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ADAPTIVE SLIDING MODE OUTPUT TRACKING VIA BACKSTEPPING FOR UNCERTAIN NONLINEAR SYSTEMS

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Abstract

An alternative adaptive scheme to achieve output tracking for nonlinear systems with parametric uncertainties is considered. The proposed approach is based upon a combination of the adaptive *backstepping* design method and a Sliding Mode Control (SMC) scheme to design dynamical adaptive sliding mode controllers and provide robust output tracking even in the presence of unknown disturbances. The validity of the proposed approach, regarding tracking objectives and robustness with respect to bounded stochastic perturbation inputs, is tested through digital computer simulations.

1 Introduction

During the last few years a series of successful Lyapunov-based adaptive nonlinear procedures [1]-[4] has been reported. These systematic *backstepping* procedures allow the recursive design of adaptive nonlinear controllers for classes of uncertain systems transformable into either the *parametric-pure* or *parametric-strict* feedback forms. Moreover, the backstepping algorithm guarantees global regulation and tracking properties when the controlled plant belongs to this latter class of systems.

An important and desirable feature for any control design method dealing with uncertainties is robustness in the presence of disturbance inputs. This aspect becomes crucial in output tracking tasks because disturbances can deteriorate the closed-loop performance in such a manner that asymptotic tracking may be lost. In [6],[8] it was shown that parameter adaptation in an adaptive backstepping algorithm is affected by unmatched disturbance inputs and, as an alternative to guarantee robust and asymptotic stabilization of uncertain systems in the presence of undesirable perturbation inputs and unmodelled dynamics, combined backstepping and sliding mode control approaches were proposed.

In [1] it was assumed that full state measurement is available for the design of output tracking controllers, whereas in [2],[4] only the output is available for feedback, requiring the use of stable filters and estimated values of the unmeasurable state coordinates. Observed-based schemes for parametric-strict feedback nonlinear systems can guarantee an input-to-state stability property ([5]).

Here we consider the output tracking problem of uncertain nonlinear systems with the assumption of full state measurement, and propose a combination of the adaptive backstepping technique and input-output linearization, in conjunction with SMC, in order to design adaptive sliding mode output tracking controllers and guarantee robustness with respect to undesirable additive perturbation inputs $\nu(t)$. Furthermore, a modified version of the non-overparameterized adaptive algorithm in [3] is proposed, which guarantees a more robust and better transient performance of parameter adaptation and achieves a more direct interpretation of the design parameters with respect to the closed-loop performance.

2 Backstepping design of adaptive sliding mode output tracking controllers

In this section we propose a backstepping-like procedure to design adaptive sliding surfaces on which a sliding regime may be induced using SMC, and achieve asymptotic output tracking. Consider the uncertain nonlinear system

$$\begin{aligned} \dot{\xi} &= f_0(\xi) + \sum_{i=1}^{p} \theta_i f_i(\xi) + g_0(\xi,\nu)u \\ y &= h(\xi) \end{aligned} \tag{1}$$

where $\xi \in \Re^n$ is the state, $u \in \Re$ the input, $y \in \Re$ the output, $\theta = [\theta_1, \ldots, \theta_p]^T$ a vector of unknown constant parameters, $h(\xi)$ a smooth function on $\Re^n$ with $h(0) = 0$, and $g_0, f_i,\ 0 \leq i \leq p$, smooth vector fields on $\Re^n$ with $g_0(\xi) \neq 0,\ \forall \xi \in \Re^n, f_i(0) = 0,\ 0 \leq i \leq p$.

