Proceedings of the 34th
Conference on Decision & Control
New Orleans, LA - December 1995

Adaptive backstepping PWM stabilization of DC-to-DC converters towards minimum or non-minimum phase equilibria

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Abstract

The linear parameterization assumption often made in adaptive regulation schemes of nonlinear systems can be regarded not only as a counterintuitive assumption but also one with little physical significance. In this paper, a first order Taylor series approximation is proposed that linearizes the natural nonlinear dependencies upon the components of the vector of unknown parameters. This approach is shown to lead to an improved, and more realistic, non-overparameterized backstepping design based on input-output considerations. The results are applied to the direct adaptive output stabilization towards minimum or non-minimum phase equilibria of average Pulse-Width-Modulation controlled dc-to-dc power converters of the Boost type.

Key words : DC-to-DC Power Converters, Adaptive Regulation, Pulse-Width-Modulation, Backstepping, Nonlinear Parametrization

1 Introduction

In a large number of practical cases, the assumption of perfect knowledge of the circuit parameters of dc-to-dc converters is invalid. Ageing effects on the components, as well as high frequency phenomena, often alter the known nominal values. A number of adaptive control techniques have been proposed for the feedback regulation of such a class of uncertain systems. One of the fundamental assumptions made in most of the available adaptive control techniques lies in the fact that the components of the vector of unknown parameters enter the description of the system in a linear fashion. In particular, a recently developed adaptive control technique, known as the backstepping technique, mainly studied by Kokotovic and his co-workers (sec Kanellakopoulos et al [1] and Krstić et al [4]), still uses this linear parameterization assumption. For switch regulated systems, like dc-to-dc power converters, the adaptive backstepping design method has been used by Sira-Ramírez *et al* [9] from an input-output viewpoint for generalized canonical forms.

Briefly, the backstepping technique consists of designing temporary stabilizing controls and update laws regulating subsystems of order 1 to $n-1$ via Lyapunov arguments. At the final step of the algorithm, the real control and the n-th overparameterized update law are designed and the nth-order system is finally regulated. The more recently proposed version of the adaptive backstepping algorithm avoiding overparameterization also reduces the order of the controller and hence brings in more desirable stability properties. This last scheme will be adopted in this paper, rather than the more classical one. It is also interesting to notice that the backstepping algorithm is suitable for a larger class of systems than those in *pure parametric feedback form*, as we will show in the development of the paper.

In the recent literature, and thanks to the work of Utkin [11] on Variable Structure Systems, some sliding adaptive schemes have been proposed with special concern for robustness (see [10] and [2]). Similarly, discontinuous adaptive feedback PWM control of dc-to-dc power supplies has been defined in [8]. In fact, a simpler alternative based on a suitable average (i.e. infinite frequency) approximation of the dc-to-dc converter is usually proposed, with proofs of validity (sec [7]). Unfortunately, the average PWM model, thus obtained, is evidently not transformable into a "pure parametric fecdback form" as control input derivatives invariably appear at the final stage of the algorithm . This is due to the fact that the relative degree of the regulated output is less than the state dimension. Hence, rather than a classical input-state backstepping strategy, an input-output viewpoint is chosen, resulting in a dynamical duty ratio adaptive controller. It should be pointed out here that if the inductor current represents the regulated output, the so-called zero-dynamics associated with the output stabilization problem leads to the minimum-phase case, i.e. a stable duty ratio dynamics when the output is forced to zero. However, when the capacitor voltage output is to be regulated, the

0-7803-2685-7/95 $4.00 © 1995 IEEE

corresponding zero-dynamics is unstable around the equilibrium point and a non-minimum phase case is at hand. A feedback resetting strategy developed recently in [5] tackles this problem.

A need has long been felt for overcoming the nonphysical character of the linear parameterization restriction (see [6], [2] and [3]), explicit in the model assumptions of the adaptive backstepping procedure. In average models of dc–to–dc power supplies, nonlinear parameterization is inherent, which directly involves the lumped circuit component values R, C, L and E. This case may now be handled by using the improved backstepping algorithm presented in this paper. Our scheme takes into account the nonlinear parameter case and proposes the use of a first-order Taylor approximation thus enlarging the class of systems to which the algorithm is applicable.

