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On Passivity-Based Sliding Mode Control of Switched DC-to-DC Power Converters *

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Abstract

In this paper we propose to incorporate, into the sliding mode controller design for the regulation of dc-todc power converters of the "Boost" type, the energy dissipation and passivity properties of the switchregulated system. It is announced that for the "traditional" sliding mode controller, the integral of the stored energy is infinite while for a mixed passivitybased sliding mode controller the same index becomes finite.

1 Introduction

Sliding mode control of switched-regulated circuits has proven to be a naturally successful method for the regulation of dc-to-dc power supplies. See Venkataramanan et al [1], Sira-Ramírez and Ilic [2] and the references therein. The sliding mode feedback control approaches, so far presented, entirely overlook the physical properties of the "plant" constituted by the power converter and, specially, they ignore the possibility of modifiying its energy dissipation characteristic. The main objective of this article is to extend the passivity-based controller methodology to include switch-regulated models of dc-to-dc power converters, without appealing to approximate average models. We thus propose a new mixed passivity-based sliding mode controller. We only treat, in detail, the regulator design for the "Boost" case, The results can be extended to other dc-to-dc power converters, such as the "Buck", "Buck-boost" and the "Ćuk" converter.

2 Sliding Mode Control of the "Boost" Converter

Consider the following switch-regulated "Boost" converter circuit model (see Figure 1) The differential Gerardo Escobar and Romeo Ortega Université de Compiègne URA C.N.R.S. S17, BP 649 60206 Compiègne, France e-mail: rortega@hds.univ-compiegne.fr

equations describing the circuit are given by

$$\dot{x}_1 = -(1-u) \frac{1}{L} x_2 + \frac{E}{L}$$

$$\dot{x}_2 = (1-u) \frac{1}{C} x_1 - \frac{1}{RC} x_2 \qquad (2.1)$$

where x_1 and x_2 represent, respectively, the input inductor current and the output capacitor voltage variables. The positive quantity E represents the constant voltage value of the external voltage source. The variable u denotes the switch position function, acting as a control input. Such a control input takes values in the discrete set $\{0, 1\}$.

In te following proposition we denote by \overline{x}_1 and \overline{x}_2 , the state variables of the system under *ideal sliding mode* conditions.

Propostion 2.1 Consider the switching line $s = x_1 - V_d^2/RE$, where $V_d > 0$ is a desired constant capacitor voltage value. The switching policy, given by

$$u = \frac{1}{2} \left[1 - \text{sign}(s) \right] = \frac{1}{2} \left[1 - \text{sign}(x_1 - V_d^2/RE) \right]$$
(2.2)

locally creates a stable sliding regime on the line s = 0with ideal sliding dynamics characterized by

$$\overline{x}_1 = \frac{V_d^2}{RE} \quad ; \quad \dot{\overline{x}}_2 = -\frac{1}{RC} \left[\overline{x}_2 - \frac{V_d^2}{\overline{x}_2} \right] \quad ; \quad u_{eq} = 1 - \frac{E}{\overline{x}_2}$$
(2.3)

Moreover, the ideal sliding dynamics behaviour of the capacitor voltage variable, described by (2.3), can be explicitly computed as

$$\bar{x}_2(t) = \sqrt{V_d^2 + \left[\bar{x}_2^2(t_h) - V_d^2\right] e^{-\frac{2}{RC}(t - t_h)}}$$
(2.4)

where t_h stands for the reaching instant of the sliding line s = 0 and $\overline{x}_2(t_h)$ is the capacitor voltage at time t_h .

