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# Lagrangian Modeling of Switch Regulated DC-to-DC Power Converters *

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## Abstract

A Lagrangian approach is used for the modeling of switch-regulated DC–to–DC Power Converters. A set of switched Euler–Lagrange (EL) parameters, is proposed which recovers the individual EL formulations of the intervening circuit topologies for each particular switch position. Switched models of the "Boost", the "Buck–Boost", and the "Ćuk" converters circuits are systematically derived from the lagrangian formalism.

## 1 Introduction

Modeling of switched regulated dc-to-dc power converters was initiated by the pioneering work of Middlebrook and Ćuk [1] and Ćuk [2] in the mid seventies. The area has undergone a wealth of practical and theoretical development as evidenced by the growing list of research monographs, and textbooks, devoted to the subject (see, for instance, Kassakian *et al* [3] ).

In this article, a Lagrangian dynamics approach is used for deriving a physically motivated model of the DC–to–DC power converters. The approach consists in establishing the Euler–Lagrange (EL) parameters of the circuits associated with each one of the topologies corresponding to the two possible positions of the regulating switch. This consideration immediately leads one to realize that some EL parameters remain invariant under the switching action while some others are definitely modified by either the addition, or annihilation, of certain quantities. A switched model of the non-invariant parts of the EL parameters can then be proposed by their suitable inclusion through the switch position parameter. This inclusion is carried out in a consistent fashion so that, under a particular switch position parameter value, the original EL parameters, corresponding to the two intervening circuit topologies, are exactly recovered.

The switched EL parameter considerations immediately lead, through the use of the classical Lagrangian dynamics equations, to traditional systems of differential equations with discontinuous right hand sides, describing the actual behavior of the treated converters. The obtained switch-regulated models entirely coincide with the *state models* of DC–to–DC Power Converters introduced in [1] and [2].

## 2 Modeling of Switched Euler–Lagrange Systems

The Euler–Lagrange formulation of dynamical systems constitutes a thoroughly studied and developed chapter of Classical Mechanics. For the particular case of electrical and electromechanical systems, the reader is refered to the book by Meisel [4].

An Euler–Lagrange system is classically characterized by the following set of nonlinear differential equations, known as *Lagrange equations*,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = -\frac{\partial \mathcal{D}}{\partial \dot{q}} + \mathcal{F}_q \qquad (2.1)$$

where $q$ is the vector of *generalized positions*, assumed to have $n$ components, represented by $q_1, \ldots, q_n$, and $\dot{q}$ is the vector of *generalized velocities*. The scalar function $\mathcal{L}$ is the *Lagrangian* of the system, defined as the difference between the *kinetic energy* of the system, denoted by $\mathcal{T}(\dot{q}, q)$, and the *potential energy* of the system, denoted by $\mathcal{V}(q)$, i.e,

$$\mathcal{L}(\dot{q}, q) = \mathcal{T}(\dot{q}, q) - \mathcal{V}(q) \qquad (2.2)$$

The function $\mathcal{D}(\dot{q})$ is the *Rayleigh dissipation cofunction* of the system. The vector $\mathcal{F}_q = (\mathcal{F}_{q_1}, \ldots, \mathcal{F}_{q_n})$ represents the ordered components of the set of *generalized forcing functions* associated with each generalized coordinate.

Euler–Lagrange systems are, thus, generally represented by the set of equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{T}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{T}}{\partial q} + \frac{\partial \mathcal{V}}{\partial q} = -\frac{\partial \mathcal{D}}{\partial \dot{q}} + \mathcal{F}_q \qquad (2.3)$$

We refer to the set of functions $(\mathcal{T}, \mathcal{V}, \mathcal{D}, \mathcal{F})$ as the *Euler–Lagrange parameters* of the system and simply

