On the Passivization of Nonlinear Systems: A Geometric Approach

Hebertt Sira–Ramírez
Departamento Sistemas de Control
Escuela de Ingeniería de Sistemas
Universidad de Los Andes
Mérida 5101, Venezuela.
e-mail: isira@ing.ula.ve

Abstract

A geometric approach is explored for the passivity based regulation of continuous time processess. Although the approach is applicable to multi-input systems only the single input case is treated with an application example from the biological process control area where, traditionally, passivity based regulation has not been considered.

1 Introduction

Passive systems constitute a particular class of systems for which a scalar *available energy storage function* can be identified such that its rate of change, along the controlled systems trajectories, is never superior to the *supply rate* represented by the product of the input and the output. As a consequence of this simple definition, several interesting properties readily emerge. Passive systems, with positive definite storage functions, are zero input stable in the sense of Lyapunov and also, if the system output is rendered zero, by means of an appropriate feedback, the remaining dynamics or *zero dynamics* is also Lyapunov stable. These properties make the intrinsic behaviour of passive systems particularly attractive and even desirable in connection with a possible controller design strategy. Rendering an arbitrary nonlinear system passive will be addressed as "passivization" of the given system. It is our purpose to show that any nonlinear system can be "passivified" by means of a suitable state-dependent input coordinate transformation. The passivization is achievable in the sense that the system operator relating the new input coordinate and an auxiliary output function, is a passive operator. A more general form of passivity is that of *dissipativity* from where, historically speaking, all known results stem. General studies about dissipation properties in nonlinear systems were first provided by Willems [1]. The extension of these results to the case of nonlinear systems, which are affine in the control input, were given in the works by Hill and Moylan [2],[3]. Passivization of nonlinear systems by means of feedback was treated in the work of Byrnes *et al* [4]. Interesting developments can be found in Kokotovic and Sussman, [6], and in the work of Lin [5]. Non trivial applications of passivity based control, ranging from robotics to synchronous motors and power electronics have been given by Ortega and his coworkers (see the many references in [7]). The reader is invited to explore a complete and clear exposition, with many historical references, in the recent book by A. van der Schaft [8].

In this article we propose a geometric approach for the characterization of a passivization process induced on nonlinear systems by means of a state dependent input coordinate transformation. A geometric characterization of passivization implies the study of the local behaviour of the system defining vector fields with respect to a manifold of constant energy storage function. A natural decomposition of the system vector fields may be obtained by means of a simple state dependent input coordinate transformation which renders lossless, with respect to the storage function, the non-dissipative component of the drift vector field. The transformed system description is shown to contain four basic terms: the growth rate of the storage function gradient in balance with the dissipative field, the energy storage invariant fields and the field associated with the passivity supply rate. This decomposition is shown to have an immediate effect on the feedback regulation design carried out

SIEEEM-96-27 Artículo recomendado y aprobado por el Comité Técnico del IEEE Sección Monterrey, y presentado en el SIEEEM96, celebrado en Monterrey, N.L., México, del 23 al 25 de Octubre de 1996.

through energy storage function modification and damping injection possibilities on the associated semi-linear state space representation of the input transformed system.

Section 2 presents the general theoretical considerations dealing with the definitions of dissipativity, losslessnes and passivity. We also present in this section the geometric aspects of the passivization scheme by means of a state dependent input coordinate transformation (i.e., feedback). Section 3 is devoted to present a nontrivial application example, drawn from the biological process control area. Section 4 contains the conclusions and suggestions for further research in this area.

2 A Geometric Approach to Passivity-Based Regulation

2.1 Background Definitions

Consider the system,

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \qquad (2.1)$$

where $x \in \mathcal{X} \subset R^n$ is the state vector, $u \in \mathcal{U} \subset R^m$ is the control input and the vector $y \in \mathcal{Y} \subset R^p$ is the output function of the system. The vector fields $f(x)$ and $g(x)$ are assumed to be smooth vector fields on $\mathcal{X}$. For simplicity we assume the existence of an isolated state $x = x_e$, of interest, where $f(x_e) = 0$.

