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Adaptive Input–Output Linearization for PWM Regulation of DC–to–DC Power Converters *

Hebertt Sira–Ramírez
Departamento Sistemas de Control
Escuela Ingeniería de Sistemas
Universidad de Los Andes
Mérida 5101, Venezuela
e-mail: isira@ing.ula.ve

Miguel Rios–Bolivar †and Alan S.I. Zinober
Applied Mathematics Section
School of Mathematics and Statistics
University of Sheffield
Sheffield S10 2TN, U.K.
e-mail: A.Zinober@sheffield.ac.uk

Abstract

Dynamical Adaptive regulation of Pulse–Width–Modulation (PWM) controlled power supplies is proposed using a suitable combination of average dynamical input–output linearization and the "backstepping" controller design method. The validity of the proposed approach, regarding control objectives and robustness with respect to unmodelled, yet unmatched, and bounded stochastic perturbation inputs, is tested through digital computer simulations.

1 Introduction

In this article, a non–overparameterized adaptive feedback control strategy is adopted which is based on a combination of the *adaptive backstepping* controller design algorithm (see [1], and [2]) and input-output dynamical feedback linearization (see [3]). The backstepping adaptation algorithm here proposed is shown to be suitable for a large class of state linearizable nonlinear systems with constant, but unknown, parameters including those systems which are not transformable to Parametric Pure and Strict Parameteric Feedback Canonical Forms. Observability of the uncertain system is demanded while inputs are allowed to appear in the intermediate steps of the procedure. Moreover, control input derivatives are invariably present at the final stage of the proposed algorithm. In Section 2 the developments leading to dynamical non–overparameterized adaptive PWM control strategies for dc–to–dc power supplies of the "Boost" types containing unknown parameter values, are presented. Computer simulations are presented in Section 3 for the assessment of the closed–loop performance of the derived dynamical adaptive controller strategies. Section 4 contains the conclusions and suggestions for further work in this area.

2 An Adaptive Controller for the "Boost" Converter

Consider the "Boost" converter, shown in Figure 1. The system is described by

$$\begin{aligned} \dot{I}(t) &= -\frac{1}{L}(1-u)V(t) + \frac{E}{L} \\ \dot{V}(t) &= \frac{1}{C}(1-u)I(t) - \frac{1}{RC}V(t) \\ y &= I(t) \end{aligned} \tag{2.1}$$

where L, C and R are respectively the inductance, capacitance and resistance values of the circuit components. The quantity E represents the constant value of the external voltage source. The control input function u is the *switch position function* taking values in the discrete set $\{0,1\}$. It is well known that, in a "boost" converter if the output capacitor voltage $V(t)$ is taken as the regulated output, then the system is non–minimum phase (see [4]).

A PWM feedback strategy for the specification of the switch position function u, is given as,

$$u = \begin{cases} 1 & \text{for } t_k \le t < t_k + \mu_a(t_k)T \\ 0 & \text{for } t_k + \mu_a(t_k)T \le t < t_k + T \end{cases}$$
$$t_k + T = t_{k+1} \;\; ; \;\; k = 0, 1, \ldots \tag{2.2}$$

where $\mu_a(t_k)$ is the value of the *actual duty ratio* function at the sampling instant t_k. The actual

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†On leave from the Universidad de Los Andes, Mérida–Venezuela

duty ratio function is obtained from the (feedback) ***computed duty ratio function***, denoted by μ, when restricted to taking values in the closed interval $[0,1]$. The sampling period, T, is assumed to be constant.

The *Average Model* for the PWM controlled Boost converter (2.1), (2.2), is obtained by formally replacing the switch position function u by the duty ratio function μ. Since the state and output variables have an average connotation we denote them differently from the original variables.

$$\begin{aligned} \dot{\zeta}_1 &= -\frac{1}{L}(1-\mu)\zeta_2 + \frac{E}{L} \\ \dot{\zeta}_2 &= \frac{1}{C}(1-\mu)\zeta_1 - \frac{1}{RC}\zeta_2 \\ \eta &= \zeta_1 \end{aligned} \tag{2.3}$$

The Average PWM model (2.3) has the enormous advantage of reducing any output stabilization problem, defined on the converter model (2.1), to a standard ***nonlinear feedback controller design problem*** in which the duty ratio function μ plays the role of the required input variable. Once the computed duty ratio function, μ, is synthesized as a feedback function for the regulation of the average system, the actual duty ratio function, μ_a, is obtained from a bounding operation on the values of μ, to the unit interval.