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express a system $\Sigma$ by the ordered cuadruple

$$\Sigma = (\mathcal{T}, \mathcal{V}, \mathcal{D}, \mathcal{F}) \tag{2.4}$$

Note that equation (2.3) can be simply rewritten in terms of the *non-conservative* Lagrangian function $\mathcal{L}_{FD}$ (see White and Woodson [5]) as follows

$$\frac{\mathrm{d}}{dt}\frac{\partial \mathcal{L}_{FD}}{\partial \dot{q}} - \frac{\partial \mathcal{L}_{FD}}{\partial q} = 0 \tag{2.5}$$

where $\mathcal{L}_{FD}$ is given by

$$\mathcal{L}_{FD} = \mathcal{T} + \int_0^t \mathcal{D} dt - \mathcal{V} + q^T \mathcal{F}_q \tag{2.6}$$

## 2.1 Switch Regulated Euler–Lagrange Systems

We are particularly interested in dynamical systems containing a single *switch*, regarded as the only *control function* of the system. The switch position, denoted by the scalar $u$, is assumed to take values on a discrete set of the form $\{0,1\}$. We assume that for each one of the switch position values, the resulting system is an Euler-Lagrange system (EL system for short) characterized by its corresponding EL parameters. In other words, we assume that when the switch position parameter takes the value, say, $u = 1$, the system, denoted by $\Sigma_1$, is characterized by a known set of EL parameters,

$$\Sigma_1 = (\mathcal{T}_1, \mathcal{V}_1, \mathcal{D}_1, \mathcal{F}_1) \tag{2.7}$$

The system $\Sigma_1$ is thus characterized by its nonconservative Lagrangian function $\mathcal{L}_{DF}^1$. Application of the traditional EL equations to $\mathcal{L}_{DF}^1$ is said to generate a *dynamical model* $\mathcal{M}_1$ of the system $\Sigma_1$

Similarly, when the switch position parameter takes the value $u = 0$, we assume that the resulting system, denoted by $\Sigma_0$ is characterized by

$$\Sigma_0 = (\mathcal{T}_0, \mathcal{V}_0, \mathcal{D}_0, \mathcal{F}_0) \tag{2.8}$$

with associated nonconservative Lagrangian function denoted by $\mathcal{L}_{DF}^0$ and dynamic model $\mathcal{M}_0$.

**Definition 2.1** *A function $\phi_u(\dot{q}, q) = \phi(\dot{q}, q, u)$, parametrized by $u$, is said to be* consistent *with the functions $\phi_0(\dot{q}, q)$ and $\phi_1(\dot{q}, q)$ whenever*

$$\phi_u|_{u=0} = \phi_0 \quad ; \quad \phi_u|_{u=1} = \phi_1 \tag{2.9}$$

**Definition 2.2** *We define a* nonconservative switched Lagrangian function $\mathcal{L}_{DF}^u$, *associated with the Lagrangian functions $\mathcal{L}_{DF}^0$ and $\mathcal{L}_{DF}^1$, as a function, parametrized by the switch position $u$, which is consistent with $\mathcal{L}_{DF}^0$ and $\mathcal{L}_{DF}^1$ for the corresponding values of the switch position parameter, $u \in \{0,1\}$.*

In correspondance with the non-conservative switched Lagrangian function $\mathcal{L}_{DF}^u$ we may also introduce the set of *switched EL parameters* $(\mathcal{T}_u, \mathcal{V}_u, \mathcal{D}_u, \mathcal{F}_u)$ as a set of functions parametrized by $u$ which are consistent, in the sense described above, with respect to the EL parameters of the systems $\Sigma_0$ and $\Sigma_1$ for each corresponding value of $u$. Similarly, the definition can also be extended, in an obvious manner, to engloble the dynamical *switched model* $\mathcal{M}_u$ of the switched system $\Sigma_u$ to be a model, parametrized by $u$, which is consistent with $\mathcal{M}_0$ and $\mathcal{M}_1$ for each corresponding value of $u$.

