A PASSIVITY-BASED SLIDING MODE CONTROLLER FOR THE BOOST CONVERTER *

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Abstract

A passivity based sliding mode controlle is proposed which achieves the output feedback stabilization of a realistic model of a switchmode DC-to-DC Power Converter of the "Boost" type. The possibilities of using a state observer of discontinuous nature is also explored in detail. Suitable "damping injections" and "energy shappings" are shown to be independently accomplished, via discontinuous feedback and discontinuous feedforward output injections, for both the regulated plant and the sliding mode observer error dynamics, respectively.

Keywords : DC-to-DC Power Converters, Sliding Modes, Passivity Based Output Feedback Regulation.

1 Introduction

In this article an output feedback controller is proposed for the stabilization of a switch regulated DC-to-DC power converter of the "boost" type. The design entitles a suitable modification of the dissipation energy properties of the desired closed-loop stabilization error dynamics and of the state reconstruction error dynamics. This is achieved through dynamical discontinuous feedback and discontinuous feedforward output injections, respectively. We first obtain, under the assumption of full state availability, a dynamical passivity-based sliding mode feedback controller. An alternative observer-based output feedback scheme is next proposed using a sliding mode observer whose design is also based on passivity considerations. In the controller and observer designs the *workless* forces, inherent in the open loop dynamics of the system and of the observer, are never cancelled through feedback nor through the feedforward output injections. In this manner, the controller-observer structure is considerably simplified with respect to other schemes based on exact linearization.

Sliding mode control of dc-to-dc power converters was first treated by Venkataramanan *et al* [1] from an approximate linearization viewpoint. The topic was later treated by Sira-Ramírez [2], from a nonlinear differential geometric viewpoint. Connections of sliding mode controllers for dc-to-dc power converters with singular perturbation techniques, involving the natural time scale separation properties of the average models of the power converters, was treated by Sira-Ramírez and Ilic-Spong in [3]. An exact linearization approach for sliding mode controlled dc-to-dc power converters was also proposed by Sira-Ramírez and Ilic-Spong in [4]. More recently, sliding mode control of dc-to-dc power converters has been approached from an Extended Linearization viewpoint in the work of Sira-Ramírez and Rios-Bolívar [5]. Passivity based controllers for Pulse-Width-Modulation controlled models of ideal dc-to-dc power converters are also proposed in Sira-Ramírez and Ortega [7].

Section 2 of this article presents a passivity-based sliding mode controller for a realistic model of a "boost" converter. The sliding mode controller, which turns out to be dynamical in nature, is derived under the assumption of full state availability for feedback. Section 3 is devoted to derive a sliding mode observer by means of passivity considerations. The feedforward injection terms act as limited control variables which enhance dissipativeness of the estimation error energy. The overall asymptotic stability of the closed loop controller is presented in Section 4. The last section is devoted to some conclusions and suggestions for further work.

2 A Passivity–Based Sliding Mode Controller

Consider the following realistic model of the "boost" converter circuit proposed in the work of Czarkowski and Kazimierczuk, [8], shown in Figure 1

$$L\dot{x}_1 + (1-u)\frac{R}{r_C+R}x_2 + r(u)x_1 = E - (1-u)V_F \qquad (2.1)$$
$$C\dot{x}_2 - (1-u)\frac{R}{r_C+R}x_1 + \frac{1}{r_C+R}x_2 = 0$$

where L and C and R are respectively the input

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inductance, the output capacitance and the load resistance of the converter circuit. The variables x_1 and x_2 represent, the inductor current and the capacitor voltage, respectively. E is the voltage of the external source and u is an ideal *switch* position function taking values on the discrete set $\{0,1\}$. The control dependent resistance $r(u)$ is given by $r(u) = r_L + ur_{DS} + (1-u)R_F(1-u)r_C||R$, where the symbol $r_C||R$ stands for the resistance of the parallel connection of r_C and R. The resistances r_L and r_C represent parasitic resistances associated with the input inductor and the output capacitor, while r_{DS} and R_F are "ON" resistances associated with the transistor and diode constituting the switching arrangement. The voltage source V_F represents a parasitic voltage appearing across the diode during the conduction stages. In order to avoid non-minimum phase problems, the output of the converter will be taken to be the input inductor current x_1. However, for the time being we assume that the entire state vector is available for measurement (see [7])

