On backstepping PWM control of DC-to-DC converters

Laurent Karsenti and Hebertt Sira-Ramírez

Laboratoire des Signaux et Systèmes,
C.N.R.S., SUPELEC,
91192 Gif-sur-Yvette Cedex,France
e-mail:karsenti@lss.supelec.fr

Departamento Sistemas de Contro
Universidad de Los Andes
Mérida 5101, Venezuela
e-mail:isira@ing.ula.ve

Abstract

An incremental approach of the adaptive backstepping regulation, based on a flatness property, has been applied to uncertain dc-to-dc power converters of the Boost type. The backstepping design is applied on the average Pulse-Width-Modulation incremental system, and gives a dynamical stabilizing control with interesting robustness features. A simulation study illustrates the good properties of such a controller.

Key words : DC-to-DC Power Converters, Adaptive Regulation, Pulse-Width-Modulatior Backstepping, Flatness

1 Introduction

The backstepping feedback controller design technique has been recently advocated as a systematic nonlinear adaptive feedback control procedure aimed at obtaining simultaneous input to state feedback linearization and stability-based controller parameter update laws. This line of work has been summarized and thoroughly explained in a recent book by Kristić, Kanellakopoulos and Kokotovic [5]. The geometric based conditions to be demanded on a given system, on which the process is to be applied, imply some restrictions, summarized in the existence of a parameter independent state coordinate transformation taking the system into the so called *pure parameter feedback canonical form*.

In this work we present a simple example, drawn from the area of power electronics, consisting on an average model of a DC-to-DC Power converter of the boost type on which the conditions in [5] are no longer valid. We emphasize, however, that following a similar philosophical approach as that used for the input-output adaptive feedback linearization procedure, based on backstepping, already presented in Sira-Ramirez *et al* [7] and Karsenti *et al* [3], one can indeed achieve input-to-state adaptive feedback linearization without need of satisfying the stringent geometric conditions derived in [5].

In this article, we deal with the dynamical input-to-state adaptive feedback linearization and stabilization of an average model of a switch regulated "boost" converter circuit. All the parameters in the circuit model are assumed to be unknown but constant. In particular, we exploit the fact that the system is *differentially flat* [1] with the nominal total stored energy as the linearizing output for the nominal system. The nominal state coordinate transformation is applied to the actual system model and an incremental parametric uncertain system is obtained, which is not in pure parameter feedback canonical form. The backstepping procedure is directly applied to the perturbed model and, as a result, a dynamical adaptive feedback controller is obtained.

Section 2 presents the problem statement for the average PWM control of a boost converter circuit. Section 3 exploits the differential flatness associated to the stored energy of the system in order to obtain a nominal state feedback transformation of the system. This transformation places the system in a suitable form for application of the adaptive backstepping design technique, in spite of the fact of not complying with any of the existing canonical forms in [5]. Section 4 applies directly the backstepping design procedure to the incremental parameter uncertain system dealing with the control input as an available additional state variable. A dynamical adaptive feedback controller is hence obtained with interesting robustness features. Section 5 tests the robustness, associated with the proposed controller, with respect to external, unmatched, stochastic perturbation signals.

2 Problem statement: the Boost DC–to–DC converter

Consider the Boost converter circuit, shown in Figure 1. The system of differential equations, describing the inductor current $I(t)$ and the capacitor voltage $V(t)$, is given by

$$\begin{aligned} \dot{I}(t) &= -\frac{1}{L}(1-u)V(t) + \frac{E}{L} \\ \dot{V}(t) &= \frac{1}{C}(1-u)I(t) - \frac{1}{RC}V(t) \end{aligned} \tag{1}$$

where L, C and R are, respectively, the inductance, capacitance and resistance values of the circuit components. Let us note the physical assumptions: L, C and R are assumed to be nonzero. The quantity E represents the constant value of the external voltage source. We denote the values of the several parameters defining the circuit equations as

$$\theta_1 = \frac{1}{L} \quad ; \quad \theta_2 = \frac{E}{L} \quad ; \quad \theta_3 = \frac{1}{C} \quad ; \quad \theta_4 = \frac{1}{RC} \tag{2}$$