A switched system arsing from the EL systems $\Sigma_0$ and $\Sigma_1$ is a switched EL system whenever it is completely characterized by its set of switched EL parameters

$$\Sigma_u = (\mathcal{T}_u, \mathcal{V}_u, \mathcal{D}_u, \mathcal{F}_u) \tag{2.10}$$

The basic problem in an EL approach to the modeling of switched systems, arising from individual EL systems, is the following: *Given two EL systems $\Sigma_0$ and $\Sigma_1$ characterized by EL parameters, $(\mathcal{T}_0, \mathcal{V}_0, \mathcal{D}_0, \mathcal{F}_0)$ and $(\mathcal{T}_1, \mathcal{V}_1, \mathcal{D}_1, \mathcal{F}_1)$, respectively, determine a consistent parametrization of the EL parameters, $(\mathcal{T}_u, \mathcal{V}_u, \mathcal{D}_u, \mathcal{F}_u)$ in terms of the switch position $u$, with corresponding nonconservative switched Lagrangian $\mathcal{F}_{DF}^u$, such that the model obtained by direct application of the EL equations (2.5) on $\mathcal{F}_{DF}^u$, results in a parametrized model $\mathcal{M}_u$, which is consistent with $\mathcal{M}_0$ and $\mathcal{M}_1$.*

# 3 A Lagrangian Viewpoint in the Modeling of DC–to–DC Converters with Ideal Switches

## 3.1 The "Boost" converter circuit

Consider the switch–regulated "Boost" converter circuit of Figure 1. The differential equations describing the circuit were derived in [1] using the classic Kirchoff laws. Such set of equations are given by

$$\begin{aligned} \dot{x}_1 &= -(1-u)\frac{1}{L}x_2 + \frac{E}{L} \\ \dot{x}_2 &= (1-u)\frac{1}{C}x_1 - \frac{1}{RC}x_2 \end{aligned} \tag{3.1}$$

where $x_1$ and $x_2$ represent, respectively, the input inductor current and the output capacitor voltage variables. The positive quantity $E$ represents the constant voltage value of the external voltage source. The parameter $u$ denotes the switch position. The switch position parameter takes values in the discrete set $\{0,1\}$.

Consider $u = 1$. In this case two separate, or decoupled, circuits are clearly obtained and the corresponding Lagrange dynamics formulation can be carried out as follows.

Define $\mathcal{T}_1(\dot{q}_L)$ and $\mathcal{V}_1(q_C)$ as the kinetic and potential energies of the circuit respectively. We denote by $\mathcal{D}_1(\dot{q}_C)$ the Rayleigh dissipation cofunction of the circuit. These quantities are readily found to be

$$\begin{aligned} \mathcal{T}_1(\dot{q}_L) &= \frac{1}{2}L(\dot{q}_L)^2 \\ \mathcal{V}_1(q_C) &= \frac{1}{2C}q_C^2 \\ \mathcal{D}_1(\dot{q}_C) &= \frac{1}{2}R(\dot{q}_C)^2 \\ \mathcal{F}^1_{q_L} &= E \;\; ; \;\; \mathcal{F}^1_{q_C} = 0 \end{aligned} \qquad (3.2)$$

where $\mathcal{F}^1_{q_L}$ and $\mathcal{F}^1_{q_C}$ are the *generalized forcing* functions associated with the coordinates $q_L$ and $q_C$, respectively.

Evidently, the Lagrange equations (2.1), or (2.3), used on these EL parameters immediately rederive equation (3.1), with $u = 1$, as it can be esasily verified.

Consider now the case $u = 0$. The corresponding Lagrange dynamics formulation is carried out in the next paragraphs.

Define $\mathcal{T}_0(\dot{q}_L)$ and $\mathcal{V}_0(q_C)$ as the kinetic and potential energies of the circuit, respectively. We denote by $D_0(\dot{q}_L, \dot{q}_C)$ the Rayleigh dissipation function of the circuit. These quantities are readily found to be,

$$\begin{aligned} \mathcal{T}_0(\dot{q}_L) &= \frac{1}{2}L(\dot{q}_L)^2 \\ \mathcal{V}_0(q_C) &= \frac{1}{2C}q_C^2 \\ \mathcal{D}_0(\dot{q}_L, \dot{q}_C) &= \frac{1}{2}R(\dot{q}_L - \dot{q}_C)^2 \\ \mathcal{F}^0_{q_L} &= E \;\; ; \;\; \mathcal{F}^0_{q_C} = 0 \end{aligned} \qquad (3.3)$$

where, $\mathcal{F}^0_{q_L}$ and $\mathcal{F}^0_{q_C}$ are the *generalized forcing* functions associated with the coordinates $q_L$ and $q_C$, respectively.