Associated with system (2.1) it is assumed to exist a *storage function*, $V : \mathcal{X} \mapsto R^+$. The storage function is such that $V(x_e) = \min_{x \in \mathcal{X}} V(x) = 0$. We define the *supply rate* function as a function $s : \mathcal{U} \times \mathcal{Y} \mapsto R$. This function is usually taken as the standard (inner) product $s(u,y) =< u, y >= u^T y$ in connection with passivity considerations.

We introduce some well-known background definitions about dissipative, lossless and passive systems (see Byrnes *et al* [4] and van der Schaft [8] for further details).

Definition 2.1 *([8]) System (2.1) is said to be* dissipative *with respect to the supply rate $s(u,y)$ if there exists a storage function $V : \mathcal{X} \mapsto R^+$, such that for all $x_0 \in \mathcal{X}$ and for all $t_1 \geq t_0$, and all input functions u, the following relation holds*

$$V(x(t_1)) - V(x(t_0)) \leq \int_{t_0}^{t_1} s(u(t), y(t))dt \qquad (2.2)$$

with $x(t_0) = x_0$ and $x(t_1)$ is the state resulting, at time t_1, from the solution of system (2.1) taking as initial condition x_0 and as control input the function $u(t)$

Definition 2.2 *([8]) System (2.1) is* passive *if it is dissipative with respect to the supply rate $s(u,y) = u^T y$. The system is* strictly input passive *if there exists $\delta > 0$ such that the system is dissipative with respect to $s(u,y) = u^T y - \delta ||u||^2$. The system is* strictly output passive *if there exist an $\epsilon > 0$ such that the system is dissipative with respect to $s(u,y) = u^T y - \epsilon ||u||^2$.*

We shall be addressing means of rendering a system of the form (2.1) passive, or at least lossless, by means of state feedback. We therefore introduce a definition of "passivifiable" system in the following terms

Definition 2.3 *System (2.2) is said to be "passivifiable" with respect to the storage function V if there exists a* regular affine feedback law *of the form*

$$u = \alpha(x) + \beta(x)v \ ; \ \alpha(x) \in R^m \ ; \ \beta(x) \in R^{m \times m} \quad (2.3)$$

where $\beta(x)$ is a nonsingular matrix, and such that the closed loop system (2.1)-(2.3) becomes passive with new vector input v.

Analogous definitions apply for the *strict input* and *strict output passivization* of systems of the form (2.1). As we shall see, in the first case, the feedback law is necessarily *nonlinear* in the new input v and, in the second case, the affine feedback law must include an output injection, or an output feedback, term.

Definition 2.4 *Consider a smooth drift vector field $\phi(x)$. Lef $L_\phi V$ stand for the* Lie derivative of V in the direction of ϕ. *In local coordiantes*

$$L_\phi V(x) = \frac{\partial V}{\partial x} \phi(x)$$

We say that the drift vector field $f(x)$ of (2.1) has a dissipative component $f_d(x)$, *with respect to the storage function V, whenever $f(x)$ can be expressed as the sum of two components*

$$f(x) = f_d(x) + f_{nd}(x)$$

such that,

1. $$L_{f_d} V(x) \leq 0 \ ; \ \forall x \in \mathcal{X}$$

   and

2. *$L_{f_{nd}} V(x)$ does not have any summand which is less than or equal to zero, in all of $\mathcal{X}$.*

2.2 Passivization by means of affine feedback

2.2.1 Single input case

Consider the single input case of the affine system (2.1) i.e., with $m = 1$ and $p = 1$. For a given control input $u = u(t)$ and any initial state x_0, the time derivative of the storage function V, along the solutions of (2.1), is given by

$$\dot{V} = \frac{\partial V}{\partial x} f(x) + \left(\frac{\partial V}{\partial x} g(x) \right) u = L_f V(x) + [L_g V(x)]\, u \tag{2.4}$$

It is assumed that V is a function of *relative degree* equals to one, i.e., $L_g V \neq 0$ in all of $\mathcal{X}$.