In Section 2 of the paper, a statement of the problem is established through the description of the nonlinearly parameterized Boost dc-to-dc converter, the extension to the Buck and Buck-Boost converters being straightforward. Section 3 presents the non-overparameterized backstepping regulation towards a minimum phase equilibrium point for the average Boost converter model. Section 4 tackles the non-minimum phase case. Finally, Section 5 presents some simulation studies that highlight the validity of the proposed adaptive control algorithm.

2 Problem statement: the Boost DC–to–DC converter

In this section, we present the switchmode and the average PWM model of the Boost dc–to–dc power converter to be treated.

Consider the Boost converter circuit, shown in Figure 1. The system of differential equations, describing the inductor current $I(t)$ and the capacitor voltage $V(t)$, is given by

$$\begin{aligned} \dot{I}(t) &= -\frac{1}{L}(1-u)V(t) + \frac{E}{L} \\ \dot{V}(t) &= \frac{1}{C}(1-u)I(t) - \frac{1}{RC}V(t) \end{aligned} \qquad (1)$$

where L, C and R are, respectively, the inductance, capacitance and resistance values of the circuit components. Let us note the physical assumptions: L, C and R are nonzero. The quantity E represents the constant value of the external voltage source.

Let us denote the values of the several parameters defining the circuit equations as

$$\theta_1 = L \;\; ; \;\; \theta_2 = C \;\; ; \;\; \theta_3 = R \;\; ; \;\; \theta_4 = E \qquad (2)$$

with $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T \in \Omega \subset R^4$. The components of the unknown parameter vector, belonging to a given compact set Ω, are assumed to be constants (meaning slowly time-varying unknown parameters).

We shall be successively considering the inductor current $I(t)$ and the capacitor voltage $V(t)$ for the regulated output function, here denoted by y.

The control input function u is the *switch position function* taking values in the discrete set $\{0, 1\}$. A PWM feedback strategy for the specification of the switch position function u, occurring at regularly sampled instants of time is usually specified as follows:

$$u = \begin{cases} 1 & for\ t_k \le t < t_k + \mu_a(t_k)T \\ 0 & for\ t_k + \mu_a(t_k)T \le t < t_k + T \end{cases}$$
$$t_k + T = t_{k+1} \;\; ; \;\; k = 0, 1, \ldots \qquad (3)$$

where $\mu_a(t_k)$ is the value of the *actual duty ratio* function at the sampling instant t_k and the sampling period, T, is assumed to be constant. This actual duty ratio determines the width of time Δt_k ($\Delta t_k = \mu_a(t_k)T$) where $u = 1$ between two sampling instants. This corresponds exactly to a Pulse Width Modulation of the control in order to achieve regulation.

The actual duty ratio function is obtained from a bounding operation carried out on the feedback *computed duty ratio function* , denoted by μ, which restricts the values of μ to the closed interval $[0, 1]$. This physical restriction results in a local stabilization result, as it is well-known in the literature.

A classical and direct output stabilization of system (1), via control (3), would be difficult to perform since it would require an *exact discretization* procedure and a nonlinear discrete time duty ratio feedback design. For this reason, a simpler alternative is usually proposed which is based on a suitable average (i.e. infinite frequency) approximation of the converter model. In spite of the fact that the average approximation deteriorates for low sampling frequencies, its validity has been well established (see Sira-Ramírez [7]).

This assumption of infinite sampling frequency results in a smooth nonlinear average system, in which μ is interpreted as the control input, i.e. the *equivalent control* in Sliding Mode theory (see Utkin [11]). Therefore the problem can be advantageously treated as a standard *nonlinear feedback controller design problem*. The *Average Model* for the PWM controlled Boost converter (1), (3), is then obtained by formally replacing the switch position function u by the duty ratio function μ, and the state variables by their average

$$\begin{aligned} \dot{\zeta}_1 &= -\frac{1}{\theta_1}(1-\mu)\zeta_2 + \frac{\theta_4}{\theta_1} \qquad \eta = \zeta_1 \\ \dot{\zeta}_2 &= \frac{1}{\theta_2}(1-\mu)\zeta_1 - \frac{1}{\theta_2\theta_3}\zeta_2 \end{aligned} \qquad (4)$$

For a constant value of the duty ratio function, corresponding to a desired set point, $\mu = U$, with $0 < U < 1$, the equilibrium values of the average PWM converter state variables are readily obtained from (2) and (4) as

$$\zeta_1(U) = \frac{\theta_4}{\theta_3(1-U)^2} \quad ; \quad \zeta_2(U) = \frac{\theta_4}{(1-U)}$$