For ease of reference we rewrite equation (2.1) in matrix form as

$$\mathcal{D}_B\dot{x} + (1-u)\mathcal{J}_B x + \mathcal{R}_B(u)x = \mathcal{E}_B(u) \qquad (2.2)$$

where

$$\begin{aligned} \mathcal{D}_B &= \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \; ; \; \mathcal{J}_B = \begin{bmatrix} 0 & \frac{R}{r_C+R} \\ -\frac{R}{r_C+R} & 0 \end{bmatrix} \\ \mathcal{R}_B(u) &= \begin{bmatrix} r(u) & 0 \\ 0 & \frac{1}{r_C+R} \end{bmatrix} \\ \mathcal{E}_B(u) &= \begin{bmatrix} E-(1-u)V_F \\ 0 \end{bmatrix} \end{aligned} \qquad (2.3)$$

The total stored energy of the system is given by

$$H = \frac{1}{2}x^T\mathcal{D}_B x = \frac{1}{2}(Lx_1^2 + Cx_2^2) \qquad (2.4)$$

The time derivative of the energy function H along the controlled trajectories of (2.1) result in

$$\begin{aligned} \dot{H}(t) &= x^T\mathcal{E}_B(u) - x^T\mathcal{R}_B(u)x \qquad (2.5) \\ &= x_1(E-(1-u)V_F) - \frac{1}{r_C+R}x_2^2 \\ &\quad -r(u)x_1^2 \\ &\leq x_1 E \end{aligned}$$

and upon integration of the previous expression on an arbitrary time interval $[0,t]$, the system is seen to be *passive* from the input source E to the output x_1 (see Ortega *et al* [9]). Note that the "forces" represented by $(1-u)\mathcal{J}_B x$ are indeed *workless* due to the skew-symmetry of $\mathcal{J}_B$

Let $x_d(t)$ denote a desired trajectory for the state variables of the "boost" converter (2.1). Denoting by $\tilde{x}(t)$ the error vector trajectory $x(t)-x_d(t)$, one obtains from (2.2) the following expression

$$\mathcal{D}_B\dot{\tilde{x}} + (1-u)\mathcal{J}_B\tilde{x} + \mathcal{R}_B\tilde{x} = \hat{\psi}(u) \qquad (2.6)$$

where

$$\hat{\psi}(u) = \mathcal{E}_B(u) - (\mathcal{D}_B\dot{x}_d + (1-u)\mathcal{J}_B x_d + \mathcal{R}_B x_d) \quad (2.7)$$

A desired damping can be achieved for the error system through the injection, to the dynamics (2.6), of a suitable term of the form

$$\mathcal{R}_{1B}\tilde{x} = \begin{bmatrix} R_1 & 0 \\ 0 & 1/R_2 \end{bmatrix}\tilde{x} \; ; \; R_1, R_2 > 0 \qquad (2.8)$$

Let the desired damping term be specified then by the $\mathcal{R}_{Bd}\tilde{x} = (\mathcal{R}_B + \mathcal{R}_{1B})\tilde{x}$. One finds immediately that

$$\mathcal{D}_B\dot{\tilde{x}} + (1-u)\mathcal{J}_B\tilde{x} + \mathcal{R}_{Bd}\tilde{x} = \psi(u) \qquad (2.9)$$

with

$$\begin{aligned} \psi(u) &= \mathcal{E}_B(u) \qquad (2.10) \\ &\quad - [\mathcal{D}_B\dot{x}_d + (1-u)\mathcal{J}_B x_d + \mathcal{R}_B x_d - \mathcal{R}_{1B}\tilde{x}] \end{aligned}$$

If $\psi(u)$ were identically zero, the resulting error dynamics, or the *desired error* dynamics

$$\mathcal{D}_B\dot{\tilde{x}} + (1-u)\mathcal{J}_B\tilde{x} + \mathcal{R}_{Bd}\tilde{x} = 0 \qquad (2.11)$$

would be asymptotically stable to zero. Indeed, the error system (2.11) has an associated total stored energy, given by: $H_d(t) = \frac{1}{2}\tilde{x}^T\mathcal{D}_B\tilde{x}$. The time derivative, along the trajectories of (2.11), satisfies, for some constant and positive scalar α, the following property,

$$\dot{H}_d(t) = -x^T\mathcal{R}_{Bd}(u)x \leq -\alpha||\tilde{x}||^2 \qquad (2.12)$$