Suppose that the vector field $f(x)$ has a dissipative component $f_d(x)$ with respect to the storage function V

The time derivative of the energy storage function is then given by

$$\dot{V} = L_{f_d} V(x) + L_{f_{nd}} V(x) + [L_g V(x)]\, u \tag{2.5}$$

Note that the previous expression may be rewritten as

$$\dot{V} = L_{f_d} V(x) + [L_g V(x)] \left[\frac{L_{f_{nd}} V(x)}{L_g V(x)} + u \right] \tag{2.6}$$

Define the following state dependent input coordinate transformation

$$v = \frac{L_{f_{nd}} V(x)}{L_g V(x)} + u \tag{2.7}$$

The time derivative of the energy storage function satisfies then the following inequality

$$\dot{V} \leq [L_g V(x)]\, v \tag{2.8}$$

In other words, if the system has a dissipative component, of the drift vector field f, with respect to the energy storage function V, then the system exhibits a *passive* behaviour between the transformed input v and the auxiliary scalar output, $[L_g V(x)]$. If, on the other hand, the system drift vector field f does *not* have a dissipative component, i.e., $f_d(x) = 0 \; \forall \; x \in \mathcal{X}$, then the transformed system is no longer passive but *lossless* between the new input v and the auxiliary output $L_g V(x)$.

We have, therefore, proven the following result:

Proposition 2.5 *System (2.1) is passivifiable with respect to the storage function V, by means of affine feedback of the form (2.3) if and only if*

- $$L_g V(x) = h(x) \; \forall \; x \in \mathcal{X}$$

  and

- *There exists a dissipative component $f_d(x)$ of the vector field f, with respect to the storage function V i.e., $f = f_d(x) + f_{nd}(x)$, such that $L_{f_d} V(x) \leq 0 \; \forall \; x \in \mathcal{X}$.*

The affine feedback law, or state dependent input coordinate transformation, that achieves passivization is given by

$$u = v - \frac{L_{f_{nd}} V(x)}{L_g V(x)}$$

If a dissipative component, $f_d(x)$, of $f(x)$ does not exist, but still $L_g V(x) = h(x)$, then the system may be rendered lossless with respect to the storage function $V(x)$, with the same affine feedback law.

The above result can be extended to strict output passivifiable systems as follows:

Proposition 2.6 *System (2.1) is strictly output passivifiable with respect to the storage function V, by means of affine feedback, and output signal injection, if and only if*

- $$L_g V(x) = h(x) \; \forall \; x \in \mathcal{X}$$

  and

- *There exists a dissipative component $f_d(x)$ of the vector field f, with respect to the storage function V.*

The affine state feedback law with output injection (i.e., output feedback) that achieves strict output passivization is given by

$$u = v - \frac{L_{f_{nd}} V(x)}{L_g V(x)} - \epsilon\, y$$

where ϵ is a strictly positive constant.

Proof

Immediate from the definition of a strict output passive system.

□

A less interesting consequence is obtained for strict input passivifiable systems

Proposition 2.7 *System (2.1) is strictly input passivifiable with respect to the storage function V, by means of a nonlinear state feedback if and only if*

- $$L_g V(x) = h(x) \; \forall \; x \in \mathcal{X}$$

  and

- *There exists a dissipative component $f_d(x)$ of the vector field f, with respect to the storage function V.*

The nonlinear state feedback law that achieves strict input passivization is given by

$$u = -\frac{L_{f_{nd}}V(x)}{L_gV(x)} + (1 - \delta\frac{v}{L_gV})v$$

where δ is a strictly positive constant.

2.2.2 A geometric interpretation of passivization by affine feedback

Suppose a system of the form (2.1) is passivifiable and an input coordinate transformation of the form $u = v - L_{f_{nd}}V(x)/L_gV(x)$ has been applied to the system.

In transformed input coordinates, the system (2.1) is given, upon some simple algebraic manipulations and use of the definition of the Lie derivative, by

$$\dot{x} = f_d(x) + \left[I - g(x)\frac{\partial V(x)/\partial x}{L_gV(x)}\right] f_{nd}(x) + g(x)v \quad (2.9)$$

The geometric interpretation of equation (2.9) is given in Figure 1.

We clearly identify three terms in the right hand side of equation (2.9). The first term is, according to its definition, the dissipative term. The second term is the *workless* term, and the third term is the term responsible for the supply rate in terms of the new control input.