3 Backstepping Regulation towards a minimum-phase equilibrium

What we first aim at via this dynamical non–overparameterized adaptive backstepping controller is to achieve the feedback regulation of the average input inductor current $\zeta_1(t)$, towards a known, constant, equilibrium value, denoted by $X_1 = \zeta_1(U)$. Thus, indirect feedback regulation of the output capacitor voltage is accomplished. Let us note that the traditional backstepping dynamical controller associated with the Parametric Pure and Parametric Strict Feedback Canonical forms, cannot be applied here, since "control" input and its first-order time derivative naturally appear in our proposed regulation procedure for the average Boost converter model (4). The approach leads to an adaptive controller which is *dynamical* in nature. We will hence proceed to apply a modified version of the backstepping algorithm to the synthesis of a non–overparameterized adaptive feedback controller. Then, once the adaptive controller expressions are found, the average state variables ζ_1, ζ_2, appearing in the feedback controller are substituted, respectively, by the actual (i.e. non-averaged) variables $I(t)$, $V(t)$.

Now, let the average PWM Boost converter model (4), where η is the average value of the output signal, i.e. the inductor current $I(t)$.

Let us recall that the parameters θ_i , $i = 1, \ldots, 4$ represent the *actual* values of the uncertain parameters, as given by (2). Instead of the true values, $\theta = [\theta_1, \ldots, \theta_p]^T$, which are unknown, a controller will be designed using parameter estimates $\widehat{\theta} = [\widehat{\theta}_1, \ldots, \widehat{\theta}_p]^T$.

Assumption A1: In the following, when functions of ζ and θ, say $\gamma(\zeta, \theta)$ are used, we will introduce the error function $\gamma(\zeta,\theta) - \gamma(\zeta,\widehat{\theta})$, and we will consider only the truncated first-order Taylor approximation $\frac{\partial\gamma}{\partial\theta}(\zeta,\widehat{\theta})(\theta - \widehat{\theta})$, assuming that the truncated term $\mathcal{O}((\theta-\widehat{\theta})^T(\theta-\widehat{\theta}))$ will not hamper the closed-loop stability of the system. This truncation will be validated in the simulation study.

Step 0

Let z_1 stand for the output variable error, defined as $z_1 = \eta - \zeta_1(U) = \zeta_1 - \zeta_1(U)$

According to the average system model equations (4) the time derivative of the output error z_1, is of unknown nature and given by

$$\dot{z}_1 = -\frac{1}{\theta_1}(1-\mu)\zeta_2 + \frac{\theta_4}{\theta_1} \qquad (5)$$

An estimate of the time derivative of the error variable z_1 may be obtained directly from (5) by replacing the components of the unknown parameter vector θ by their estimated values $\widehat{\theta}$,

$$\widehat{\dot{z}}_1 = -\frac{1}{\widehat{\theta}_1}(1-\mu)\zeta_2 + \frac{\widehat{\theta}_4}{\widehat{\theta}_1} = \gamma_1(\zeta,\widehat{\theta},\mu) \qquad (6)$$

By using expression (6) and the first-order Taylor approximation assumption, we can rewrite expression (5) as follows :

$$\dot{z}_1 = \widehat{\dot{z}}_1 + (\theta-\widehat{\theta})^T \frac{\partial\gamma_1}{\partial\widehat{\theta}}^T \qquad (7)$$

$$\text{with: } \frac{\partial\gamma_1}{\partial\theta} = \left[\begin{array}{cccc} \frac{(1-\mu)\zeta_2-\widehat{\theta}_4}{\widehat{\theta}_1^2} & 0 & 0 & \frac{1}{\widehat{\theta}_1} \end{array} \right]$$

Step 1

Let us impose the pseudo-controller error z_2:

$$z_2 = \widehat{\dot{z}}_1 + c_1 z_1 \qquad (8)$$

where c_1 is a positive design constant.

In fact, in the previous literature, z_2 corresponds to the error between ζ_2, which temporarily plays the role of subsystem (5) control, and the stabilizing control α_1, given by:

$\alpha_1(\zeta_1,\zeta_2,\widehat{\theta}) = \zeta_2 - c_1(\zeta_1 - \zeta_1(U)) + \frac{(1-\mu)\zeta_2}{\widehat{\theta}_1} - \frac{\widehat{\theta}_4}{\widehat{\theta}_1}$.