with $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T \in \Omega \subset \mathbf{R}^4$. The components of the unknown parameter vector, belonging to a given compact set Ω, are assumed to be constants.
We take the inductor current $I(t)$ as the regulated output function, here denoted by y. In this case, we know that the zero dynamics are stable, which corresponds to the minimum-phase case. On the other hand, if y is the output capacitor voltage $V(t)$, we obtain a non minimum-phase equilibrium (see Llanes-Santiago and Sira-Ramirez [6]).
The control input function u is the *switch position function* taking values in the discrete set $\{0, 1\}$.
A PWM feedback strategy for the specification of the switch position function u, occurring at regularly sampled instants of time is usually specified as follows:

$$u = \begin{cases} 1 & \text{for } t_k \leq t < t_k + \mu_a(t_k)T \\ 0 & \text{for } t_k + \mu_a(t_k)T \leq t < t_k + T \end{cases}$$
$$t_k + T = t_{k+1} \quad ; \quad k = 0, 1, \ldots \tag{3}$$

where $\mu_a(t_k)$ is the value of the *actual duty ratio* function at the sampling instant t_k and the sampling period, T, is assumed to be constant.
The actual duty ratio function is obtained from a bounding operation carried out on the feedback *computed duty ratio function*, denoted by μ, which restricts the values of μ to the closed interval $[0, 1]$. This physical restriction results of course in a local stabilization.
If we assume to have an infinite sampling frequency, it results in a smooth nonlinear average system, in which μ is interpreted as the control input, i.e. the *equivalent control* in Sliding Mode theory. Therefore the problem can be advantageously treated as a standard *nonlinear feedback controller design problem*. The *Average Model* for the PWM controlled Boost converter (1), (3), is then obtained by formally replacing the switch position function u by the duty ratio function μ, and the state variables by their average

$$\begin{aligned} \dot{\zeta}_1 &= -\theta_1(1-\mu)\zeta_2 + \theta_2 \qquad y = \zeta_1 \\ \dot{\zeta}_2 &= \theta_3(1-\mu)\zeta_1 - \theta_4\zeta_2 \end{aligned} \tag{4}$$

For a constant value of the duty ratio function, corresponding to a desired set point, $\mu = U$, with $0 < U < 1$, the equilibrium values of the average PWM converter state variables are readily obtained from (2) and (4) as

$$\zeta_{1n} = \frac{\theta_4\theta_2}{\theta_3\theta_1(1-U)^2} \quad ; \quad \zeta_{2n} = \frac{\theta_2}{\theta_1(1-U)}$$

3 Partial linearization via flatness

Let us consider incremental variations around known nominal values of the parameters, as follows :

$$\theta_i = \bar{\theta}_i + \Delta\theta_i \quad \text{for} \quad i = 1 \ldots 4, \tag{5}$$

where the $\bar{\theta}_i$ are the known nominal values.
Let us consider the following change of coordinates, exhibiting a flat output z_1 for the nominal system (see Sira-Ramirez and Ilic-Spong [8]) :

$$\left\{ \begin{array}{ll} z_1 & = \frac{1}{2}(\frac{1}{\bar{\theta}_1}\zeta_1^2 + \frac{1}{\bar{\theta}_3}\zeta_2^2) \\ \\ z_2 & = \frac{\bar{\theta}_2}{\bar{\theta}_1}\zeta_1 - \frac{\bar{\theta}_4}{\bar{\theta}_3}\zeta_2^2 \end{array} \right. \tag{6}$$

The resulting system in the new z-coordinates is then given by :

$$\left\{ \begin{array}{ll} \dot{z}_1 & = z_2 + \Delta\theta^T(f_1(z) + g_1(z)\mu) \\ \\ z_2 & = h_2(z,\mu) + \Delta\theta^T(f_2(z) + g_2(z)\mu) \end{array} \right. \tag{7}$$

with

$$f_1(z) = \begin{pmatrix} -\frac{\zeta_1\zeta_2}{\bar{\theta}_1} \\ \frac{\zeta_1}{\bar{\theta}_1} \\ \frac{\zeta_1\zeta_2}{\bar{\theta}_3} \\ -\frac{\zeta_2^2}{\bar{\theta}_3} \end{pmatrix} \qquad g_1(z) = \begin{pmatrix} \frac{\zeta_1\zeta_2}{\bar{\theta}_3} \\ 0 \\ -\frac{\zeta_1\zeta_2}{\bar{\theta}_1} \\ 0 \end{pmatrix}$$