We denote the values of the several parameters defining the circuit equations as

$$\theta_1 = \frac{1}{L} \quad ; \quad \theta_2 = \frac{1}{C} \quad ; \quad \theta_3 = \frac{1}{RC} \quad ; \quad \theta_4 = \frac{E}{L} \tag{2.4}$$

The actual values of these parameters are assumed to be totally unknown, hence the need for controller adaptation.

Under the assumption of a constant value of the duty ratio function $\mu = U$, with $0 < U < 1$, the equilibrium values of the average PWM converter models are readily obtained from (2.3) and (2.4) as

$$\zeta_1(U) = \frac{\theta_3\theta_4}{\theta_2\theta_1(1-U)^2} \quad ; \quad \zeta_2(U) = \frac{\theta_4}{\theta_1(1-U)} \tag{2.5}$$

The primary objective of our adaptive duty ratio synthesizer is in the feedback regulation of the average input inductor current $\zeta_1(t)$, towards a known, constant, equilibrium value, denoted by $X_1 = \zeta_1(U)$. This value corresponds to some constant value, U, of the (actual) duty ratio function.

2.1 Non–Overparameterized Adaptive Controller for the Boost Converter

Consider the average PWM Boost converter model (2.3). We denote an estimate of the unknown parameter values by,

$$\hat{\theta}_i \ , \ i = 1,2,3,4 \tag{2.6}$$

Step 0
We let z_1 stand for the output variable error, defined as

$$z_1 = \eta - \zeta_1(U) = \zeta_1 - \zeta_1(U) \tag{2.7}$$

The time derivative of the output error z_1, is of unknown nature and given by

$$\dot{z}_1 = -\theta_1(1-\mu)\zeta_2 + \theta_4 \tag{2.8}$$

An estimate of $\dot{z}_1$ may be obtained as,

$$\hat{\dot{z}}_1 = -\hat{\theta}_1(1-\mu)\zeta_2 + \hat{\theta}_4 \tag{2.9}$$

We can rewrite the expression (2.8) as

$$\dot{z}_1 = \hat{\dot{z}}_1 + (\theta - \hat{\theta})^T \begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{2.10}$$

Step 1
Let the estimated value of $\dot{z}_1$ satisfy,

$$\hat{\dot{z}}_1 = -c_1 z_1 \quad ; \quad c_1 > 0 \tag{2.11}$$

where c_1 is a positive scalar design constant. We call the following form of expression (2.11), the "pseudo–controller" equation.

$$-\hat{\theta}_1(1-\mu)\zeta_2 + \hat{\theta}_4 = -c_1(\zeta_1 - \zeta_1(U)) \tag{2.12}$$

This equation represents a desired algebraic relation by which an effective stabilization of the output error would be possible when used in combination with a suitable estimation update law for the unknown parameters θ.
If the pseudo–controller relation (2.12) were valid, then from (2.11) and (2.10), the output error z_1 would satisfy

$$\dot{z}_1 = -c_1 z_1 + (\theta - \hat{\theta})^T \begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{2.13}$$

Choosing Γ to be a positive definite diagonal matrix whose elements γ_{ii} , $i = 1,2,3,4$, will be called *parameter adaptation gains,* we could consider next a scalar positive definite Lyapunov function of the form

$$V_1 = \frac{1}{2}\left[z_1^2 + (\theta - \widehat{\theta})^T\Gamma^{-1}(\theta - \widehat{\theta})\right] \qquad (2.14)$$

The time derivative of V_1 would result in

$$\begin{aligned} \dot{V}_1 &= -c_1 z_1^2 + \\ &(\theta - \widehat{\theta})^T\Gamma^{-1}\left(-\dot{\widehat{\theta}} + z_1\,\Gamma\begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right) \end{aligned} \qquad (2.15)$$

However, the pseudo-controller expression (2.12) is not generally valid from the outset. Thus a pseudo-controller error must be defined and our control effort in the second step of the algorithm should be geared towards forcing such an error to zero.

