NONLINEAR CONTROL OF CHUA'S CIRCUIT

Richard Márquez
Escuela de Ingeniería de Sistemas
Universidad de Los Andes
Mérida 5101
Venezuela

Hebertt Sira-Ramírez
Departamento Sistemas de Control
Escuela de Ingeniería de Sistemas
Universidad de Los Andes
Mérida 5101
Venezuela

ABSTRACT

It is shown that Chua's circuit is a *differencially flat system*. This feature makes the linearizing feedback controller design a task particularly simple. This note presents a smooth nonlinear controller and a dynamical discontinuous strategy which stabilize the system around an admissible trajectory $y_d(t)$. Computer simulations are presented to illustrate the performance of the proposed controllers.

Keywords : Chaos Control, Differentially Flat Systems, Dynamical Discontinuous Control, Nonlinear Control Systems.

I. INTRODUCTION

'Controlling chaos' is actually an active area of research in which the physics, mathematics and the engineering communities have been very interested in the last few years ([1], [2], cf [3], [4] and references therein). The control strategies used so far have been restricted to tools obtained from dynamical system theory (Ott-Grebori-Yorke approach, resonant methods, entrainment and migration control, etc) mostly developed by physicists and mathematicians with a clear tendency to use non-feedback control techniques. Only recently an ***engineering control approach***, represented by linear feedback strategies, Chen and Dong [5], and frequency response methods, Genesio *et al* [6], has received considerably attention to solve this problem.

One of the most popular nonlinear electronic circuits used to explain and experiment with chaotic dynamics is *Chua's circuit* [7], [8], [9], [10]. In this note, it is shown that a controlled version of this circuit is included in the broad class of *differentially flat systems*. Differentially flat systems were introduced by Fliess *et al* in [11] using differential algebraic tools. The *flatness property* possibly represents the best nonlinear extension of Kalman's controllability. A large class of dynamical control systems is indeed differentially flat: linear controllable systems, systems linearizable by state coordinates transformations and static state feedback are flat. Mechanical systems with nonholonomic velocity constraints are also flat, etc (cf [12], [13]). Flat systems are equivalent to nonlinear control systems linearizable by a dynamical *endogenous* feedback. This feature makes the linearizing feedback controller design a task particularly simple (cf [14], [12]).

Based on the flatness property of Chua's Circuit, we present the design of a smooth nonlinear feedback controller (cf [15], [16]), and a dynamical discontinuous nonlinear feedback strategy (cf [17], [13]). Given one of these control policies, the chaotic response of *Chua's* circuit can be stabilized around a desired admissible periodic orbit $y_d(t)$, or, alternatively, towards a constant equilibrium point.

This note is organized as follows. In Section II we prove that Chua's circuit is a differentially flat system Section III presents the design of a smooth nonlinear feedback controller and a dynamical discontinuous nonlinear feedback strategy. Section IV is devoted to conclusions and suggestions for further research.

II. DIFFERENTIAL FLATNESS OF CHUA'S CIRCUIT

Consider the following single input nonlinear control system

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x \in \Re^n$ are the states variables and $u \in \Re$ is the control input.

Definition 1 *[12] A system (1) is a* ***differentially flat*** *system if there exist one* ***differentially*** *independent output y such that*

1. *x and u are differential functions of y, i.e., functions of y and a finite number of its time derivatives.*

2. *y is differential function of x.*

The output y is called **linearizing** or **flat** *output.*

□

Chua's Circuit model

The following state equations model Chua's circuit (called Chua's circuit if $Ro = 0$ and Chua's oscillator if $Ro \neq 0$):

$$\begin{aligned} \frac{dv_1}{dt} &= \frac{1}{C_1}[G(v_2 - v_1) - f(v_1)] \\ \frac{dv_2}{dt} &= \frac{1}{C_2}[G(v_1 - v_2) + i_3] \\ \frac{di_3}{dt} &= -\frac{1}{L}(v_2 + R_0 i_3) \end{aligned} \tag{2}$$

where v_1, v_2, i_3 are the state variables as shown in Figure 1, $G = 1/R$. $f(v_1)$ represents the v–i characteristic of the nonlinear resistor N_R, which is called *Chua's diode*.

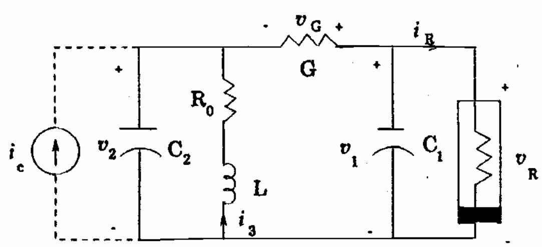

Figure 1: Chua's circuit

The function $f(v_1)$ is generally taken to be a piecewise linear function (see Figure 2). For our purpose, $f(v_1)$ will be approximated by a smooth locally invertible nonlinear function like $f(x) = c_0x + c_1x^3$, or a 'sigmoid' $f(x) = ax + b(\exp(cx) - 1)/(\exp(cx) + 1)$.