$$f_2(z) = \begin{pmatrix} -\frac{\bar{\theta}_2\zeta_2}{\bar{\theta}_1} \\ \frac{\bar{\theta}_2}{\bar{\theta}_1} \\ -\frac{2\bar{\theta}_4\zeta_1}{\bar{\theta}_3} \\ \frac{2\bar{\theta}_4\zeta_2}{\bar{\theta}_3} \end{pmatrix} \qquad g_2(z) = \begin{pmatrix} \frac{\bar{\theta}_2\zeta_2}{\bar{\theta}_1} \\ 0 \\ \frac{2\bar{\theta}_4\zeta_1}{\bar{\theta}_3} \\ 0 \end{pmatrix}$$

and

$$h_2(z,\mu) = -\bar{\theta}_2\zeta_2(1-\mu) + \frac{\bar{\theta}_2^2}{\bar{\theta}_1} - 2\bar{\theta}_4\zeta_1\zeta_2(1-\mu) + \frac{2\bar{\theta}_4^2\zeta_2^2}{\bar{\theta}_3}$$

After the previous state transformation, the resulting system (5) is in a form resembling the pure parametric feedback form, introduced by Kanellakopoulos et al. in [2]. In fact, there is a slight difference due to the input dependance in the control μ in the first equation. Moreover, in order that a modified backstepping algorithm could be applied to this form of systems, the control has to be continuous. Indeed, at the second step of the procedure, we differentiate the control. That is why, the discontinuous control u, which takes discrete values in $\{0, 1\}$, has been approximated by its average (namely infinite sampling frequency) PWM value.

Remark :
Notice that the state coordinates transformation linking the original average state variables (ζ_1, ζ_2) to the new coordinates (z_1, z_2) is locally invertible, if the Jacobian matrix of this transformation:

$$\frac{\partial z}{\partial \zeta} = \left[\begin{array}{cc} \frac{1}{\bar{\theta}_1}\zeta_1 & \frac{1}{\bar{\theta}_3}\zeta_2 \\ \frac{\bar{\theta}_2}{\bar{\theta}_1} & -2\frac{\bar{\theta}_4}{\bar{\theta}_3}\zeta_2 \end{array} \right]$$

is non–singular. This is equivalent to :

$$\zeta_2(2\bar{\theta}_4\zeta_1 + \bar{\theta}_2) \neq 0.$$

This is a non-restrictive assumption since the physical values of the state variables and the nominal parameters are strictly positive.
Note that the non-singularity of the Jacobian matrix is equivalent to the observability of the average system (4) (see H.Sira-Ramírez and P.Lischinsky-Arenas [9]).
Hence, the local invertibility of the state transformation is ensured.

4 Backstepping design on the PWM average converter

Let us now apply a modified version of the backstepping procedure , introduced by Kanellakopoulos, Kokotovic and Morse in [2], and then improved by Krstić, Kanellakopoulos and Kokotovic in [4] , in the sense of a non overparameterized version of the previous one. In fact, we are extending the algorithm of [4] to systems allowing input dependancies in all its equations. The design is developed in detail in the following.

Step 1 :
We first perform the following change of coordinates :

$$\begin{aligned} \eta_1 &= z_1 - z_{1n} \\ \eta_2 &= z_2 - \alpha_1 \end{aligned} \tag{8}$$

with

$$z_{1n} = \frac{1}{2}\left(\frac{\zeta_{1n}^2}{\theta_1} + \frac{\zeta_{2n}^2}{\theta_3}\right),$$

where ζ_{1n} and ζ_{2n} are the nominal inductor current and capacitor voltage values towards which we want to regulate the system.
Note that η_1 can be defined as a regulation error and η_2 as a pseudo-control error.
The first equation of (7) is then rewritten in these new coordinates as follows :

$$\dot{\eta}_1 = \eta_2 + \alpha_1 + (f_1 + g_1\mu)^T\widehat{\Delta\theta} + (f_1 + g_1\mu)^T\widetilde{\Delta\theta}, \tag{9}$$

with the parametric error $\widetilde{\Delta\theta} = \Delta\theta - \widehat{\Delta\theta}$.
We now derive the *stabilizing function* α_1 as if it were a control variable and use it to stabilize the one-dimensional sub-system (9), taking as a Lyapunov function candidate the quantity:

$$V_1 = \frac{1}{2}\eta_1^2 + \frac{1}{2}\widetilde{\Delta\theta}^T\Gamma^{-1}\widetilde{\Delta\theta},$$

with Γ a diagonal positive definite matrix of *adaptation gains*.
Then, the derivative of V_1 is given by:

$$\dot{V}_1 = \eta_1\eta_2 + \eta_1[\alpha_1 + (f_1 + g_1\mu)^T\widehat{\Delta\theta}] + \widetilde{\Delta\theta}^T\Gamma^{-1}[\Gamma(f_1 + g_1\mu)\eta_1 - \dot{\widehat{\Delta\theta}}].$$