In [1] the necessary and sufficient conditions are given to transform (1) globally into the following *parametric-strict-feedback normal form*, for $\nu = 0$,

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \theta^T \gamma_i(x_1, \ldots, x_i, x^r) \quad 1 \leq i \leq \rho - 1 \\ \dot{x}_\rho &= \gamma_0(x) + \theta^T \gamma_\rho(x) + \beta_0(x)u \end{aligned} \tag{2}$$

$$\dot{x}^r = \Phi_0(y, x^r) + \sum_{i=1}^{p} \theta_i \Phi_i(y, x^r)$$
$$y = x_1$$

where ρ is the relative degree of (1), i.e. ρ is the integer for which the following conditions are satisfied for all $\xi \in \Re^n$

$$L_{g_0} L_{f_0}^i h \equiv 0, \quad 0 \le i \le \rho - 2 \tag{3}$$
$$L_{g_0} L_{f_0}^{\rho-1} h \neq 0 \tag{4}$$

and the x^r-subsystem is the $(n-\rho)$-dimensional nonlinear part of (1) that cannot be transformed into a chain of integrators. Assuming that the x^r-subsystem has the bounded-input bounded-state property with respect to y as its input, the problem of tracking a bounded reference signal $y_r(t)$, with its first ρ derivatives known and bounded, was solved by an overparameterized adaptive backstepping algorithm.

We develop here a recursive procedure, similar to that in [3], to design non-overparameterized adaptive sliding mode output tracking control and provide robustness even in the presence of disturbance inputs.

A drawback of the control design procedure in [3] is associated with the use of the design parameters c_i at intermediate steps of the algorithm because the design parameter-dependent coordinate transformation yields, in the original coordinates, products of the c_i's at subsequent steps. This generates high gains for the parameter adaptation and can cause large variations of the estimated parameters. In contrast to this approach, the algorithm proposed here has a design parameter-independent coordinate transformation and employs the design parameters only at the final step of the algorithm. Thus, the resulting adaptive control system exhibits better transient performance and convergence properties as well as a more robust behaviour.

For simplicity, we present here the control design algorithm to achieve asymptotic output tracking for the system (1) in input-output linearizable form, i.e. the relative degree ρ is equal to the system order n, and (2) takes the form

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \theta^T \gamma_i(x_1, \ldots, x_i) \quad 1 \le i \le n-1 \\ \dot{x}_n &= \gamma_0(x) + \theta^T \gamma_n(x) + (\beta_0(x) + \nu) u \\ y &= x_1 \end{aligned} \tag{5}$$

where the γ_i and β_0 are smooth nonlinear functions of their arguments, with $\beta_0(x) \neq 0, \ \forall x \in \Re^n$.

2.1 Backstepping algorithm

STEP 1. The tracking error function z_1 is defined as

$$z_1 := y(t) - y_r(t) = x_1(t) - y_r(t) \tag{6}$$

where $y_r(t)$ is a known and bounded reference signal with n bounded and known derivatives $y_r^{(i)}, i = 1, \ldots, n$. The time derivative of z_1 is

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r = x_2 + \theta^T \gamma_1 - \dot{y}_r \tag{7}$$

Defining $\hat{\theta}$ as the estimated values of θ, we can rewrite $\dot{z}_1$ as

$$\dot{z}_1 = x_2 + \hat{\theta}^T \gamma_1 - \dot{y}_r + \tilde{\theta}^T \gamma_1 \tag{8}$$

with $\tilde{\theta} := \theta - \hat{\theta}$ as the estimation error. Consider the Lyapunov function

$$V_1(z_1, \tilde{\theta}) = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{9}$$

with $\Gamma = \Gamma^T > 0$, and its corresponding time derivative

$$\dot{V}_1 = z_1 [x_2 + \hat{\theta}^T \gamma_1 - \dot{y}_r] + \tilde{\theta}^T \Gamma^{-1} (-\dot{\hat{\theta}} + \Gamma z_1 \gamma_1) \tag{10}$$

We can eliminate $\tilde{\theta}$ from $\dot{V}_1$ using the tuning function

$$\dot{\hat{\theta}} = \tau_1 = \Gamma z_1 \gamma_1 = \Gamma z_1 \omega_1 \tag{11}$$

and, if x_2 were the control, we would achieve $\dot{V}_1 = -z_1^2$ with the virtual control $x_2 = \alpha_1$ defined as

$$\alpha_1 = -\hat{\theta}^T \gamma_1 + \dot{y}_r - z_1 \tag{12}$$

Since x_2 is not the control and, therefore cannot be chosen arbitrarily, we define the second error variable

$$z_2 = x_2 - \alpha_1 = x_2 + \hat{\theta}^T \gamma_1 - \dot{y}_r + z_1 \tag{13}$$

as the deviation of the state variable x_2 from its desired trajectory. Thus, the closed-loop form of $\dot{z}_1$ is

$$\dot{z}_1 = -z_1 + z_2 + \tilde{\theta}^T \omega_1 \tag{14}$$

and the time derivative of V_1 is

$$\dot{V}_1 = -z_1^2 + z_1 z_2 + \tilde{\theta}^T \Gamma^{-1} (-\dot{\hat{\theta}} + \tau_1) \tag{15}$$