Evidently, the Lagrange equations associated with these definitions immediately rederive equation (3.1), with $u = 0$, as it can be esasily verified.

The EL parameters of the two situations generated by the different switch position values result in identical kinetic and potential energies. The switching action merely changes the Rayleigh dissipation cofunction between the values $\mathcal{D}_0(\dot{q}_C)$ and $\mathcal{D}_1(\dot{q}_L, \dot{q}_C)$. Therefore, the *dissipation structure* of the system is the only one affected by the switch position. One may then regard the switching action as a "damping injection", performed through the inductor current.

$$\begin{aligned} \mathcal{T}_u(\dot{q}_L) &= \frac{1}{2}L(\dot{q}_L)^2 \\ \mathcal{V}_u(q_C) &= \frac{1}{2C}q_C^2 \\ \mathcal{D}_u(\dot{q}_L, \dot{q}_C) &= \frac{1}{2}R\left[(1-u)\dot{q}_L - \dot{q}_C\right]^2 \\ \mathcal{F}^u_{q_L} &= E \;\; ; \;\; \mathcal{F}^u_{q_C} = 0 \end{aligned} \qquad (3.4)$$

Note that in the cases where $u$ takes the values $u = 1$ and $u = 0$, one recovers, respectively, the dissipation cofunctions $\mathcal{D}_1(\dot{q}_C)$ in (3.2) and $\mathcal{D}_0(\dot{q}_L, \dot{q}_C)$ in (3.3) from the proposed dissipation cofunction, $\mathcal{D}_u(\dot{q}_L, \dot{q}_C)$, of equation (3.4). The proposed EL parameters are therefore *consistent.*

The switched lagrangian function associated with the above defined EL parameters is given by

$$\mathcal{L}_u = \mathcal{T}_u(\dot{q}_L) - \mathcal{V}_u(q_C) = \frac{1}{2}L(\dot{q}_L)^2 - \frac{1}{2C}q_C^2 \qquad (3.5)$$

One then proceeds, using the Lagrange equations (2.1), to formally obtain the switch-position parametrized differential equations defining the switch regulated system which corresponds to the proposed switched EL parameters (3.4). Such equations are given by

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}_u}{\partial \dot{q}_L}\right) - \frac{\partial \mathcal{L}_u}{\partial q_L} &= -\frac{\partial \mathcal{D}_u}{\partial \dot{q}_L} + \mathcal{F}_{q_L} \\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}_u}{\partial \dot{q}_C}\right) - \frac{\partial \mathcal{L}_u}{\partial q_C} &= -\frac{\partial \mathcal{D}_u}{\partial \dot{q}_C} + \mathcal{F}_{q_C} \end{aligned} \qquad (3.6)$$

Use of (3.6) on (3.5),(3.4) result in the following set of differential equations

$$\begin{aligned} L\ddot{q}_L &= -(1-u)R\left[(1-u)\dot{q}_L - \dot{q}_C\right] + E \\ \frac{q_C}{C} &= R\left[(1-u)\dot{q}_L - \dot{q}_C\right] \end{aligned} \qquad (3.7)$$

which can be rewritten, after substitution of the second equation into the first, as

$$\begin{aligned} \ddot{q}_L &= -(1-u)\frac{q_C}{LC} + \frac{E}{L} \\ \dot{q}_C &= -\frac{1}{RC}q_C + (1-u)\dot{q}_L \end{aligned} \qquad (3.8)$$

Using $x_1 = \dot{q}_L$ and $x_2 = q_C/C$ one obtains

$$\begin{aligned} \dot{x}_1 &= -(1-u)\,\frac{1}{L}x_2 + \frac{E}{L} \\ \dot{x}_2 &= (1-u)\,\frac{1}{C}x_1 - \frac{1}{RC}x_2 \end{aligned} \qquad (3.9)$$

The proposed switched dynamics (3.9) coincides with the classical state model developed in [1] and [2].

