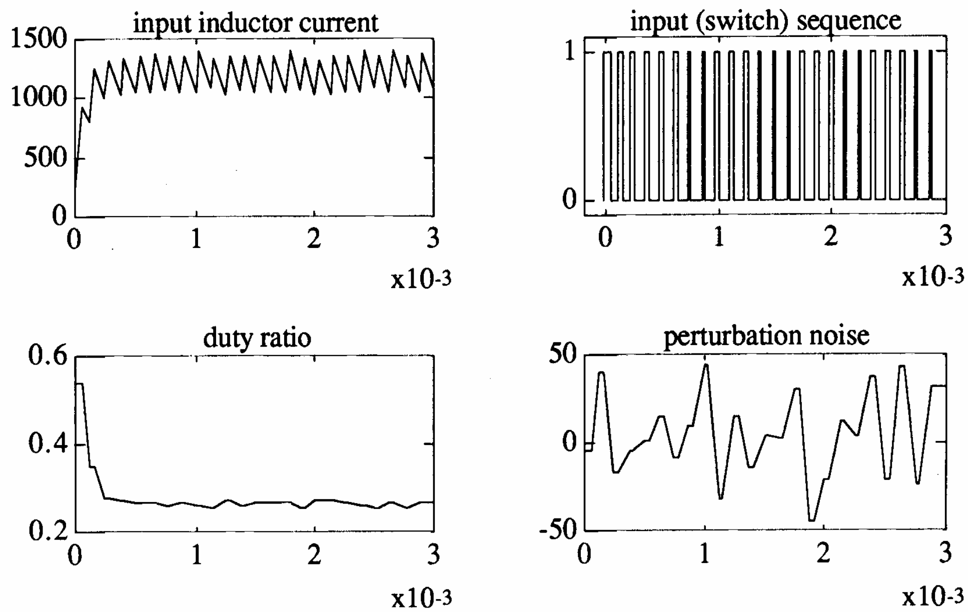

Figure 5: Simulation Results of PWM Regulation of Perturbed Buck–derived Converter

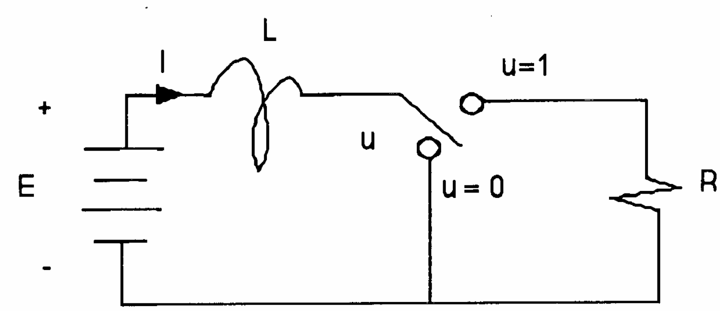

Figure 6: The Boost-derived Converter

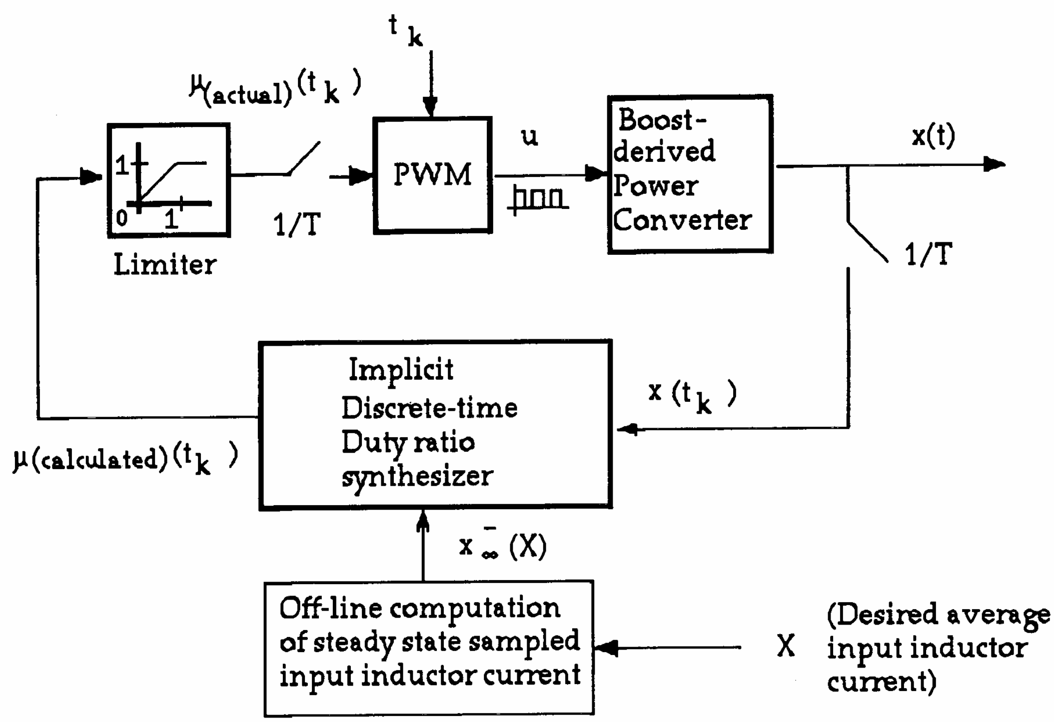

Figure 7: A PWM Feedback Regulation Scheme for the Boost–derived Converter Based on Exact Discretization

Figure 8: Simulation Results of PWM Regulation of Perturbed Boost–derived Converter

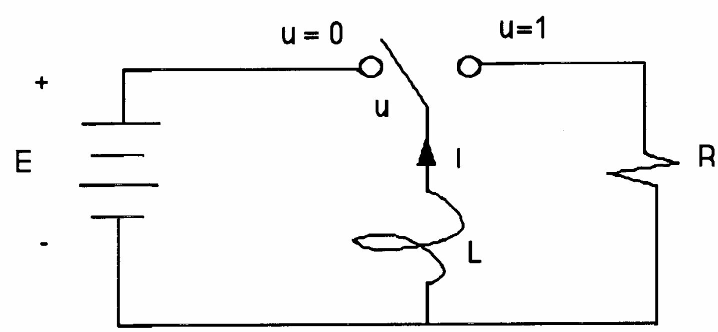

Figure 9: The Buck–Boost–derived Converter