STEP k $(2 \le k \le n-1)$. The time derivative of the error variable z_k is

$$\dot{z}_k = x_{k+1} - \sum_{i=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial x_i} x_{i+1} + \theta^T \omega_k - \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - y_r^{(k)} \tag{16}$$

with

$$\omega_k = \gamma_k - \sum_{i=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial x_i} \gamma_i \tag{17}$$

We rewrite $\dot{z}_k$ as

$$\dot{z}_k = x_{k+1} - \sum_{i=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial x_i} x_{i+1} + \hat{\theta}^T \omega_k - \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - y_r^{(k)} + \tilde{\theta}^T \omega_k \tag{18}$$

which can stabilized with respect to the augmented Lyapunov function

$$V_k = V_{k-1} + \frac{1}{2} z_k^2 \tag{19}$$

The time derivative of V_k is

$$\begin{aligned} \dot{V}_k = z_k \Big[& z_{k-1} + x_{k+1} - \sum_{i=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial x_i} x_{i+1} + \hat{\theta}^T \omega_k \\ & - \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - y_r^{(k)} \Big] \\ & - \sum_{i=1}^{k-1} z_i^2 - \sum_{i=1}^{k-2} z_{i+1} \frac{\partial \alpha_i}{\partial \hat{\theta}} (\dot{\hat{\theta}} - \tau_{k-1}) \\ & + \tilde{\theta}^T \Gamma^{-1} (-\dot{\hat{\theta}} + \tau_{k-1} + \Gamma z_k \omega_k) \end{aligned} \tag{20}$$

We can eliminate $\tilde{\theta}$ from $\dot{V}_k$ using the tuning function

$$\dot{\hat{\theta}} = \tau_k = \tau_{k-1} + \Gamma z_k \omega_k = \Gamma \sum_{i=1}^{k} z_i \omega_i \tag{21}$$

and noting that

$$\dot{\hat{\theta}} - \tau_{k-1} = \dot{\hat{\theta}} - \tau_k + \tau_k - \tau_{k-1} = \dot{\hat{\theta}} - \tau_k + \Gamma z_k \omega_k \tag{22}$$

we rewrite $\dot{V}_k$ as

$$\begin{aligned} \dot{V}_k = &-\sum_{i=1}^{k-1} z_i^2 + \Big(\sum_{i=1}^{k-2} z_{i+1}\frac{\partial \alpha_i}{\partial \hat{\theta}} + \tilde{\theta}^T\Gamma^{-1}\Big)(-\dot{\hat{\theta}} + \tau_k) \\ &+ z_k\Big[z_{k-1} + x_{k+1} - \sum_{i=1}^{k-1}\frac{\partial \alpha_{k-1}}{\partial x_i}x_{i+1} - \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \\ &+ \Big(\hat{\theta}^T - \sum_{i=1}^{k-2} z_{i+1}\frac{\partial \alpha_i}{\partial \hat{\theta}}\Gamma\Big)\omega_k - y_r^{(k)}\Big] \end{aligned} \tag{23}$$

Now, if x_{k+1} were the control we would achieve $\dot{V}_k = -\sum_{i=1}^{k} z_i^2$ with the virtual control $x_{k+1} = \alpha_k$ defined as

$$\begin{aligned} \alpha_k \;=\;& -z_{k-1} + \sum_{i=1}^{k-1}\frac{\partial \alpha_{k-1}}{\partial x_i}x_{i+1} + \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\tau_k \\ &- \Big(\hat{\theta}^T - \sum_{i=1}^{k-2} z_{i+1}\frac{\partial \alpha_i}{\partial \hat{\theta}}\Gamma\Big)\omega_k + y_r^{(k)} - z_k \end{aligned} \tag{24}$$