The condition $\psi(u) = 0$ can be explicitly expressed as

$$\begin{aligned} & L\dot{x}_{1d} + r(u)x_{1d} + (1-u)\frac{R}{r_C+R}x_{2d} \\ & \quad -R_1(x_1 - x_{1d}) = E - (1-u)V_F \\ & C\dot{x}_{2d} - (1-u)\frac{R}{r_C+R}x_{1d} + \frac{1}{r_C+R}x_{2d} \\ & \quad -\frac{1}{R_2}(x_2 - x_{2d}) = 0 \end{aligned} \qquad (2.13)$$

Let $x_{1d} = I_d$, be the desired constant steady state value of the input inductor current. It is easy to see that, under this circumstance, the first expression in (2.13) is somewhat contradictory. Indeed, note that u is a variable that takes values in the discrete set $\{0,1\}$ while the functions x_1 and x_{2d} are at least continuous functions of time. In other words, the above pair of equations can only be exactly satisfied as long as u itself is a continuous function, which is a contradiction unless u is constant. However regarding u as a constant destroys all possibilities of *feedback* regulation. Hence, the expressions (2.13) must be regarded as holding valid in an *equivalent control* sense, i.e., in an average sense (see Utkin [10]). We therefore denote u as an equivalent control u_{eq} and x_{2d} as z_{2d}.

From the first expression in (2.13) one then obtains the value of the *virtual control* input, u_{eq}, that renders the first component of ψ identically zero in an average sense.

$$u_{eq} = \frac{a(x_1, z_{2d})}{b(z_{2d})} \tag{2.14}$$

with

$$\begin{aligned} a(x_1, z_{2d}) &= R_1(x_1 - I_d) + E - V_F \\ &\quad -(r_L + R_F + r_C||R)I_d - \frac{R}{r_C + R} z_{2d} \\ b(z_{2d}) &= (r_{DS} - R_F - r_C||R)I_d \\ &\quad - \frac{R}{r_C + R} z_{2d} - V_F \end{aligned} \tag{2.15}$$

A well known necessary and sufficient condition for the existance of a sliding regime is constituted by the following condition, which was rigorously obtained in [2],

$$0 < u_{eq} = \frac{a(x_1, z_{2d})}{b(z_{2d})} < 1 \tag{2.16}$$

The above conditions actually delimits a time-varying region in the space of the variable z_{2d} where such a sliding regime exists.

The second expression in (2.13), can also be regarded as being valid in an average sense, when evaluated for the obtained equivalent control input $u = u_{eq}$. The resulting differential equation actually constitutes an *ideal sliding dynamics*. Such dynamics is consistent with both the desire of having $x_{1d} = I_d$ and also with the fact that the relation $\psi = 0$ holds valid in an average sense.

$$\begin{aligned} C\dot{z}_{2d} - (1 - u_{eq})\frac{R}{r_C + R} I_d + \frac{1}{r_C + R} z_{2d} \\ - \frac{1}{R_2}(x_2 - z_{2d}) = 0 \end{aligned} \tag{2.17}$$

In fact, the expression (2.17) qualifies as a dynamically generated *duty ratio* function corresponding to the "infinite switching frequency" pulse width modulation feedback strategy, which is equivalent to the proposed ideal sliding mode behaviour (see [2] for details).

From the previous developments, it is clear, at least in an average or ideal slidng mode sense, that the "controller" state variable x_{2d} satisfies, $x_{2d} \to z_{2d}$. From the passivity analysis of the closed loop system we also have that, $x_2 \to x_{2d}$. Hence, it is also true that, $x_2 \to z_{2d}$, while $x_1 \to x_{1d} = I_d$. Let us denote by V_d the steady state value of z_{2d}. The equivalent control, u_{eq}, is then seen to asymptotically converge towards a constant value, denoted here by U, and given by

$$U = \frac{E - V_F - (r_L + R_F + r_C||R)I_d - \frac{R}{r_C+R} V_d}{(r_{DS} - R_F - r_C||R)I_d - \frac{R}{r_C+R} V_d - V_F} \tag{2.18}$$

The steady state value V_d of z_{2d}, written in terms of U may be obtained from (2.17) as

$$V_d = (1 - U)RI_d \tag{2.19}$$

Eliminating U from the previous expressions we obtain the steady state value of the output capacitor voltage V_d in terms of the desired input inductor current I_d. This elementary computation is left for the interested reader.