Note that the matrix

$$M(x) = \left[I - g(x)\frac{\partial V(x)/\partial x}{L_gV(x)}\right] \quad (2.10)$$

is a *projection operator* onto the tangent space to the level surface $V(x) = constant$, along the distribution span $\{g\}$.

Indeed, it is easy to verify that $M(x)$ satisfies the following properties:

- $$M(x)g(x) = 0 \;\; \forall\, x \in \mathcal{X} \quad (2.11)$$

- $$dVM(x) = 0 \;\; \forall\, x \in \mathcal{X} \quad (2.12)$$

- $$M^2(x) = M(x) \;\; \forall\, x \in \mathcal{X} \quad (2.13)$$

2.3 Feedback controller design via passivization

Suppose system (2.1) is passivifiable and assume $f_d(x)$ is a dissipative component of $f(x)$ with respect to the storage function $V(x)$. Suppose furthermore that $V(x)$ is given in the form

$$V(x) = \frac{1}{2}x^Tx \quad (2.14)$$

Then, the state space representation (2.9) can be further specialized to

$$\dot{x} = f_d(x) + \left[I - g(x)\frac{x^T}{x^Tg(x)}\right] f_{nd}(x) + g(x)v \quad (2.15)$$

From the fact that the time derivative of $V(x)$ is computed as

$$\begin{aligned} x^T\dot{x} &= x^Tf_d(x) + x^T\left[I - g(x)\frac{x^T}{x^Tg(x)}\right] f_{nd}(x) \\ &\quad + x^Tg(x)v \\ &= x^Tf_d(x) + x^Tg(x)v \end{aligned} \quad (2.16)$$

with the first term in the sum being strictly negative or at most zero, it is easily seen that the above system (2.15) may always be rewritten in the following form

$$\dot{x} = -R(x)x - J(x)x + g(x)v \quad (2.17)$$

with the following trivial identifications

$$f_d(x) = -R(x)x \;\; ; \;\; \left[I - g(x)\frac{x^T}{x^Tg(x)}\right] f_{nd}(x) = -J(x)x \quad (2.18)$$

with $R(x)$ being a positive semidefinite matrix in $\mathcal{X}$, and $J(x)$ is an anti-symmetric matrix.

A passivity based controller can be proposed for systems of the form (2.17) by considering the following *modified storage function*

$$V_d(x, x_d) = \frac{1}{2}(x - x_d)^T(x - x_d) \quad (2.19)$$

where x_d is an auxiliary state vector to be defined later.

Along the solutions of the system (2.17), the function $V_d(x, x_d)$ exhibits the following time derivative

$$\dot{V}_d(x, x_d) = (x - x_d)^T\left[-R(x)x - J(x)x + g(x)v - \dot{x}_d\right] \quad (2.20)$$

Completing squares in the right hand side and adding a *damping injection* term of the form $-R_{di}(x)x$, so that $R_m(x) = R(x) + R_{di}(x)$ is a negative definite matrix for all $x \in \mathcal{X}$, as follows,

$$\begin{aligned} \dot{V}_d(x, x_d) &= (x - x_d)^T[-(R(x) + R_{di}(x))(x - x_d) \\ &\quad - J(x)(x - x_d) - \dot{x}_d - R(x)x_d \\ &\quad - J(x)x_d + R_{di}(x)(x - x_d) + g(x)v] \end{aligned} \quad (2.21)$$

Note that if we let the auxiliary vector x_d, satisfy the following system of differential equations

$$\dot{x}_d = -R(x)x_d - J(x)x_d + R_{di}(x)(x - x_d) + g(x)v \quad (2.22)$$

then the time derivative of $V_d(x, x_d)$ satisfies

$$\begin{aligned} \dot{V}_d(x, x_d) &= -(x - x_d)^T R_m(x)(x - x_d) \\ &\leq -\frac{\alpha}{\beta}(x - x_d)^T (x - x_d) \\ &= -\frac{\alpha}{\beta} V(x, x_d) \leq 0 \end{aligned} \quad (2.23)$$

where

$$\alpha = \sup_{x \in \mathcal{X}} \lambda_{max}(R_m(x)) > 0$$
$$\beta = \inf_{x \in \mathcal{X}} \lambda_{min}(R_m(x)) > 0$$

By Lypunov stability arguments it follows that the vector $x(t)$ exponentially asymptotically converges towards the auxiliary vector trajectory $x_d(t)$.