Since x_{k+1} is not the control and, therefore cannot be chosen arbitrarily, we define the $(k+1)$-th error variable $z_{k+1} = x_{k+1} - \alpha_k$

$$\begin{aligned} z_{k+1} \;=\;& x_{k+1} + z_{k-1} - \sum_{i=1}^{k-1}\frac{\partial \alpha_{k-1}}{\partial x_i}x_{i+1} - \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\tau_k \\ &+ \Big(\hat{\theta}^T - \sum_{i=1}^{k-2} z_{i+1}\frac{\partial \alpha_i}{\partial \hat{\theta}}\Gamma\Big)\omega_k - y_r^{(k)} + z_k \end{aligned} \tag{25}$$

as the deviation of the state variable x_{k+1} from its desired trajectory. Thus, the closed-loop form of $\dot{z}_k$ is

$$\begin{aligned} \dot{z}_k \;=\;& -z_{k-1} - z_k + z_{k+1} + \tilde{\theta}^T\omega_k - \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}(\dot{\hat{\theta}} - \tau_k) \\ &+ \sum_{i=1}^{k-2} z_{i+1}\frac{\partial \alpha_i}{\partial \hat{\theta}}\Gamma\omega_k \end{aligned} \tag{26}$$

and the time derivative of V_k is calculated as

$$\dot{V}_k = -\sum_{i=1}^{k} z_i^2 + z_k z_{k+1} + \Big(\sum_{i=1}^{k-2} z_{i+1}\frac{\partial \alpha_i}{\partial \hat{\theta}} + \tilde{\theta}^T\Gamma^{-1}\Big)(-\dot{\hat{\theta}} + \tau_k) \tag{27}$$

STEP n. By using the definition for $z_n := x_n - \alpha_{n-1}$, and adding and subtracting the estimated values $\hat{\theta}$, we obtain the time derivative of the error variable z_n as

$$\begin{aligned} \dot{z}_n \;=\;& \gamma_0 + \beta_0 u - \sum_{i=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_i}x_{i+1} + \hat{\theta}^T\omega_n - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \\ &- y_r^{(n)} + \tilde{\theta}^T\omega_n \end{aligned} \tag{28}$$

with

$$\omega_n = \gamma_n - \sum_{i=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_i}\gamma_i \tag{29}$$

We have transformed (5) into

$$\begin{aligned} \dot{z}_k \;=\;& -z_{k-1} - z_k + z_{k+1} + \tilde{\theta}^T\omega_k - \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}(\dot{\hat{\theta}} - \tau_k) \\ &+ \sum_{i=1}^{k-2} z_{i+1}\frac{\partial \alpha_i}{\partial \hat{\theta}}\Gamma\omega_k, \qquad 1 \le k \le n-1 \\ \dot{z}_n \;=\;& \gamma_0 + \beta_0 u - \sum_{i=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_i}x_{i+1} + \hat{\theta}^T\omega_n - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \\ &- y_r^{(n)} + \tilde{\theta}^T\omega_n \end{aligned} \tag{30}$$

$$\dot{\hat{\theta}} = \tau_{n-1} = \tau_{n-2} + \Gamma z_{n-1}\omega_{n-1} = \Gamma\sum_{i=1}^{n-1} z_i\omega_i \tag{31}$$

We consider now the following sliding surface, expressed in terms of the error variables z_i

$$\sigma = c_1 z_1 + c_2 z_2 + \ldots + c_{n-1}z_{n-1} + z_n = 0 \tag{32}$$

where the scalar coefficients $c_i > 0, i = 1, \ldots, n-1$, are chosen in such a manner that the polynomial

$$p(s) = c_1 + c_2 s + \ldots + c_{n-1}s^{n-2} + s^{n-1} \tag{33}$$

in the complex variable s, is Hurwitz. We impose a discontinuous control law on the dynamics of (30) in order to generate a stable sliding regime on the prescribed surface (32).