The stored error energy of the controlled system is defined as

$$H(t) = \frac{1}{2} \left[L \left(x_1(t) - \frac{V_d^2}{RE} \right)^2 + C \left(x_2(t) - V_d \right)^2 \right]$$
(2.5)

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A measure of the performance of the sliding mode controlled system, described above, is obtained by using the integral of the stored stabilization error energy. This quantity is given by

$$\mathcal{I}_B = \int_0^\infty H(\tau) \mathrm{d} \tau =$$
$$\int_0^\infty \frac{1}{2} \left[L \left(x_1(\tau) - \frac{V_d^2}{RE} \right)^2 + C \left(x_2(\tau) - V_d \right)^2 \right] \mathrm{d} \tau$$
(2.6)

We simply address such an index as the "WISSSE" index.

Propostion 2.2 The WISSSE index, computed along the sliding mode controlled trajectories of the "Boost" converter, is unbounded for all initial conditions of the converter.

3 Passivity-Based Sliding Mode Controller

Consider then the following auxiliary system

$$L\dot{x}_{1d} + (1-u)x_{2d} - R_1(x_1 - x_{1d}) = E$$

$$C\dot{x}_{2d} - (1-u)x_{1d} + \frac{1}{R}x_{2d} = 0$$
(3.1)

The following proposition depicts the most important features of a passivity based sliding currentmode regulation policy of the auxiliary system (3.1), and the plant, towards the desired constant equilibrium state $(x_{1d}(\infty), x_{2d}(\infty)) = (V_d^2/RE, V_d)$ of the "Boost" converter.

Propostion 3.1 Consider the switching line $s = x_{1d} - V_d^2/RE$, where $V_d > 0$ is a desired constant capacitor voltage value for x_{2d} and for the converter's capacitor voltage x_2 . The switching policy, given by

$$u = \frac{1}{2} [1 - \text{sign}(s)] = \frac{1}{2} [1 - \text{sign}(x_{1d} - V_d^2/RE)]$$
(3.2)

locally creates a sliding regime on the line s = 0. Moreover, the converter state trajectory x(t) converges towards the auxiliary state trajectory $x_d(t)$ and, in turn, $x_d(t)$ converges towards the desired equilibrium state. i.e.,

$$(x_1, x_2) \rightarrow (x_{1d}, x_{2d}) \rightarrow \left(\frac{V_d^2}{RE}, V_d\right)$$

Theorem 3.2 Consider the WISSSE performance index (2.6), the passivity-based sliding current-mode controller described in Proposition 3.1 yields a finite WISSSE index, provided the initial states of the converter satisfy $x_{1d}(0) \neq x_1(0)$ and $x_{2d}(0) \leq V_d$. Moreover, it can be easily established that the "input error" energy associated with the new controller satisfies the following property, depicted by the next proposition.

Propostion 3.3 The total energy associated with the difference between the control input u(t) and its constant average steady state value, $u_{eq}(\infty)$, given by

$$W_{d}(t) = \frac{1}{2} \int_{0}^{\infty} \left[u(\tau) - u_{eq}(\infty) \right]^{2} \mathrm{d}\tau$$

$$= \int_{0}^{t_{h}} \left[\frac{1}{2} \mathrm{sign} \left(x_{1d}(\tau) - \frac{V_{d}^{2}}{RE} \right) - \left(1 - \frac{E}{V_{d}} \right) \right]^{2} \mathrm{d}\tau$$

$$+ \int_{t_{h}}^{t} \left[u_{eq}(\tau) - \left(1 - \frac{E}{V_{d}} \right) \right]^{2} \mathrm{d}\tau$$

(3.3)

satisfies

$$\lim_{t\to\infty}W_d(t)=\infty$$

References

- V. VENKATARAMANAN, A. SABANOVIC, AND S. CÚK, "Sliding Mode Control of DC-to-DC Converters". In Proc. IECON'85 251-258, 1985.
- [2] H. SIRA-RAMÍREZ AND M. ILIC-SPONG, "A Geometric Approach to the Feedback Control of Switchmode DC-to-DC Power Supplies," *IEEE Transactions on Circuits and Systems*. Vol. CAS-35, No. 10, pp. 1291-1298, 1988.





Figure 1: The "Boost" Converter Circuit.