## 3.2 The "Buck–Boost" converter circuit

We summarize all the relevant formulae, and equations, leading to the switched model of the "Buck–Boost" converter circuit, through and EL formulation, in Table 1, at the end of the article. The circuit of the "Buck-Boost" converter is shown in Figure 2

**Remark 3.1** *The lagrangian approach to modeling of the "Buck-Boost" converter reveals that only the "dissipation strucutre" and the "external forcing functions" are non-invariant with respect to the switching action.*

### 3.3 The "Ćuk" converter circuit

We summarize all the relevant formulae and equations leading to the switched model of the "Ćuk" converter circuit through our proposed EL formulation in Table 2, at the end of the article. The Ćuk converter model is shown in Figure 3.

As inferred from Table 2, the lagrangian approach to modeling of the "Ćuk" converter reveals that only the "potential energy" structure of the system is non-invariant with respect to the switching action.

## 4 Conclusions

In this article we have shown that well-known models of DC–to–DC Power Converters constitute a special class of Euler–Lagrange systems with switch-dependent Euler-Lagrange parameters. Ideal switching devices were first considered and the corresponding switched models of the traditional converters structures were derived by appropriately combining the Euler Lagrange parameters associated with the intervening circuit topologies. The lagrangian formalism may also be extended to handle multivariable versions of switched-regulated power converters and realistic models of traditional switch–regulated power supplies including parasitic resistances and parasitic voltage sources. The nature of the lagrangian formulation is highly appealing and consistent with recent trends in Automatic Control theory whereby a *passivity* based approach is emerging as an advantageous physically motivated controller design technique which exploits the energy structure of Euler-Lagrange systems (see Ortega *et al* [6] and the references therein). This article, thus, must be regarded as an initial step towards the formalizaton and development of a systematic nonlinear feedback controller design methodology, based on the passivity approach, for a variety of switched regulated models of DC–to–DC Power Converters.

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# FIGURES

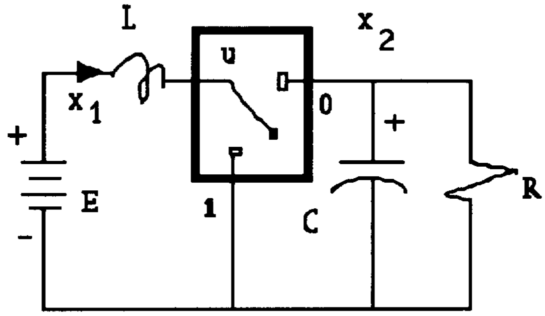


Figure 1: The "Boost" Converter Circuit.

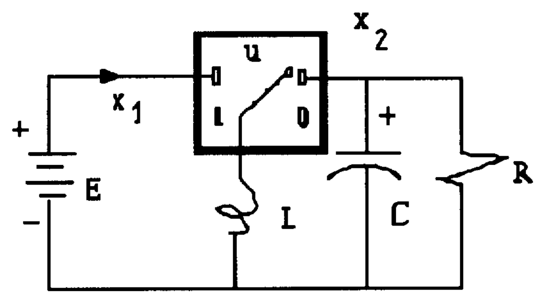


Figure 2: The "Buck–Boost" converter circuit.

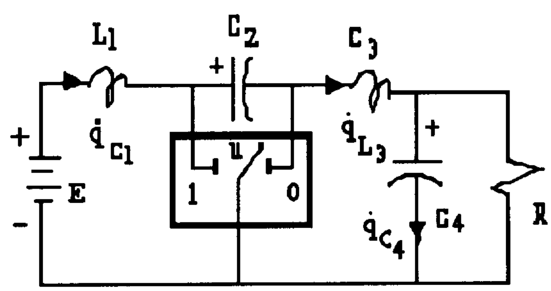


Figure 3: The "Ćuk" converter circuit.