Choosing Γ to be a positive definite diagonal matrix whose elements will be called *parameter adaptation gains*, let us then consider a scalar positive definite Lyapunov function of the form

$V_1 = \frac{1}{2}\left[z_1^2 + (\theta-\widehat{\theta})^T\Gamma^{-1}(\theta-\widehat{\theta})\right]$

with respect to which one wants to stabilize the one-dimensional subsystem derived from (7) and (8):

$$\dot{z}_1 = z_2 - c_1 z_1 + (\theta-\widehat{\theta})^T \frac{\partial\gamma_1}{\partial\widehat{\theta}}^T \qquad (9)$$

The time derivative of V_1 is then given by:

$\dot{V}_1 = z_1 z_2 - c_1 z_1^2 + (\theta-\widehat{\theta})^T\Gamma^{-1}\left(-\dot{\widehat{\theta}} + z_1\,\Gamma\,\frac{\partial\gamma_1}{\partial\widehat{\theta}}^T\right)$

Let us denote the temporary update law, the so-called tuning function τ_1, as follows:

$\tau_1(\zeta,\widehat{\theta},\mu) = z_1\,\Gamma\,\frac{\partial\gamma_1}{\partial\widehat{\theta}}^T$

By using this tuning function as the update law: $\dot{\widehat{\theta}} = \tau_1$, and by considering no pseudo controller error (i.e.: $z_2 = 0$), subsystem (9) is stabilized around the equilibrium point: $z_1 = 0$, i.e. $\zeta_1 = \zeta_1(U)$,

since the Lyapunov function derivative is reduced to $\dot{V}_1 = -c_1 z_1^2$.

However, as τ_1 is not the actual update law (i.e. $\dot{\widehat{\theta}} \neq \tau_1$) and as $z_2 \neq 0$, we hence obtain:

$$\begin{aligned} \dot{z}_1 &= z_2 - c_1 z_1 + (\theta-\widehat{\theta})^T \frac{\partial\gamma_1}{\partial\widehat{\theta}}^T \\ \dot{V}_1 &= z_1 z_2 - c_1 z_1^2 + (\theta-\widehat{\theta})^T\Gamma^{-1}\left(\tau_1 - \dot{\widehat{\theta}}\right) \end{aligned}$$

Step 2

At the second and final stage, the control and the actual update law are simultaneously designed in order to stabilize the whole system with respect to the new Lyapunov function V_2 given by:

$V_2 = V_1 + \frac{1}{2}z_2^2 = \frac{1}{2}\left[z_1^2 + z_2^2 + (\theta-\widehat{\theta})^T\Gamma^{-1}(\theta-\widehat{\theta})\right]$

The expression of z_2 is derived from (6) and (8):

$$z_2 = c_1 z_1 - \frac{(1-\mu)\zeta_2}{\widehat{\theta}_1} + \frac{\widehat{\theta}_4}{\widehat{\theta}_1}. \qquad (10)$$

Then, the pseudo-controller error z_2 dynamics is given by:

$$\dot{z}_2 = c_1\gamma_1(\zeta,\theta,\mu) + \frac{\dot{\widehat{\theta}}_4}{\widehat{\theta}_1} - \frac{\widehat{\theta}_4\dot{\widehat{\theta}}_1}{\widehat{\theta}_1^2} + \frac{\dot{\mu}\zeta_2}{\widehat{\theta}_1} + \frac{(1-\mu)\zeta_2\dot{\widehat{\theta}}_1}{\widehat{\theta}_1^2} - \frac{(1-\mu)\gamma_2(\zeta,\theta,\mu)}{\widehat{\theta}_1}$$

with : $\gamma_2(\zeta,\theta,\mu) = \frac{(1-\mu)\zeta_1}{\widehat{\theta}_2} - \frac{\zeta_2}{\widehat{\theta}_2\widehat{\theta}_3}$

And, as for $\widehat{\dot{z}_1}$ defined in (6), the estimate $\widehat{\dot{z}_2}$ is written as follows:

$$\widehat{\dot{z}_2} = c_1\gamma_1(\zeta,\widehat{\theta},\mu) + \frac{\dot{\widehat{\theta}}_4}{\widehat{\theta}_1} - \frac{\widehat{\theta}_4\dot{\widehat{\theta}}_1}{\widehat{\theta}_1^2} + \frac{\dot{\mu}\zeta_2}{\widehat{\theta}_1} + \frac{(1-\mu)\zeta_2\dot{\widehat{\theta}}_1}{\widehat{\theta}_1^2} - \frac{(1-\mu)\gamma_2(\zeta,\widehat{\theta},\mu)}{\widehat{\theta}_1} \quad (11)$$