At the final step we extend the Lyapunov function as follows

$$V_n = V_{n-1} + \frac{1}{2}\sigma^2 = \frac{1}{2}\sum_{i=1}^{n-1} z_i^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{34}$$

and using (27), for $k = n-1$, the time derivative of V_n is

$$\begin{aligned} \dot{V}_n = &-\sum_{i=1}^{n-1} z_i^2 + z_{n-1}z_n - \sum_{i=1}^{n-3} z_{i+1}\frac{\partial \alpha_i}{\partial \hat{\theta}}(\dot{\hat{\theta}} - \tau_{n-1}) \\ &+ \tilde{\theta}^T\Gamma^{-1}\Big(-\dot{\hat{\theta}} + \tau_{n-1} + \Gamma\sigma\big(\omega_n + \sum_{i=1}^{n-1} c_i\omega_i\big)\Big) \\ &+ \sigma\Big[\gamma_0 + \beta_0 u - \sum_{i=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_i}x_{i+1} + \hat{\theta}^T\omega_n - y_r^{(n)} \\ &- \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \sum_{i=1}^{n-1} c_i(-z_{i-1} - z_i + z_{i+1}) \\ &- \sum_{i=1}^{n-1} c_i\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}(\dot{\hat{\theta}} - \tau_i) \\ &+ \sum_{i=1}^{n-1} c_i\sum_{j=1}^{i-2} z_{j+1}\frac{\partial \alpha_j}{\partial \hat{\theta}}\Gamma\omega_i\Big] \end{aligned} \tag{35}$$

To eliminate $\tilde{\theta}$ from $\dot{V}_n$ we choose the update law

$$\begin{aligned}\dot{\hat{\theta}} &= \tau_n = \tau_{n-1} + \Gamma\sigma\big(\omega_n + \sum_{i=1}^{n-1} c_i\omega_i\big) \\ &= \Gamma\Big(\sum_{i=1}^{n-1} z_i\omega_i + \sigma\big(\omega_n + \sum_{i=1}^{n-1} c_i\omega_i\big)\Big) \end{aligned} \tag{36}$$

and noting that

$$\dot{\hat{\theta}} - \tau_{n-1} = \tau_n - \tau_{n-1} = \Gamma\sigma\big(\omega_n + \sum_{i=1}^{n-1} c_i\omega_i\big) \tag{37}$$

we rewrite $\dot{V}_n$ as

$$\begin{aligned}\dot{V}_n = &-\sum_{i=1}^{n-1} z_i^2 + z_{n-1}z_n + \\ &\sigma\Big[\gamma_0 + \beta_0 u - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}x_{i+1} + \hat{\theta}^T\omega_n - y_r^{(n)} \\ &-\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\tau_n + \sum_{i=1}^{n-1} c_i(-z_{i-1} - z_i + z_{i+1}) \\ &-\sum_{i=1}^{n-1} c_i\Big(\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}(\tau_n - \tau_i) - \sum_{j=1}^{i-2} z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}\Gamma\omega_i\Big) \\ &-\sum_{i=1}^{n-3} z_{i+1}\frac{\partial\alpha_i}{\partial\hat{\theta}}\Gamma\big(\omega_n + \sum_{j=1}^{n-1} c_j\omega_j\big)\Big] \end{aligned} \tag{38}$$

and finally choose the control law

$$\begin{aligned} u = \frac{1}{\beta_0}\Big[&-\gamma_0 + \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}x_{i+1} - \hat{\theta}^T\omega_n + y_r^{(n)} \\ &+\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\tau_n - \sum_{i=1}^{n-1} c_i(-z_{i-1} - z_i + z_{i+1}) \\ &+\sum_{i=1}^{n-1} c_i\Big(\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}(\tau_n - \tau_i) - \sum_{j=1}^{i-2} z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}\Gamma\omega_i\Big) \\ &+\sum_{i=1}^{n-3} z_{i+1}\frac{\partial\alpha_i}{\partial\hat{\theta}}\Gamma\big(\omega_n + \sum_{j=1}^{n-1} c_j\omega_j\big) \\ &-\kappa(\sigma + W\text{sgn}(\sigma))\Big] \end{aligned} \tag{39}$$

to obtain

$$\dot{V}_n = -\sum_{i=1}^{n-1} z_i^2 + z_{n-1}z_n - \kappa\sigma^2 - \kappa W|\sigma| \tag{40}$$