Since u is a discrete valued variable, we proceed to force the actual dynamical system in the second expression of (2.13) to behave as (2.17) in a *sliding mode* sense. Let e_{2d} denote the error of the controller state x_{2d} with respect to the ideal sliding dynamics state z_{2d}. In other words, $e_{2d} = x_{2d} - z_{2d}$. The error dynamics is readily obtained from (2.13) and (2.17) as.

$$\dot{e}_2 = -\frac{1}{C}\left(\frac{1}{R_2} + \frac{1}{r_C + R}\right) e_2 - \frac{RI_d}{C(r_C + R)}(u - u_{eq}) \tag{2.20}$$

From the fact that the equivalent control may be assumed to take values in the open interval $(0, 1)$, it is easy to see that the error e_{2d} is guaranteed to approach the condition $e_{2d} = 0$, in finite time, from any arbitrary initial condition. The switching policy that achieves such a sliding region is of the following form

$$u = \frac{1}{2}[1 + \text{sign}(x_{2d} - z_{2d})] \tag{2.21}$$

i.e., $u = 1$ for $e_2 > 0$ and $u = 0$ for $e_2 < 0$.

We have thus proven the following proposition (see also Figure 2)

Proposition

Given a desired constant value I_d for the input inductor current x_1. The following dynamically synthesized switching policy achieves asymptotically stable state trajectories, for the switch regulated plant, towards the desired equilibrium $x_1 = I_d$,

$$u = \frac{1}{2}[1 + \text{sign}(x_{2d} - z_{2d})]$$

where the variables x_{2d} and z_{2d} are obtained as the solutions of the following time-varying nonlinear differential equations with arbitrary initial conditions,

$$\begin{aligned} \dot{z}_{2d} &= (1 - u_{eq})\frac{R}{(r_C + R)C} I_d - \frac{1}{(r_C + R)C} z_{2d} \\ &\quad + \frac{1}{R_2 C}(x_2 - z_{2d}) \end{aligned}$$

and

$$\begin{aligned} \dot{x}_{2d} &= (1 - u)\frac{R}{(r_C + R)C} x_{1d} - \frac{1}{(r_C + R)C} x_{2d} \\ &\quad - \frac{1}{R_2 C}(x_2 - x_{2d}) \end{aligned}$$

with u_{eq} as given by (2.14) and (2.15).

3 A Passivity Based Sliding Mode Observer

In this section we assume that the only available state is constituted by the input inductor current $y = x_1$

We proceed to synthesize a sliding mode observer for the plant dynamics, rewritten, just for convenience, as

$$\begin{aligned}\dot{x}_1 &= -(1-u)\frac{R}{L(r_C+R)}x_2 - \frac{r(u)}{L}x_1 \\ &\quad + \frac{E-(1-u)V_F}{L} \\ \dot{x}_2 &= (1-u)\frac{R}{(r_C+R)C}x_1 - \frac{1}{(r_C+R)C}x_2 \\ y &= x_1 \end{aligned} \quad (3.1)$$

Consider, then, the following dynamical observer for the switched system (3.1)

$$\begin{aligned}\dot{\hat{x}}_1 &= -(1-u)\frac{R}{L(r_C+R)}\hat{x}_2 - \frac{r(u)}{L}\hat{x}_1 \\ &\quad + \frac{E}{L} - (1-u)\frac{V_F}{L} + \frac{h_1}{L}(y-\hat{y}) \\ \dot{\hat{x}}_2 &= (1-u)\frac{R}{(r_C+R)C}\hat{x}_1 - \frac{1}{(r_C+R)C}\hat{x}_2 \\ &\quad + \frac{h_2}{C}(y-\hat{y}) \\ \hat{y} &= \hat{x}_1 \end{aligned} \quad (3.2)$$

where h_1 and h_2 are scalar nonlinear functions representing output reconstruction error "injections" into the observer dynamics. The state reconstruction error, defined as $e = [e_1\ e_2]^T = [x_1\ x_2]^T - [\hat{x}_1\ \hat{x}_2]^T$ is seen to satisfy, after some rearrangement, the following dynamics in matrix form

$$\mathcal{D}_B\dot{e} + (1-u)\mathcal{J}_B e + \mathcal{R}_B(u)e + \mathcal{H}_B(e_1) = 0 \quad (3.3)$$

where $\mathcal{H}_B(e_1) = [h_1(e_1)\ \ h_2(e_1)]^T$.