Finally, we derive from (11) and the first-order Taylor approximation assumption the expression of the derivative of the pseudo–controller error z_2:

$$\dot{z}_2 = \widehat{\dot{z}_2} + (\theta-\widehat{\theta})^T\left(c_1\frac{\partial\gamma_1}{\partial\theta}^T - \frac{(1-\mu)}{\widehat{\theta}_1}\frac{\partial\gamma_2}{\partial\theta}^T\right) \quad (12)$$

with : $\frac{\partial\gamma_2}{\partial\theta}^T = \left[0 \quad \frac{\zeta_2-\widehat{\theta}_3\zeta_1(1-\mu)}{\widehat{\theta}_3\widehat{\theta}_2^2} \quad \frac{\zeta_2}{\widehat{\theta}_3^2\widehat{\theta}_2} \quad 0\right]$

The derivative of V_2 is then derived from (12):

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2\\ &= z_1z_2 - c_1z_1^2 + z_2\widehat{\dot{z}}_2(\theta-\widehat{\theta})^T\Gamma^{-1}\\ &\left[\tau_1 + \Gamma z_2\left(c_1\frac{\partial\gamma_1}{\partial\widehat{\theta}}^T - \frac{(1-\mu)}{\widehat{\theta}_1}\frac{\partial\gamma_2}{\partial\widehat{\theta}}^T\right) - \dot{\widehat{\theta}}\right]\end{aligned}$$

Then, we impose the pseudo–control error dynamics:

$$\widehat{\dot{z}_2} = -c_2z_2 \quad ; \quad c_2 > 0 \quad (13)$$

Moreover, the update law $\dot{\widehat{\theta}}$ is chosen in order to make the expression of $\dot{V}_2$ independent of $(\theta-\widehat{\theta})$

$\dot{\widehat{\theta}} = \Gamma\left[(z_1+c_1z_2)\frac{\partial\gamma_1}{\partial\theta}^T - z_2\frac{(1-\mu)}{\widehat{\theta}_1}\frac{\partial\gamma_2}{\partial\theta}^T\right]$,

which gives in the original coordinates (ζ_1,ζ_2) the expressions:

$$\dot{\widehat{\theta}} = \Gamma\begin{bmatrix} A\left(\frac{(1-\mu)\zeta_2-\widehat{\theta}_4}{\widehat{\theta}_1}\right)\\ B\left(\frac{\zeta_2-\widehat{\theta}_3(1-\mu)\zeta_1}{\widehat{\theta}_2}\right)\\ B\frac{\zeta_2}{\widehat{\theta}_3}\\ A\end{bmatrix} \quad (14)$$

$$\text{with} \quad A(\zeta,\widehat{\theta},\mu) = \frac{1}{\widehat{\theta}_1^2}[(1+c_1^2)(\zeta_1-\zeta_1(U))\widehat{\theta}_1 + c_1\widehat{\theta}_4 - c_1(1-\mu)\zeta_2]$$

$$B(\zeta,\widehat{\theta},\mu) = -(1-\mu)\frac{(\zeta_1-\zeta_1(U))c_1\widehat{\theta}_1 + \widehat{\theta}_4 - (1-\mu)\zeta_2}{\widehat{\theta}_2\widehat{\theta}_3\widehat{\theta}_1^2},$$

and the matrix of adaptation gains: $\Gamma = diag\{\Gamma_1,\Gamma_2,\Gamma_3,\Gamma_4\}$.