In order to eliminate the unknown term $\widetilde{\Delta\theta}$ from $\dot{V}_1$, we impose the following update law, which is called the *tuning function* in the work of Krstic et al [4] :

$$\dot{\widehat{\Delta\theta}} = \tau_1(\eta_1, \eta_2, \mu, \widehat{\Delta\theta}) = \Gamma(f_1(z) + g_1(z)\mu)\eta_1. \tag{10}$$

Then, if z_2 is our actual control, we would let $\eta_2 = 0$ or $z_2 = \alpha_1$, and in order to make $\dot{V}_1 = -c_1\eta_1^2 \leq 0$, for some positive constant c_1, we would choose the function $\alpha_1(z_1, z_2, \mu, \widehat{\Delta\theta})$ as follows:

$$\alpha_1(z_1, z_2, \mu, \widehat{\Delta\theta}) = -c_1\eta_1 - (f_1 + g_1\mu)^T \widehat{\Delta\theta}. \tag{11}$$

Now, since z_2 is not our control, namely the pseudo-control error $\eta_2 \neq 0$, and since the true parameter update law $\dot{\widehat{\Delta\theta}}$ is different from τ_1, we must rewrite the expressions of $\dot{z}_1$ and $\dot{V}_1$:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 - c_1\eta_1 + (f_1 + g_1\mu)^T \widehat{\Delta\theta} \\ \dot{V}_1 &= -c_1\eta_1^2 + \eta_1\eta_2 + \widetilde{\Delta\theta}^T \Gamma^{-1}(\tau_1 - \dot{\widehat{\Delta\theta}}). \end{aligned}$$

However, we retain τ_1 given by (10) as a tuning function which is a first approximation of the update law of parameters, and α_1 as a stabilizing function of (9), given by (11).

Step2 :
Let us set $\quad (\eta_1, \eta_2) = (z_1 - z_{1d}, z_2 - \alpha_1)$.
The sub-system (z_1, z_2) of (7) becomes

$$\begin{cases} \dot{\eta}_1 = & \eta_2 - c_1\eta_1 + (f_1 + g_1\mu)^T \widetilde{\Delta\theta} \\ \dot{\eta}_2 = & (1 - \frac{\partial\alpha_1}{\partial z_2})(h_2 + (f_2 + g_2\mu)^T \widehat{\Delta\theta}) - \frac{\partial\alpha_1}{\partial z_1}(z_2 + (f_1 + g_1\mu)^T \widehat{\Delta\theta}) - \frac{\partial\alpha_1}{\partial\widehat{\Delta\theta}}\dot{\widehat{\Delta\theta}} \\ & -\frac{\partial\alpha_1}{\partial\mu}\dot{\mu} + [(1 - \frac{\partial\alpha_1}{\partial z_2})(f_2 + g_2\mu)^T - \frac{\partial\alpha_1}{\partial z_1}(f_1 + g_1\mu)^T]\widetilde{\Delta\theta} \end{cases} \tag{12}$$

In order to stabilize the two-dimensional system (12), let us use α_2 as a control and

$$V_2 = V_1 + \frac{1}{2}\eta_2^2 = \frac{1}{2}\eta_1^2 + \frac{1}{2}\eta_2^2 + \frac{1}{2}\widetilde{\Delta\theta}^T \Gamma^{-1} \widetilde{\Delta\theta}$$

as a Lyapunov function candidate.
As previously, we eliminate $\widetilde{\Delta\theta}$ from $\dot{V}_2$ with the following update law :

$$\dot{\widehat{\Delta\theta}} = \tau_1(\eta, \mu, \widehat{\Delta\theta}) + \Gamma\left((1 - \frac{\partial\alpha_1}{\partial z_2})(f_2 + g_2\mu) - \frac{\partial\alpha_1}{\partial z_1}(f_1 + g_1\mu)\right)\eta_2. \tag{13}$$