Consider then the pseudo-controller error

$$z_2 = \widehat{\dot{z}}_1 + c_1 z_1 = -\widehat{\theta}_1(1-\mu)\zeta_2 + \widehat{\theta}_4 + c_1(\zeta_1 - \zeta_1(U)) \qquad (2.16)$$

Using (2.16), the actual expression for the output error derivative becomes,

$$\dot{z}_1 = -c_1 z_1 + z_2 + (\theta - \widehat{\theta})^T\begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (2.17)$$

As a consequence, the derivative of the Lyapunov function (2.14) is given by

$$\begin{aligned} \dot{V}_1 &= -c_1 z_1^2 + z_2 + \\ &(\theta - \widehat{\theta})^T\Gamma^{-1}\left(-\dot{\widehat{\theta}} + z_1\,\Gamma\begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right) \end{aligned} \qquad (2.18)$$

Step 2

The time derivative of z_2 may then be written as:

$$\begin{aligned} \dot{z}_2 &= \theta_1\left(-c_1(1-\mu)\zeta_2\right) + \theta_2\left(-\widehat{\theta}_1(1-\mu)^2\zeta_1\right) + \\ &\quad \theta_3\left(\widehat{\theta}_1(1-\mu)\zeta_2\right) + \theta_4\left(c_1\right) \\ &\quad -\dot{\widehat{\theta}}_1(1-\mu)\zeta_2 + \widehat{\theta}_1\dot{\mu}\zeta_2 + \dot{\widehat{\theta}}_4 \end{aligned} \qquad (2.19)$$

An estimate of the pseudo-controller error derivative (2.19) is then obtained as,

$$\begin{aligned} \widehat{\dot{z}}_2 &= \widehat{\theta}_1\left(-c_1(1-\mu)\zeta_2\right) + \widehat{\theta}_2\left(-\widehat{\theta}_1(1-\mu)^2\zeta_1\right) + \\ &\quad \widehat{\theta}_3\left(\widehat{\theta}_1(1-\mu)\zeta_2\right) + \widehat{\theta}_4\left(c_1\right) \\ &\quad -\dot{\widehat{\theta}}_1(1-\mu)\zeta_2 + \widehat{\theta}_1\dot{\mu}\zeta_2 + \dot{\widehat{\theta}}_4 \end{aligned} \qquad (2.20)$$

One can easily rewrite the derivative of the pseudo-controller error z_2 as

$$\dot{z}_2 = \widehat{\dot{z}}_2 + (\theta - \widehat{\theta})^T\begin{bmatrix} -c_1(1-\mu)\zeta_2 \\ -\widehat{\theta}_1(1-\mu)^2\zeta_1 \\ \widehat{\theta}_1(1-\mu)\zeta_2 \\ c_1 \end{bmatrix} \qquad (2.21)$$

We now let the estimate of the pseudo-controller error derivative satisfy,

$$\widehat{\dot{z}}_2 = -c_2 z_2 \quad ; \quad c_2 > 0 \qquad (2.22)$$

If the time derivatives of the estimated parameters were already known, then expression (2.22) would constitute a *controller equation.* We proceed then to find the update laws for the estimation of the unknown parameters.

Substituting (2.22) in (2.21), the closed-loop behaviour of the pseudo-controller error z_2 is found to be governed by

$$\dot{z}_2 = -c_2 z_2 + (\theta - \widehat{\theta})^T\begin{bmatrix} -c_1(1-\mu)\zeta_2 \\ -\widehat{\theta}_1(1-\mu)^2\zeta_1 \\ \widehat{\theta}_1(1-\mu)\zeta_2 \\ c_1 \end{bmatrix} \qquad (2.23)$$