To prove the asymptotic stability consider

$$\dot{V}_n = -z^T Q z - \kappa W|\sigma| \tag{41}$$

with

$$Q = \begin{bmatrix} 1+\kappa c_1^2 & \dots & \kappa c_1 c_{n-1} & \kappa c_1 \\ \kappa c_1 c_2 & \dots & \kappa c_2 c_{n-1} & \kappa c_2 \\ \vdots & \dots & \vdots & \vdots \\ \kappa c_1 c_{n-1} & \dots & 1+\kappa c_{n-1}^2 & -\frac{1}{2}+\kappa c_{n-1} \\ \kappa c_1 & \dots & -\frac{1}{2}+\kappa c_{n-1} & \kappa \end{bmatrix}$$

The principal minors of Q have the value

$$1 + \kappa\sum_{i=1}^{d} c_i^2 > 0 \ ; \quad 1 \le d \le n-1 \tag{42}$$

Thus, a sufficient condition on the design parameters to achieve asymptotic tracking can be obtained from

$$\det(Q) = -\frac{1}{4} + \kappa\Big(1 + c_{n-1} - \frac{1}{4}\sum_{i=1}^{n-2} c_i^2\Big) > 0 \tag{43}$$

So $\dot{V}_n \le -z^T Q z < 0$ and therefore, since

$$\lim_{t\to\infty} z_1(t) = y(t) - y_r(t) = 0 \ , \tag{44}$$

asymptotic tracking is achieved. The convergence of the state trajectories towards the sliding surface can be established from the LaSalle invariance theorem.

Note that the algorithm proposed here also achieves global asymptotic tracking for the class of uncertain nonlinear systems transformable into (5). Moreover, it can be extended to a broader class of nonlinear systems that cannot be transformed into either parametric-pure or parametric-strict feedback form by parameter-independent state coordinate transformation [9]. A similar approach was used in [9] for PWM regulation of DC-to-DC power converters by a suitable combination of dynamical input-output linearization and a systematic backstepping-like procedure, whereas the asymptotic output tracking problem, without parametric uncertainties, has been solved in [7] by dynamical SMC strategies.

3 Dynamical adaptive sliding mode tracking control of the Buck-Boost converter

Consider the average Buck-Boost converter model defined on the input inductor current x_1 and the output capacitor voltage x_2

$$\begin{aligned} \dot{x}_1 &= \theta_1(1-\mu)x_2 + (\theta_4 + \nu)\mu \\ \dot{x}_2 &= -\theta_2(1-\mu)x_1 - \theta_3 x_2 \\ y &= x_1 \end{aligned} \tag{45}$$

with

$$\theta_1 = \frac{1}{L} \ ; \quad \theta_2 = \frac{1}{C} \ ; \quad \theta_3 = \frac{1}{RC} \ ; \quad \theta_4 = \frac{E}{L} \tag{46}$$

where L, C and R are respectively the inductance, capacitance and resistance values of the circuit components, while E is the constant external voltage source. These four circuit components define the set of unknown parameters $\theta \in \Re^4$. The control input function μ takes values in the interval $[0, 1]$ and the regulated output function is the input inductor current x_1, which must follow a desired output reference signal $y_r(t)$. ν is an external stochastic bounded perturbation input.

For $\mu = U$ constant, with $0 \le U \le 1$, the equilibrium values model are readily obtained from (45), for $\nu = 0$, as

$$X_1(U) = \frac{\theta_3\theta_4 U}{\theta_1\theta_2(1-U)^2} \qquad X_2(U) = -\frac{\theta_4 U}{\theta_1(1-U)} \quad (47)$$

Our primary objective is to design a dynamical adaptive sliding mode control for tracking a known reference signal $y_r(t)$. In particular, we are interested in driving the input inductor current x_1 to follow a smooth trajectory between two operating equilibrium points X_1, X_1^*

$$y_r(t) = \begin{cases} X_1 & 0 \le t < t_1 \\ X_1^* + (X_1 - X_1^*)\exp(-k(t-t_1)^2) & t \ge t_1 \end{cases} \quad (48)$$

Note that (45) is not transformable into the parametric-pure or parametric-strict feedback forms by parameter-independent state coordinate transformations, therefore adaptive backstepping design, under conditions given in [3], is not applicable.