Hence, $\dot{\widehat{\Delta\theta}}$ is equal to $\tau_1(\eta, \mu, \widehat{\Delta\theta})$ plus a corrective term.
Then, in order to obtain, for some positive constant c_2 :

$$\dot{V}_2 = -c_1\eta_1^2 - c_2\eta_2^2 \leq 0,$$

we choose the following dynamical control law :

$$\begin{aligned} \dot{\mu} = \ & (\widehat{\Delta\theta}^T g_1)^{-1}[-c_2\eta_2 - \eta_1 - (1 - \tfrac{\partial\alpha_1}{\partial z_2})(h_2 + (f_2 + g_2\mu)^T \widehat{\Delta\theta}) + \tfrac{\partial\alpha_1}{\partial z_1}(z_2 + (f_1 + g_1\mu)^T \widehat{\Delta\theta}) \\ & + \tfrac{\partial\alpha_1}{\partial\widehat{\Delta\theta}}\Gamma\left((1 - \tfrac{\partial\alpha_1}{\partial z_2})(f_2 + g_2\mu)\eta_2 - \tfrac{\partial\alpha_1}{\partial z_1}(f_1 + g_1\mu)\eta_2\right) + (f_1 + g_1\mu)\eta_1] \end{aligned} \tag{14}$$

Then, the stability analysis is quite similar to the one done by Kanellakopoulos et. al. in [2].
Since $\dot{V} = -\sum_{k=1}^{2} c_k z_k^2$, with positive c_k, it is straightforward that :

$$\dot{V} \leq -c_{min}\|z\|^2.$$

with c_{min} the minimum of the c_i, $1 \leq i \leq 2$.
This proves the uniform stability of the equilibrium: $\eta = 0, \hat{\theta} = \theta$ of the approximate adaptive system (12), according to Lyapunov arguments.
An estimate $\Omega_a \subset \Omega$ of the region of attraction of this equilibrium is obtained as follows. It is straightforward that the point $\eta = 0, \hat{\theta} = \theta$ coincides with the point $z = z_n, \hat{\theta} = \theta$. Let $\Omega_a(c)$ be the invariant set of (12) defined by $V < c$, and let c^* be the largest constant c such that $\Omega_a(c) \subset \Omega$. Then, an estimate of the region of attraction is given by:

$$\Omega_a = \Omega_a(c^*) = \{(\eta, \hat{\theta}) / V(\eta, \hat{\theta}) < c^*\} \quad \text{with} \quad c^* = \arg \sup_{\Omega_a(c) \subset \Omega} \{c\}.$$

Finally, using the LaSalle invariance principle, it is easily shown that the closed-loop system is such that $\forall(\eta(0), \hat{\theta}(0)) \in \Omega_a$, we have $\lim_{t \to \infty} \eta(t) = 0$, i.e. we obtain the regulation of system (12) towards the point $\eta = 0$.
Inductively, and as in [2], it can be concluded that system (7) is locally regulated around the equilibrium point $z = z_n$.

Remarks :

- Note that this dynamical controller exists if the following non-singularity condition is satisfied:

  $$\widehat{\Delta\theta}^T g_1(z) \neq 0$$

  which corresponds to the simpler expression :

  $$(\frac{\hat{L}}{L} - \frac{\hat{C}}{C})\zeta_1 \zeta_2 \neq 0.$$

  We can point out that the state variables ζ_1 and ζ_2 are bounded away from zero.

- The input dependent state coordinates transformation linking the original average state variables (z_1, z_2) to the new coordinates (η_1, η_2), respectively, the output error and the pseudo-controller error, given by:
  $\eta_1 = z_1 - z_{1n}$
  $\eta_2 = z_2 + c_1 z_1 + \widehat{\Delta\theta}^T (f_1(z) + g_1(z)\mu)$
  is locally invertible, if and only if the Jacobian matrix of this transformation is non-singular, namely :

  $$\frac{\partial \eta}{\partial z} = \begin{bmatrix} 1 & 0 \\ c_1 + \beta\eta_2 + \frac{\widehat{\Delta\theta_2}}{\theta_1} & 1 + \beta\eta_1 - 2\eta_2 \frac{\widehat{\Delta\theta_1}}{\theta_1} \end{bmatrix}$$

  with $\beta = (\mu - 1)(\frac{\widehat{\Delta\theta_1}}{\theta_1} - \frac{\widehat{\Delta\theta_2}}{\theta_1})$.