Finally, the actual control is readily obtained in an implicit manner, as a solution of a nonlinear time-varying differential equation, derived from (10), (11) and (13):

$$\begin{aligned}\dot{\mu} = \ & \frac{\widehat{\theta}_1}{\zeta_2}\{-c_1c_2(\zeta_1-\zeta_1(U)) + \frac{1}{\widehat{\theta}_1}[c_1+c_2\\ &+\Gamma_1A(\frac{\widehat{\theta}_4-(1-\mu)\zeta_2}{\widehat{\theta}_1^2})]((1-\mu)\zeta_2-\widehat{\theta}_4)\\ &-\Gamma_4\frac{A}{\widehat{\theta}_1} - (1-\mu)(\frac{\zeta_2-\widehat{\theta}_3(1-\mu)\zeta_1}{\widehat{\theta}_1\widehat{\theta}_2\widehat{\theta}_3})\}\end{aligned} \quad (15)$$

Remarks :

- **A2**: A non-restrictive assumption, which has a physical meaning since generally $\zeta_2(U) \neq 0$, is derived from dynamical expression (15) of the control and is given by : $\zeta_2(t) \neq 0 \quad \forall t$. Hence, this singularity of the controller can always be conveniently avoided with a judicious choice of the initial conditions of the system.

- **A3**: The input dependent state coordinates transformation linking the original average state variables ζ_1 and ζ_2 to the new coordinates (z_1,z_2), respectively, the output error and the pseudo–controller error, given by: $z_1 = \zeta_1 - \zeta_1(U)$

$z_2 = \frac{\widehat{\theta}_4-(1-\mu)\zeta_2}{\widehat{\theta}_1} + c_1(\zeta_1-\zeta_1(U))$

is locally invertible, since the Jacobian matrix of this transformation is non–singular everywhere except at persistently saturated values of the duty ratio function $\mu = 1$: $\quad \frac{\partial z}{\partial\zeta} = \begin{bmatrix} 1 & 0\\ c_1 & -\frac{(1-\mu)}{\widehat{\theta}_1}\end{bmatrix}$

Note that the local non–singularity of the Jacobian matrix is equivalent to the local observability of the average system (4) (see [7]).

Finally, provided that assumptions **A1**, **A2** and **A3** are satisfied, and with the additional assumption **A4**: $4c_1c_2 > 1$ – which corresponds to a certain design of the parameters c_1 and c_2 –, if one uses the parameter update law (14) and the dynamical control (15), the time derivative of the Lyapunov function V_2, is given by: $\dot{V}_2 = z_1z_2 - c_1z_1^2 - c_2z_2^2$, and $\dot{V}_2$ is a negative definite function.

Then, an asymptotically stable behaviour to zero can be guaranteed for both the output error z_1 and the pseudo–controller error z_2 while achieving bounded evolution of the parameter estimates $\widehat{\theta}$ (see proof in [1]). The regulation around the original average equilibrium point (ζ_1,ζ_2) follows immediately. Moreover, note that the proof of local asymptotical stability for the initial exact system (1) (instead of the approximated system with **A1**) is given in [3].

Finally, the actual duty ratio synthesizer for the PWM regulated system is obtained by bounding the computed controller μ to the closed interval $[0,1]$, and by replacing the average state variables ζ_1, ζ_2, appearing in the controller expressions, respectively, with the actual state variables $I(t)$ and $V(t)$. The PWM feedback strategy is then evidently derived from (3).

Note that in this case of the inductance current output, and when the estimated parameters equal the true ones (non-adaptive case), the *zero-dynamics* – i.e. when $z_1 = 0$ – , derived from the Fliess Generalized Observability Canonical form, are stable around $\mu = U$ and given by (see [9]):

$$\dot{\mu} = \frac{1}{\widehat{\theta}_2\widehat{\theta}_3(1-U)^2}(1-\mu)(2-U-\mu).$$

4 Backstepping Regulation towards a non minimum-phase equilibrium

In this section, we briefly give the main results for the regulation of the Boost dc-to-dc converter output capacitor voltage. It is well known that if the output capacitor voltage $V(t)$ is taken as the regulated output, then the system is non-minimum phase (see [7]). In such a non-minimum phase case, our proposed method leads to an *unstable* adaptive controller. Then, as in a paper of Sira-Ramírez *et al* (see [5]) , a solution consists of a controller resetting strategy.