Using then a scalar Lyapunov function of the form

$$V_2 = \frac{1}{2}\left[z_1^2 + z_2^2 + (\theta - \widehat{\theta})^T\Gamma^{-1}(\theta - \widehat{\theta})\right] = V_1 + \frac{1}{2}z_2^2 \qquad (2.24)$$

one finds that the time derivative of V_2 satisfies

$$\begin{aligned} \dot{V}_2 &= -c_1 z_1^2 + z_1 z_2 - c_2 z_2^2 \qquad (2.25) \\ &+(\theta - \widehat{\theta})^T\Gamma^{-1}\left(-\dot{\widehat{\theta}} + z_1\Gamma\begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \right. \\ &\qquad \left. z_2\,\Gamma\begin{bmatrix} -c_1(1-\mu)\zeta_2 \\ -\widehat{\theta}_1(1-\mu)^2\zeta_1 \\ \widehat{\theta}_1(1-\mu)\zeta_2 \\ c_1 \end{bmatrix}\right) \end{aligned}$$

If the update law for the estimated value of the parameters is now chosen as

$$\begin{aligned} \dot{\widehat{\theta}} &= z_1\Gamma\begin{bmatrix} -(1-\mu)\zeta_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \\ &\qquad z_2\,\Gamma\begin{bmatrix} -c_1(1-\mu)\zeta_2 \\ -\widehat{\theta}_1(1-\mu)^2\zeta_1 \\ \widehat{\theta}_1(1-\mu)\zeta_2 \\ c_1 \end{bmatrix} \end{aligned} \qquad (2.26)$$

then an asymptotically stable behaviour to zero can be guaranteed for both the output error and the pseudo–controller error while achieving bounded evolution of the parameter estimates $\hat{\theta}$. Indeed, the output derivative of the Lyapunov function (2.25) is obtained as

$$\dot{V} = \frac{1}{2}\left(-c_1 z_1^2 + z_1 z_2 - c_2 z_2^2\right) \qquad (2.27)$$

which is a negative definite function provided the design parameters c_1 and c_2 satisfy $4c_1c_2 > 1$. From (2.20), (2.22) and (2.26) the duty ratio function μ can be readily obtained in an implicit manner, as the solution of a nonlinear time–varying differential equation. One obtains, after some straightforward manipulations

$$\begin{aligned} \dot{\mu} \;=\; & \frac{1}{\widehat{\theta}_1\zeta_2}\Big\{ - c_1c_2(\zeta_1 - X_1) - (c_1+c_2)\times \\ & \left[-\widehat{\theta}_1(1-\mu)\zeta_2 + \widehat{\theta}_4\right] \\ & +\widehat{\theta}_1(1-\mu)\left[\widehat{\theta}_2(1-\mu)\zeta_1 - \widehat{\theta}_3\zeta_2\right] \\ & - \left[\gamma_{44} + \gamma_{11}(1-\mu)^2\zeta_2^2\right]\Big[(\zeta_1 - X_1) + \\ & c_1\left(-\widehat{\theta}_1(1-\mu)\zeta_2 + \widehat{\theta}_4 + c_1(\zeta_1 - X_1)\right)\Big]\Big\} \end{aligned} \qquad (2.28)$$

It should be stressed that the output μ of the dynamical controller (2.28) constitutes the *computed* duty ratio function. The *actual* duty ratio function, denoted by μ_a, is obtained from the following bounding operation on μ,

$$\mu_a(t) = \begin{cases} 1 & \text{for } \mu(t) \geq 1 \\ \mu(t) & \text{for } 0 < \mu(t) < 1 \\ 0 & \text{for } \mu(t) < 0 \end{cases} \qquad (2.29)$$

The above physical restriction on the values of the computed duty ratio function results in local stabilization of the average input inductor current.

3 Simulation Results

We used the following stochastically perturbed model for simulations

$$\begin{aligned} \dot{I}(t) &= -\frac{1}{L}(1-u)V(t) + \frac{E+\nu(t)}{L} \\ \dot{V}(t) &= \frac{1}{C}(1-u)I(t) - \frac{1}{RC}V(t) \\ y &= I(t) \end{aligned} \qquad (3.1)$$

The following "unknown" values of the circuit parameters, were used,

$$C = 181.82\ \mu\text{F}\ ;\ L = 0.27\ \text{mH}\ ;\ R = 2.44\Omega\ ;$$

$$E = 14.667\ \text{Volts}$$

The sampling frequency was set to be 100KHz and the computer gernated random noise amplitude was set to be 2.44 Volts (16 % of the value of E). Figure 2 depicts the dynamic adaptively regulated state responses of the Boost converter. Figure 3 and 4 show the estimated parameters. Figure 5 depicts the duty ratio function trajectory and Figure 6 represents the perturbation noise. The regulated output variable, $I(t)$, is seen to converge asymptotically towards the desired equilibrium value set to be $I(t) = 15.75$, with corresponding $V = 23.77$Volts and $\mu = U = 0.38$.