5 Simulation results

Simulations were performed for the Boost converter model in conjunction with the dynamical adaptive PWM controller described in Section 4 for the regulation of the input inductor current variable $I(t)$ of the converter. The nominal and perturbed versions of this converter model have been studied in simulation. The perturbation consisted of an unmodelled stochastic but bounded noise –denoted by $\nu(t)$– acting on the circuit through the external source voltage E. Hence, $\nu(t)$ is an unmatched additive disturbance, as shown in the following perturbed model, used in simulations:

$$\begin{aligned} \dot{I}(t) &= -\frac{1}{L}(1-u)\,V(t) + \left(\frac{E+\nu(t)}{L}\right) \qquad y = I(t) \\ \dot{V}(t) &= \frac{1}{C}(1-u)\,I(t) - \frac{1}{LC}V(t) \end{aligned}$$

The following "unknown" values of the circuit parameters were used for simulation purposes $C = 181.8\mu$F; $L = 0.27$mH; $R = 2.44\Omega$; $E = 14.66$V .
The sampling frequency was set to 100K Hz which corresponds to about 5000 clock pulses (by using a 50 MHz normal PC) for each sampling period . One could even have better computational allowances by decreasing the switch frequency but this may increase chattering. Let us also note that the MOSFET transistor is able to switch at this frequency value and is adequate to the corresponding power range (which is about 375 Watts).
As for the random input noise (expressing the electromagnetic interference, the resistive effect, the sensor error...), its amplitude was set to 0.4 Volts (2.7 % of the value of E) in Figure 3 and 2 Volts (13.6 % of the value of E) in Figure 4.
value of E).
The desired equilibrium value for the average input inductor current was set to $I(t) = 15.75$ amp. The obtained steady-state equilibrium value for the average output capacitor voltage was $V = 23.77$Volts. The duty ratio function corresponding to this equilibrium is $\mu = U = 0.38$.
The regulated output variable, $I(t)$, is seen to converge asymptotically towards the desired equilibrium value in the nominal and perturbed cases (respectively in Figure 2-3 and 4-5), exhibiting good performance and robustness. Note that the time scaling is the millisecond in the figures. The bounded evolution of the parameter estimates, a small portion of the switchings actions as well as the duty ratio function and the perturbation noise are also shown in the figures.

References

[1] M. Fliess, J. Lévine, P. Martín and P. Rouchon, "Flatness and Defect of Nonlinear Systems, Introductory Theory and Examples," *International J. of Control*, Vol. 61, pp. 1327-1361, 1995.

[2] I.Kanellakopoulos, P.V.Kokotovic and S.Morse, "Systematic design of adaptive controllers for feedback linearizable systems", *IEEE Aut. Control*, Vol 36, 1991, pp. 1241–1253.

[3] Karsenti, L., Sira-Ramírez, H., and Lamnabhi-Lagarrigue, F., Backstepping adaptive PWM stabilization of DC-to-DC converters towards minimum or non-minimum phase equilibria, *Proceedings 34th IEEE CDC*, New Orleans, 1995.

[4] M.Krstić, I.Kanellakopoulos and P.V.Kokotovic, "Adaptive nonlinear control without over-parameterization", *Systems & Control Letters*, Vol 19, 1992, pp. 177–185.

[5] M. Kristić, I. Kanellakopoulos and P. K. Kokotovic, *Nonlinear and Adaptive Control Design*, John Wiley & Sons 1995.

[6] O.Llanes-Santiago and H.Sira–Ramírez, "A Controller Resetting Strategy for the Stabilization of DC-to-DC Power Converters towards Non-minimum Phase Equilibria", *Proceedings 33d IEEE CDC*, Florida, Dec 1994.

[7] H. Sira-Ramírez, M.Garcia-Esteban and A. S. I. Zinober, "Dynamical Adaptive Pulse Width Modulation Control of DC-to-DC Power Converters", *International J. of Control* (accepted for publication, to appear).

[8] Sira-Ramírez, H., and Ilic-Spong, M., 1989, Exact linearization in switched-mode DC-to-DC power converters, *International Journal of Control*, Vol. **50**, pp. 511-524.

[9] H.Sira-Ramirez and P.Lischinsky–Arenas, "The Differential Algebraic Approach in Non-linear Dynamical Compensator Design for Dc–to–Dc Power Converters", *Int. Journal of Control*, Vol. 54, 1991, pp. 111–133.