| **BUCK – BOOST CONVERTER** | | |
|---|---|---|
| Euler – Lagrange Parameters for Possible Switch Positions | | |
| $u$ | $u=0$ | $u=1$ |
| Kinetic Energy | $\mathcal{T}_0(\dot{q}_L)=\frac{1}{2}L\,(\dot{q}_L)^2$ | $\mathcal{T}_1(\dot{q}_L)=\frac{1}{2}L\,(\dot{q}_L)^2$ |
| Potential Energy | $\mathcal{V}_0(q_C)=\frac{1}{2C}q_C^2$ | $\mathcal{V}_1(q_C)=\frac{1}{2C}q_C^2$ |
| Rayleigh Dissipation | $\mathcal{D}_0(\dot{q}_L,\dot{q}_C)=\frac{1}{2}R\,(\dot{q}_L+\dot{q}_C)^2$ | $\mathcal{D}_1(\dot{q}_C)=\frac{1}{2}R\,(\dot{q}_C)^2$ |
| Forcing Functions | $\mathcal{F}^0_{q_L}=0\ ;\ \mathcal{F}^0_{q_C}=0$ | $\mathcal{F}^1_{q_L}=E\quad;\quad\mathcal{F}^1_{q_C}=0$ |
| Switched Euler – Lagrange Parameters | | |
| Kinetic Energy | $\mathcal{T}_u(\dot{q}_L)=\frac{1}{2}L\,(\dot{q}_L)^2$ | |
| Potential Energy | $\mathcal{V}_u(q_C)=\frac{1}{2C}q_C^2$ | |
| Rayleigh Dissipation | $\mathcal{D}_u(\dot{q}_L,\dot{q}_C)=\frac{1}{2}R[(1-u)\dot{q}_L+\dot{q}_C]^2$ | |
| Forcing Functions | $\mathcal{F}^u_{q_L}=u\,E\quad;\quad\mathcal{F}^u_{q_C}=0$ | |
| Lagrangian for the "Buck – Boost" Converter Model | | |
| $\mathcal{L}_u=\mathcal{T}_u(\dot{q}_L)-\mathcal{V}_u(q_C)=\frac{1}{2}L\,(\dot{q}_L)^2-\frac{1}{2C}q_C^2$ | | |
| Switched model in Generalized Coordinates | | |
| $L\ddot{q}_L=-(1-u)R\,[(1-u)\dot{q}_L+\dot{q}_C]+uE$ <br> $\frac{q_C}{C}=-R\,[(1-u)\dot{q}_L+\dot{q}_C]$ | | |
| Definition of State Variables | | |
| $x_1=\dot{q}_L\quad;\quad x_2=\frac{q_C}{C}$ | | |
| Switched Model for the "Buck – Boost" Converter | | |
| $\dot{x}_1=(1-u)\,\frac{1}{L}x_2+u\,\frac{E}{L}$ <br> $\dot{x}_2=-(1-u)\,\frac{1}{C}x_1-\frac{1}{RC}x_2$ | | |

**Table 1** An EL approach for the Modeling of the "Buck-Boost" Converter

## ĆUK CONVERTER

Euler – Lagrange Parameters for Possible Switch Positions

| $u$ | $u=0$ | $u=1$ |
|---|---|---|
| Kinetic Energy | $\mathcal{T}_0(\dot{q}_{L_1},\dot{q}_{L_3}) = \frac{1}{2}L_1\,(\dot{q}_{L_1})^2 + \frac{1}{2}L\,(\dot{q}_{L_3})^2$ | $\mathcal{T}_1(\dot{q}_{L_1},\dot{q}_{L_3}) = \frac{1}{2}L_1\,(\dot{q}_{L_1})^2 + \frac{1}{2}L\,(\dot{q}_{L_3})^2$ |
| Potential Energy | $\mathcal{V}_0(q_{L_1},q_{C_4}) = \frac{1}{2C_2}q_{L_1}^2 + \frac{1}{2C_4}q_{C_4}^2$ | $\mathcal{V}_1(q_{L_3},q_{C_4}) = \frac{1}{2C_2}q_{L_3}^2 + \frac{1}{2C_4}q_{C_4}^2$ |
| Rayleigh Dissipation | $\mathcal{D}_0(\dot{q}_{L_3},\dot{q}_{C_4}) = \frac{1}{2}R\,(\dot{q}_{L_3}-\dot{q}_{C_4})^2$ | $\mathcal{D}_1(\dot{q}_{L_3},\dot{q}_{C_4}) = \frac{1}{2}R\,(\dot{q}_{L_3}-\dot{q}_{C_4})^2$ |
| Forcing Functions | $\mathcal{F}^0_{q_{L_1}} = E \;;\; \mathcal{F}^0_{q_{L_3}} = 0 \;;\; \mathcal{F}^0_{q_{C_4}} = 0$ | $\mathcal{F}^1_{q_{L_1}} = E \;;\; \mathcal{F}^1_{q_{L_3}} = 0 \;;\; \mathcal{F}^1_{q_{C_4}} = 0$ |