In order to implement the algorithm presented in the previous section, we rewrite (45) as

$$\begin{aligned} \dot{x}_1 &= \theta^T\gamma_1(x_1, x_2, \mu) \\ \dot{x}_2 &= \theta^T\gamma_2(x_1, x_2, \mu) \\ y &= x_1 \end{aligned} \quad (49)$$

with

$$\begin{aligned} \theta &= [\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4]^T & (50) \\ \gamma_1 &= [(1-\mu)x_2 \quad 0 \quad 0 \quad \mu]^T & (51) \\ \gamma_2 &= [0 \quad -(1-\mu)x_1 \quad -x_2 \quad 0]^T & (52) \end{aligned}$$

First we define the tracking error function z_1 as in (6) and obtain the following:

$$z_2 = \hat{\theta}^T\gamma_1 - \dot{y}_r + z_1 \quad (53)$$

$$\dot{z}_1 = -z_1 + z_2 + \tilde{\theta}^T\omega_1 \quad (54)$$

$$\dot{V}_1 = -z_1^2 + z_1 z_2 + \tilde{\theta}^T\Gamma^{-1}(-\dot{\hat{\theta}} + \tau_1) \quad (55)$$

At the second step, we define the sliding surface $\sigma = c_1 z_1 + z_2$ and the augmented Lyapunov function $V_2 = V_1 + \frac{1}{2}\sigma^2$. The time derivative of V_2 is

$$\begin{aligned} \dot{V}_2 = & -z_1^2 + z_1 z_2 + \tilde{\theta}\Gamma^{-1}\big(-\dot{\hat{\theta}} + \tau_1 + \Gamma\sigma(\omega_2 + c_1\omega_1)\big) \\ & + \sigma\Big[\hat{\theta}^T\omega_2 + \dot{\hat{\theta}}\gamma_1 + (\hat{\theta}_4 - \hat{\theta}_1 x_2)\dot{\mu} \\ & - \dot{y}_r - \ddot{y}_r + c_1(-z_1 + z_2)\Big] \end{aligned} \quad (56)$$

To eliminate $\tilde{\theta}$ from $\dot{V}_2$ we choose the update law

$$\dot{\hat{\theta}} = \tau_2 = \tau_1 + \Gamma\sigma(\omega_2 + c_1\omega_1) = \Gamma\big[z_1\omega_1 + \sigma(\omega_2 + c_1\omega_1)\big] \quad (57)$$

The control function μ can be readily obtained in an implicit manner, as the solution of the following nonlinear time-varying differential equation

$$\begin{aligned} \dot{\mu} = \frac{1}{(\hat{\theta}_4 - \hat{\theta}_1 x_2)}\Big[& -\hat{\theta}^T\omega_2 - \tau_2^T\gamma_1 + \dot{y}_r + \ddot{y}_r \\ & - c_1(-z_1 + z_2) - \kappa\left(\sigma + W\mathrm{sgn}(\sigma)\right)\Big] \end{aligned} \quad (58)$$

and

$$\dot{V}_2 = -z_1^2 + z_1 z_2 - \kappa\sigma^2 - \kappa W|\sigma| \quad (59)$$

From (43), the sufficient condition on the design parameters to guarantee asymptotic tracking is

$$\kappa(1 + c_1) > \frac{1}{4} \quad (60)$$

An important advantage arises from the dynamical adaptive variable structure controller represented by (58): *the output tracking error function $z_1(t)$ asymptotically approaches zero with substantially reduced chattering.*

Simulations were carried out to assess the adaptively controlled tracking behaviour of the average Buck-Boost converter model. The following nominal values of the circuit parameters were used: $C = 181.82\,\mu F$, $L = 0.27\,mH$, $R = 2.44\Omega$, $E = 14.667$ Volts. These values of yield the model parameters $\theta_1 = 3.6 \times 10^3$, $\theta_2 = 5.5 \times 10^3$, $\theta_3 = 2.25 \times 10^3$, $\theta_4 = 52.8 \times 10^3$. The design parameters were $c_1 = 3$, $\kappa = 15$, $W = 10$, $\Gamma = I_4$.