In the sequel, Chua's circuit is modified to include a control variable represented by the current i_c added to node 1 of this circuit. This transforms the state equations (2) into

$$\begin{aligned} \frac{dv_1}{dt} &= \frac{1}{C_1}[G(v_2 - v_1) - f(v_1)] \\ \frac{dv_2}{dt} &= \frac{1}{C_2}[G(v_1 - v_2) + i_3 + i_c] \\ \frac{di_3}{dt} &= -\frac{1}{L}(v_2 + R_0 i_3) \end{aligned} \tag{3}$$

or in dimensionless form

$$\begin{aligned} \frac{dx_1}{d\tau} &= \alpha(x_2 - x_1 - f(x_1)) \end{aligned}$$

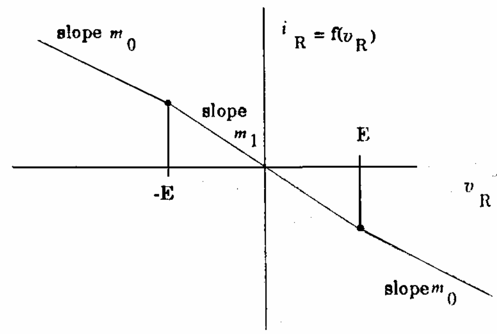

Figure 2: Chua's diode: Typical v–i characteristic

$$\begin{aligned} \frac{dx_2}{d\tau} &= x_1 - x_2 + x_3 + u \\ \frac{dx_3}{d\tau} &= -\beta x_2 - \gamma x_3 \end{aligned} \tag{4}$$

where $x_1 \doteq v_1$, $x_2 \doteq v_2$, $x_3 \doteq i_3/G$ are the state variables, $\alpha \doteq C_2/C_1$, $\beta \doteq C_2/LG^2$, $\gamma \doteq C_2R_o/LG$ are the essential parameters, $\tau \doteq G/C_2t$ is the time scaling, $u \doteq i_c/G$ is the control input, and $f(\cdot) \doteq f(\cdot)/G$ is the new locally invertible nonlinear function.

Flatness

It is easy to show that (4) is linearizable by state coordinates transformation and static state feedback therefore it is differentially flat. Using the methods found in [15], [16] one immediately obtains the linearizing output y:

$$y := x_1/\alpha + x_3/\beta$$

The time derivative of the flat output is given by

$$\dot{y} = -x_1 - f(x_1) - \gamma x_3/\beta$$

It is necessary to solve the following equation in terms of x_1:

$$\dot{y} + \gamma y = -x_1 - f(x_1) + \frac{\gamma}{\beta}x_1$$

Setting $F(x_1) = -x_1 - f(x_1) + \gamma x_1/\beta$, where F is locally invertible by the assumptions on $f(x_1)$, results

$$x_1 = F^{-1}(\dot{y}(\tau) + \gamma y(\tau))$$

After some algebraic manipulations, state and input variables can be expressed as differential functions of the linearizing output y.

III. CONTROLLER DESIGN

Continuous nonlinear feedback controller

It is possible to achieve an input–output linearization of system (4). Let the system output be $y \doteq x_1$, the control law

$$\begin{aligned} u &= \alpha(-x_1 + x_2 - f(x_1)) - x_1 + x_2 - x_3 + \dots \\ &+ \frac{\partial f(x_1)}{\partial x_1}\alpha(-x_1 + x_2 - f(x_1)) + \frac{v}{\alpha} \end{aligned} \tag{5}$$

gives the following *linear* input–output system

$$\ddot{y} = v$$

It should be clear that the full state (x_1, x_2, x_3) must be available to implement this nonlinear feedback control law.

Taking the control objective to be the tracking of a prescribed reference trajectory $y_d(t) \doteq x_1^d(t)$ by the output $y(t)$, we may use the following state-space transformation

$$\begin{aligned} q_1(t) &= x_1(t) - x_1^d(t) = y(t) - y_d(t) \\ q_2(t) &= \dot{x}_1(t) - \dot{x}_1^d(t) \\ &= \alpha(x_2(t) - x_1(t) - f(x_1(t))) - \dot{x}_1^d(t) \\ z(t) &= x_3(t) \end{aligned}$$

Setting v according to

$$v = \ddot{x}_1^d + k_d(\dot{x}_1^d - \dot{x}_1) + k_p(x_1^d - x_1) \tag{6}$$

results in the following dynamics for the tracking error

$$\ddot{q}_1 + k_d\dot{q}_1 + k_p q_1 = 0$$

where $k_d, k_p > 0$ so that $s^2 + k_d s + k_p$ is Hurwitz.