Average model with V as output

$$\begin{aligned} \dot{\zeta}_1 &= \frac{1}{\theta_2}(1-\mu)\zeta_2 - \frac{1}{\theta_2\theta_3}\zeta_1 \qquad \eta = \zeta_1 \\ \dot{\zeta}_2 &= -\frac{1}{\theta_1}(1-\mu)\zeta_1 + \frac{\theta_4}{\theta_1} \\ \text{with } \theta_1 &= L \ ; \ \theta_2 = C \ ; \ \theta_3 = R \ ; \ \theta_4 = E \\ \zeta_1 &= V_{av} \ ; \ \zeta_2 = I_{av} \text{ and } \mu = u_{av}. \end{aligned}$$

Parameter estimation update law

$$\dot{\widehat{\theta}} = \Gamma \begin{bmatrix} A(\frac{(1-\mu)\zeta_1 - \widehat{\theta}_4}{\widehat{\theta}_1}) \\ B(\frac{\zeta_1 - \widehat{\theta}_3(1-\mu)\zeta_2}{\widehat{\theta}_2}) \\ B\frac{\zeta_1}{\widehat{\theta}_3} \\ A \end{bmatrix} \qquad (16)$$

$$\begin{aligned} \text{with } A(\zeta,\widehat{\theta},\mu) = & \frac{1}{\widehat{\theta}_1\widehat{\theta}_2}(1-\mu)[(c_1 - \frac{1}{\widehat{\theta}_2\widehat{\theta}_3})\zeta_1 \\ & - c_1\zeta_1(U) + \frac{1}{\widehat{\theta}_2}(1-\mu)\zeta_2] \\ \text{and } B(\zeta,\widehat{\theta},\mu) = & \frac{1}{\widehat{\theta}_2\widehat{\theta}_3}[(1 + c_1(c_1 - \frac{1}{\widehat{\theta}_2\widehat{\theta}_3}))(\zeta_1 - \zeta_1(U)) \\ & - \frac{1}{\widehat{\theta}_2\widehat{\theta}_3}(c_1 - \frac{1}{\widehat{\theta}_2\widehat{\theta}_3})(\zeta_1 - \widehat{\theta}_3(1-\mu)\zeta_2)]. \end{aligned}$$

Adaptive non-overparameterized duty ratio

$$\begin{aligned} \dot{\mu} = & \frac{\widehat{\theta}_2}{\zeta_2}[-\frac{B\zeta_2}{\widehat{\theta}_2^3}\Gamma_2(1-\mu)(\zeta_1 - \widehat{\theta}_3(1-\mu)\zeta_2) \\ & + \frac{(1-\mu)}{\widehat{\theta}_1\widehat{\theta}_2}(\widehat{\theta}_4 - (1-\mu)\zeta_1) - c_1(c_1 - \frac{1}{\widehat{\theta}_2\widehat{\theta}_3})(\zeta_1 - \zeta_1(U)) \\ & + (c_1(\zeta_1 - \zeta_1(U)) - (\frac{\zeta_1 - \widehat{\theta}_3(1-\mu)\zeta_2}{\widehat{\theta}_2\widehat{\theta}_3}))(c_1 + c_2 - \frac{1}{\widehat{\theta}_2\widehat{\theta}_3}) \\ & + \frac{B\zeta_1}{\widehat{\theta}_2\widehat{\theta}_3}(\Gamma_2\frac{\zeta_1 - \widehat{\theta}_3(1-\mu)\zeta_2}{\widehat{\theta}_2^2} + \Gamma_3\frac{\zeta_1}{\widehat{\theta}_3^2})] \end{aligned} \qquad (17)$$

Remark : The same assumptions **A1**, **A2**, **A3** and **A4** as in the previous Section must be satisfied in this regulation scheme.

Zero-dynamics derived from the F.G.O.C. Form: $\dot{\mu} = -\frac{1}{\widehat{\theta}_1}\widehat{\theta}_3(1-\mu)^2(U-\mu)$.

Following easily from the phase-diagram, this zero-dynamics is unstable around $\mu = U$ and $\mu = 1$, which are therefore non-minimum phase equilibrium points. A control resetting strategy is then proposed in order to tackle this case, and consists in producing a quasi-sliding motion around the desired equilibrium point $\mu = U$:

Let $0 < \delta < \epsilon$. The resetting strategy is as follows :

$$\mu(t) = \begin{cases} \text{if for some } t, \ \mid \mu(t) - U \mid = \epsilon, \\ \quad \text{then } \mu(t^+) = U - \delta \ \text{ sign}(\dot{\mu}) \\ \\ \text{otherwise, } \ \mu(t) \ \text{ obeys equation(17)} \end{cases}$$