4 Conclusions

A robust adaptive Pulse–Width–Modulation feedback regulation approach, based on input–output linearization and a direct version of the backstepping algorithm, has been developed which results in the effective input inductor current stabilization for parameter uncertain dc–to–dc power converters. The approach requires no transformations to either Parametric Pure, or Parametric Strict, Feeedback Canonical Forms.

References

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FIGURES

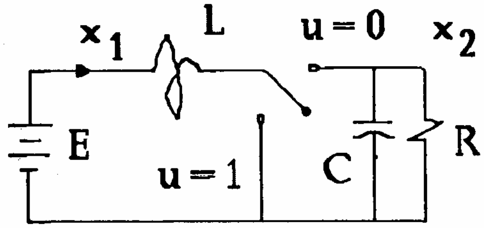

Figure 1: "Boost" Converter Circuit.

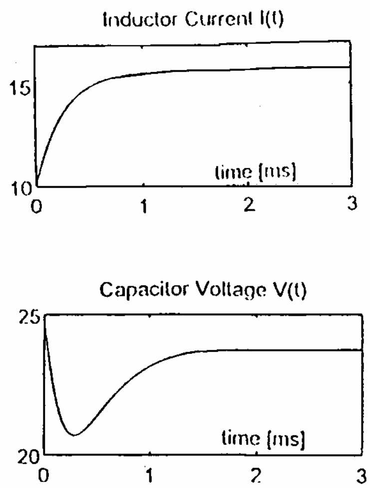

Figure 2: Dynamic adaptively controlled state trajectories of perturbed Boost converter.

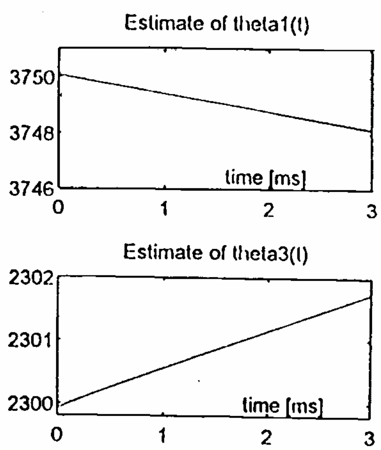

Figure 3: Estimated parameters

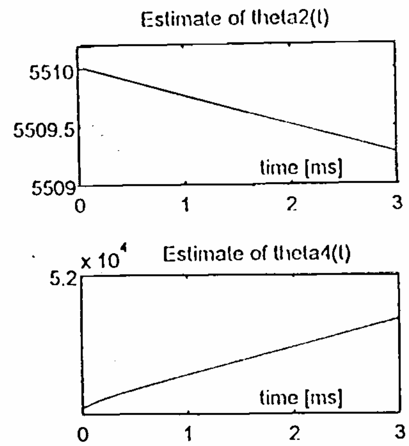

Figure 4: Estimated parameters

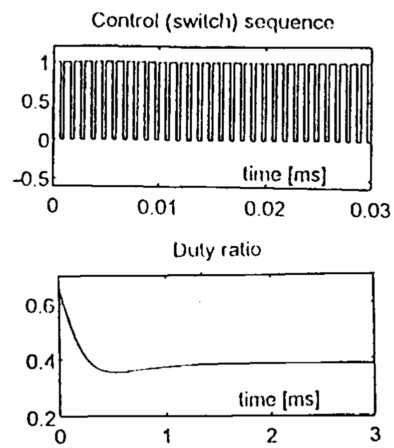

Figure 5: Duty ratio function trajecto

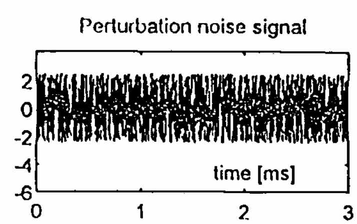

Figure 6: Perturbation noise