Figure 1 depicts the dynamic adaptively regulated tracking of the inductor current x_1 for a smooth transition between $X_1 = 22.5$ amps and $X_1^* = 10$ amps, corresponding to $U = 0.6$ and $U = 0.4695$ respectively, as well as the time evolution of the controlled capacitor voltage x_2. The regulated output variable $x_1(t)$ is seen to exhibit asymptotic tracking to the desired reference input $y_r(t)$. The figure also shows the control input function and the time evolution of the adaptive sliding surface. Figure 2 shows the estimated parameter values $\hat{\theta}$ obtained from the updating law, and an example of the perturbation noise input.

4 Conclusions

An adaptive sliding mode control design approach, based on the adaptive backstepping procedure and the VSC scheme, has been developed for the effective output tracking control of linearly parameterized uncertain nonlinear systems. The algorithm sets up a design parameter-independent state coordinate transformation and yields adaptive sliding surfaces on which stable sliding regimes can be generated. A sufficient condition on the design parameters to guarantee asymptotic tracking has been analyzed. A dynamical extension of the control design algorithm can be applied to uncertain nonlinear systems that are not transformable into the parametric-pure or parametric-strict feedback forms. The proposed control strategy has been applied to the output tracking control of the average Buck-Boost converter model and it has been shown to be remarkably robust with respect to external stochastic bounded perturbation input signals.

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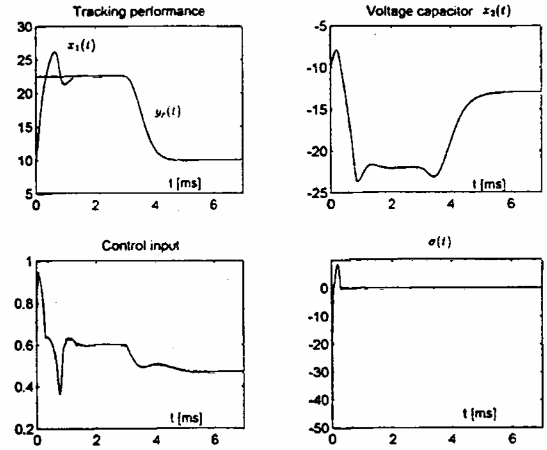

Figure 1: Dynamic adaptively regulated tracking and capacitor voltage evolution of the Buck-Boost converter, control input function and adaptive evolution of the sliding surface.

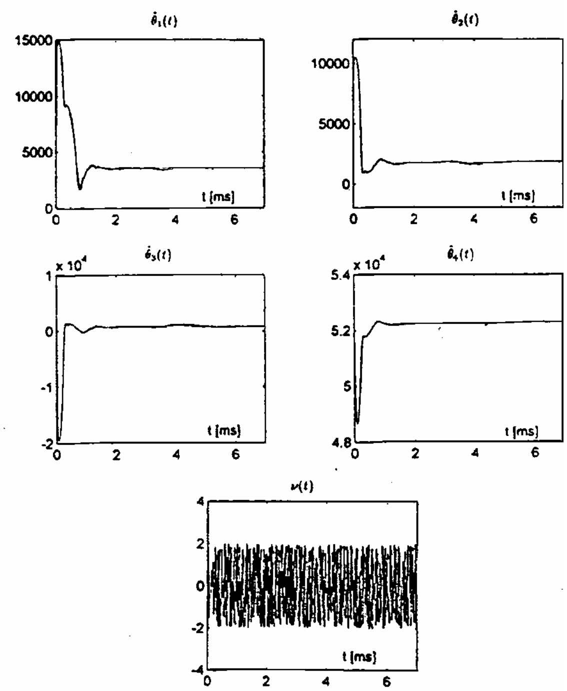

Figure 2: Parameter estimates and perturbation noise signal.