Then, it is easy to prove that, with the simple choice of the sliding surface $s = \mu - U$, the attractivity condition of this surface at these instants t^+ is now satisfied and given by:

$$s\dot{s} = -\delta \mid \dot{\mu} \mid < 0.$$

5 Simulation results

Simulations were done for the Boost converter model in conjunction with the dynamical adaptive PWM controller described in Section 3 for the regulation of the input inductor current variable $I(t)$ of the converter. This kind of converters is used in the welding industry or elsewhere a constant voltage supply is needed. The nominal and perturbed versions of this converter model have been studied in simulation. The perturbation has consisted of an unmodelled stochastic but bounded noise -denoted by $\nu(t)$- acting on the circuit through the external source voltage E. Hence, $\nu(t)$ is an unmatched additive disturbance, as shown in the following perturbed model, used in simulations:

$$\begin{aligned} \dot{I}(t) &= -\frac{1}{L}(1-u)\,V(t) + \left(\frac{E+\nu(t)}{L}\right) \qquad y = I(t) \\ \dot{V}(t) &= \frac{1}{C}(1-u)\,I(t) - \frac{1}{LC}V(t) \end{aligned}$$

The following "unknown" values of the circuit parameters were used for simulation purposes

$C = 181.8\mu F; L = 0.27mH; R = 2.44\Omega; E = 14.66V$

The sampling frequency was set to $100KHz$ which corresponds to about 500 clock pulses (by using a 50 MHz normal PC) for each sampling period . One could even have better computational allowances by decreasing the switch frequency but this may increase chattering. Let us also note that the MOSFET transistor is able to switch at this frequency value and is resistant to this power value (about 375 Watts).

As for the random input noise (expressing the electromagnetical pollution, the resistive effect, the sensor error...), its amplitude was set to 0.5 Volts.

The desired equilibrium value for the average input inductor current was set to $I(t) = 15.75$ amp. The obtained steady-state equilibrium value for the average output capacitor voltage was $V = 23.77Volts$. The duty ratio function corresponding to this equilibrium is $\mu = U = 0.38$.

The regulated output variable, $I(t)$, is seen to converge asymptotically towards the desired equilibrium value in the nominal and perturbed cases (respectively in Figure 2 and 3), pointing out good performances in robustness. The bounded evolution of the parameter estimates, a small portion of the switchings actions as well as the duty ratio function and the perturbation noise are also shown in the figures.

Conclusion : An input-output adaptive PWM feedback regulation scheme has been applied on a Boost converter model taking into account nonlinear parameterizations, relaxing by the way the nonphysical linear parameterization assumption. This scheme has been validated in a simulation study, as well as in a theoretical work [3].

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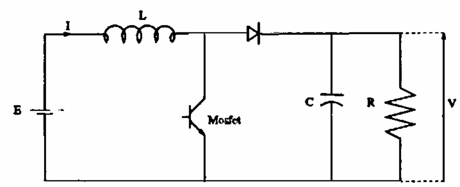

FIG. 1- Boost converter model

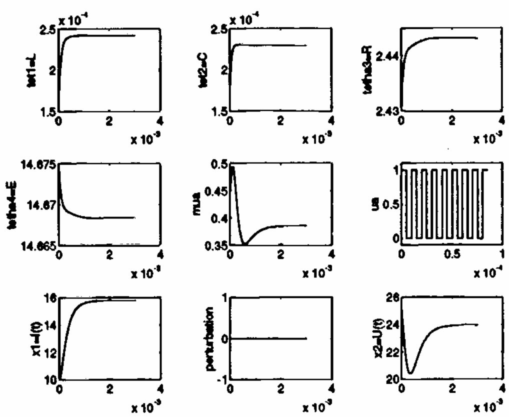

FIG. 2- Regulation of nominal Boost converter

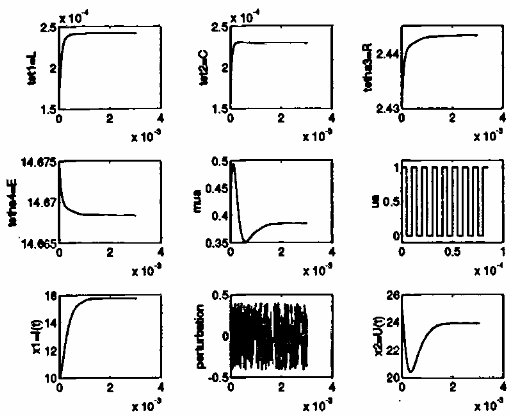

FIG. 3- Regulation of perturbed Boost converter