Take as an energy storage function the quantity $V_O(e) = \frac{1}{2}e^T\mathcal{D}_B e$. The time derivative of such a scalar function results in the following expression

$$\dot{V}_O(e) = -e^T\mathcal{R}_B(u)e - e_1h_1(e_1) - e_2h_2(e_1) \quad (3.4)$$

In order to enhance the dissipation properties of the observer dynamics while bestowig some robustness to the observer, we use a discontinuous feedforward output error injection term in combination with a linear damping term. The following choice of the output error injection terms seems then natural

$$h_1(e_1) = R_3e_1 + W\text{sign}e_1 \ \ ; \ \ h_2(e_1) = 0 \quad (3.5)$$

where W and R_3 are any strictly positive constant gains to be chosen at will.

The previous choice (3.5) of the feedforward injection terms results in a strictly negative time derivative of the energy storage function. This quantity is given by

$$\dot{V}_O = -[r(u)+R_3]e_1^2 - \frac{1}{r_C+R}e_2^2 - W\,|e_1| \quad (3.6)$$

In other words, through the limited options offered by the nature of the output error injection functions, one may still enhance the dissipation structure of the reconstruction error dynamics and thus obtain an asymptotically stable state reconstruction error behaviour.

Remark

It is interesting to note that if an ideal "boost" converter model is considered, i.e., one without parasitic resistances and voltages, then the corresponding sliding mode controller may be entirely synthesized on the basis of the output variable $y = x_1$ with no need for the output capacitor voltage variable x_2. In such a case, no need exists for an observer and, thus, the sliding mode controller is truly an output feedback controller. Notice that this is also the case in the realistic model treated above if one does not insist on providing some additional damping to the output capacitor voltage closed loop dynamics through the term $1/R_2$. Thus, leaving untouched the already valid energy dissipation properties of the output circuit, results in a substantial simplification of the feedback controller with no need for the derived observer (see equation (2.17) with $R_2 = \infty$.

4 Closed Loop Stability Assessment

In this section we provide a sketch of the proof of asymptotic stability of the closed loop system.

1. Write down, in matrix form, the equations of the composite system, constituted by the plant, the observer and the dynamical sliding mode controller, including the ideal sliding dynamics generator.
2. Substract from the plant dynamics rows the desired state dynamics and then proceed to substract the observer equations from the plant equations to form the state estimation error system. Finally, substract from the desired state dynamics the dynamics of the ideal sliding dynamics. All this may be accomplished through a single non-singular state coordinate transformation of the original system written in matrix form.
3. Note that the designed controller, the ideal sliding dynamics and the observation error equations guarantee that the right hand side of the matrix system is identically zero. The composite error system, in matrix form, already contains the suitable modifications of the energy dissipation characteristics of the original subsystems, properly introduced through dynamical feedback and output reconstruction error injections.
4. Take as a Lyapunov function candidate the sum of the storage functions associated to the error subsystems with states given by $x - x_d$, $x - \hat{x}$ and $x_d - z_d$.
5. It is easy to verify that the time derivative of this Lyapunov function candidate is, with due thanks

to the presence of workless forces, negative definite for the two possible values of the contol action u.

6. The final argument is a slightly different version of the fact that a feedback interconnection of passive systems (as it is the case for the plant, the observer and the dynamical controller) renders an overall passive system.

5 Conclusions

In this article we have combined a sliding mode control option with passivity based controllers in a manner that may significantly enhance the robustness properties of the dynamical feedback control based solely on passivity considerations. The control scheme was also shown to be extendable to a discontinuous output feedback option including a variable structure observer. The design of the observer was also carried out using passivity considerations. The overall stability of the closed loop system was proved in a straightforward manner using standard Lyapunov stability arguments.

A complete sliding mode-passivity feedback controller design methodology can be developed for the particular, but important, class of switch-regulated systems. For this task, a general Euler-Lagrange system formulation, such as that already given in [9], may be adopted as a convenient starting point.

Passivity based regulators can be extended to adaptive schemes for systems with unknown but constant parameters. The combination of adaptation and sliding modes in a passivity based approach seems challenging and, due to its various apparent advantages, it certainly deserves attention in future works.

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FIGURES

Figure 1: A realistic "Boost" converter model