The complete linearized control system is given by

$$\begin{aligned} \dot{q}_1 &= q_2 \\ \dot{q}_2 &= -k_p q_1 - k_d q_2 & (7) \\ \dot{z} &= -\frac{\beta}{\alpha}(q_2 + \dot{x}_d(t)) - \dots \\ &- \beta(q_1 + x_d(t)) - \beta f(q_1 + x_d(t)) - \gamma z \\ &= \Psi(q; z) & (8) \\ y &= q_1 & (9) \end{aligned}$$

with $q = [q_1, q_2]^T$.

The *zero dynamics*

$$\dot{z} = \Psi(0, z) = -\frac{\beta}{\alpha}\dot{x}_d(t) - \beta x_d(t) - \beta f(x_d(t)) - \gamma z$$

is globally exponentially stable for $(x_d, \dot{x}_d) = (0, 0)$, or z is bounded if x_d and $\dot{x}_d$ both are bounded. By definition, this system is globally minimum phase since its zero dynamics is globally asymptotically stable. Figure 3 shows the close loop state trajectories and the control input. As it can be seen from the numerical simulation, the proposed controller achieved the desired stabilization of the output voltage around the desired equilibrium value.

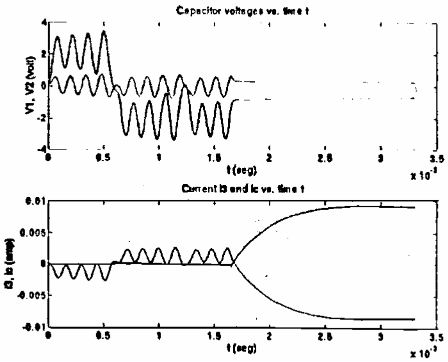

Figure 3: Controlled behavior of v_1, v_2 and i_3 using a continuous control

Dynamical discontinuous nonlinear feedback controller

Set $q_1 = x_1$, $q_2 = \dot{x}_1$, and $q_3 = \ddot{x}_1$. The system (4) has the following Fliess' Generalized Controller Canonical Form

$$\begin{aligned} \dot{q}_1 &= q_2 \\ \dot{q}_2 &= q_3 \\ \dot{q}_3 &= -\alpha\beta q_1 - \alpha\beta f(q_1) - \alpha\gamma f(q_1) - \alpha\frac{\partial f(q_1)}{\partial q_1}q_2 - \dots \\ &- \alpha\gamma\frac{\partial f(q_1)}{\partial q_1}q_2 - \gamma q_2 - \beta q_2 - \alpha\gamma q_2 - \dots \\ &- \alpha\frac{\partial^2 f(q_1)}{\partial q_1^2}q_2^2 - \alpha q_3 - q_3 - \gamma q_3 + \alpha\gamma u + \alpha\dot{u} \end{aligned} \tag{10}$$

The dynamical sliding mode controller comes from imposing the discontinuous dynamics

$$\dot{\sigma} = -W\mathrm{sign}(\sigma) \tag{11}$$

where σ is the *input dependent sliding surface*

$$\sigma = c_1 q_1 + c_2 q_2 + q_3$$

The scalar coefficients c_i $(i = 1, 2)$ are chosen in such a way that $c_1 + c_2 s + s^2$ is Hurwitz (cf [17]).

It is well known that trajectories of σ converge to a sliding regime on $\sigma = 0$ in finite time $T = W^{-1}|\sigma(0)|$.

The expression for the dynamical implicit sliding mode controller is given by

$$\begin{aligned} \dot{u} &= -\frac{c_1}{\alpha}q_2 - \frac{c_2}{\alpha}q_3 + \beta q_1 + \beta f(q_1) + \gamma f(q_1) - \gamma u + \dots \\ &+ \frac{\gamma}{\alpha}q_2 + \frac{\beta}{\alpha}q_2 + \gamma q_2 + \frac{\partial^2 f(q_1)}{\partial q_1^2}q_2^2 + q_3 + \frac{1}{\alpha}q_3 + \dots \\ &+ \frac{\gamma}{\alpha}q_3 + \frac{\partial f(q_1)}{\partial q_1}q_2 + \gamma\frac{\partial f(q_1)}{\partial q_1}q_2 - \frac{W}{\alpha}\mathrm{sign}(\sigma) \end{aligned} \tag{12}$$

We have implemented and tested both strategies, static and dynamical state feedback, using a 486 PC for data acquisition and control. The whole system was realized programming in assembler code. The hardware implementation is based on a recent work of Hayes *et al* [19].

From the computational point of view, the proposed control laws can locally stabilize the circuit response towards a constant equilibrium point. For tracking, the close loop system don't have the desired behavior due to the electrical noise of circuits elements.

IV. CONCLUSIONS

It was presented in this note that a *controlled* Chua's circuit is included in the broad class of differentially flat systems. This feature makes the linearizing feedback controller design a task particularly simple. In a rough sense, flatness implies controllability.

The proposed nonlinear control laws have the property of state asymptotic stabilization around a desired admissible trajectory or equilibrium point, this was illustrated through a numerical simulation.

It is possible to exploit differential flatness for adaptive control purpose using a novel technique called *control of the clock* [18]. This topic will be the subject of a forthcomming publication.

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