Switched Euler – Lagrange Parameters

| | |
|---|---|
| Kinetic Energy | $\mathcal{T}_u(\dot{q}_{L_1},\dot{q}_{L_3}) = \frac{1}{2}L_1\,(\dot{q}_{L_1})^2 + \frac{1}{2}L\,(\dot{q}_{L_3})^2$ |
| Potential Energy | $\mathcal{V}_u(q_{L_1},q_{L_3},q_{C_4}) = \frac{1}{2C_2}\left[(1-u)q_{L_1} + uq_{L_3}\right]^2 + \frac{1}{2C_4}q_{C_4}^2$ |
| Rayleigh Dissipation | $\mathcal{D}_u(\dot{q}_{L_3},\dot{q}_{C_4}) = \frac{1}{2}R\,(\dot{q}_{L_3}-\dot{q}_{C_4})^2$ |
| Forcing Functions | $\mathcal{F}^u_{q_{L_1}} = E \;;\; \mathcal{F}^u_{q_{L_3}} = 0 \;;\; \mathcal{F}^u_{q_{C_4}} = 0$ |

Lagrangian for the "Ćuk" Converter Model

$$\begin{aligned}\mathcal{L}_u(\dot{q}_{L_1},\dot{q}_{L_3},q_{L_1},q_{L_3},q_{C_4}) &= \mathcal{T}_u(\dot{q}_{L_1},\dot{q}_{L_3}) - \mathcal{V}_u(q_{L_1},q_{L_3},q_{C_4})\\ &= \frac{1}{2}L_1\,(\dot{q}_{L_1})^2 + \frac{1}{2}L\,(\dot{q}_{L_3})^2 - \frac{1}{2C_2}\left[(1-u)q_{L_1} + uq_{L_3}\right]^2 - \frac{1}{2C_4}q_{C_4}^2\end{aligned}$$

Switched model in Generalized Coordinates

$$\begin{aligned}L_1\ddot{q}_{L_1} &= -(1-u)\frac{1}{C_2}\left[(1-u)q_{L_1} + uq_{L_3}\right] + E\\ L_3\ddot{q}_{L_3} &= -u\frac{1}{C_2}\left[(1-u)q_{L_1} + uq_{L_3}\right] - R\,(\dot{q}_{L_3}-\dot{q}_{C_4})\\ \frac{q_{C_4}}{C_4} &= R\,(\dot{q}_{L_3}-\dot{q}_{C_4})\end{aligned}$$

Definition of State Variables

$$x_1 = \dot{q}_{L_1} \;;\; x_2 = \frac{1}{C_2}\left[(1-u)q_{L_1} + uq_{L_3}\right] \;;\; x_3 = \dot{q}_{L_3} \;;\; x_4 = \frac{q_4}{C_4}$$

Switched Model for the "Ćuk" Converter

$$\begin{aligned}\dot{x}_1 &= -(1-u)\,\frac{1}{L_1}x_2 + \frac{E}{L_1}\\ \dot{x}_2 &= (1-u)\,\frac{1}{C_2}x_1 + u\frac{1}{C_2}x_3\\ \dot{x}_3 &= -u\frac{1}{L_3}x_2 - \frac{1}{L_3}x_4\\ \dot{x}_4 &= \frac{1}{C_4}x_3 - \frac{1}{RC_4}x_4\end{aligned}$$

**Table 2** An EL approach for the Modeling of the "Ćuk" Converter