Hence one must judiciously choose a predetermined trajectory for $x_d(t)$ which renders a desired equilibrium value for $x_d(t)$, say $x_d(t) \to x_e$. This may be done in several manners with the help of the set of auxiliary equations (2.22).

3 Application to the Regulation of a Hemostat System

Consider the following hemostat system, thoroughly discussed in Buivolova and Kolmanovskii in [9].

$$\begin{aligned} \dot{x}_1 &= u + P - \frac{x_1 x_2}{x_1 + Q} + a x_2 \\ \dot{x}_2 &= -b x_2 + \frac{x_1 x_2}{x_1 + Q} \\ y &= x_1 \end{aligned} \quad (3.1)$$

where x_1 and x_2 are the concentrations of food mass and microorganisms at the time t, respectively. The system parameters P. Q and a and b are assumed to be known constants. The equilibrium point, corresponding to a constant value $\overline{u}$ of the input variable u is given by

$$\overline{x}_1 = \frac{bQ}{1-b} \quad ; \quad \overline{x}_2 = \frac{\overline{u} + P}{b - a}$$

In terms of the vector fields description (2.1) for the above affine system we have

$$f(x) = \begin{bmatrix} P - \frac{x_1 x_2}{x_1 + Q} + a x_2 \\ -b x_2 + \frac{x_1 x_2}{x_1 + Q} \end{bmatrix} \quad ; \quad g(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Consider the following energy storage function

$$V = \frac{1}{2}\left(x_1^2 + x_2^2\right) \quad (3.2)$$

The time derivative of V along the controlled motions of the system is given by

$$\begin{aligned} \dot{V} &= x_1 \left(u + P - \frac{x_1 x_2}{x_1 + Q} + a x_2 \right) \\ &\quad + x_2 \left(-b x_2 + \frac{x_1 x_2}{x_1 + Q} \right) \\ &= -b x_2^2 - \frac{x_2 x_1^2}{x_1 + Q} \\ &\quad + x_1 \left(u + P + a x_2 + \frac{x_2^2}{x_1 + Q} \right) \\ &\leq x_1 \left(u + P + a x_2 + \frac{x_2^2}{x_1 + Q} \right) \end{aligned} \quad (3.3)$$

where the last inequality is obtained under the assumption that the variables x_1 and x_2 are strictly positive for all times, which is, evidently, a physically meaningful restriction. In other words, a decomposition of the vector field $f(x)$ is possible, including a locally dissipative component $f_d(x)$ in the region of the state space where x_1 and x_2 are both strictly positive (we take $\mathcal{X} = \{(x_1, x_2) \in R^2 \text{ s.t. } x_1, x_2 > 0\} \subset R^2$). The decomposition of f in $\mathcal{X}$ into dissipative and non-dissipative components is clearly given by

$$f_d(x) = \begin{bmatrix} -\frac{x_1 x_2}{x_1 + Q} \\ -b x_2 \end{bmatrix} \quad ; \quad f_{nd}(x) = \begin{bmatrix} P + a x_2 \\ \frac{x_1 x_2}{x_1 + Q} \end{bmatrix}$$

The system is, thus, passivifiable with respect to the storage function $V(x)$.

Define a state-dependent input coordinate transformation of the form:

$$v = u + \frac{L_{f nd} V}{L_g V} = u + P + a x_2 + \frac{x_2^2}{x_1 + Q} \quad (3.4)$$

Transformation (3.4) yields a passive system between the transformed input v and the output variable x_1. Indeed, integrating the inequality (3.3) on obtains the following passivity relationship

$$V(x(t)) - V(x(0)) \leq \int_0^t x_1(\sigma) v(\sigma) d\sigma \quad (3.5)$$

Under the above defined input coordinate transformation the system is readily rewritten as

$$\begin{aligned} \dot{x}_1 &= v - \frac{x_1 x_2}{x_1 + Q} - \frac{x_2^2}{x_1 + Q} \\ \dot{x}_2 &= -b x_2 + \frac{x_1 x_2}{x_1 + Q} \end{aligned} \quad (3.6)$$

In matrix notation, the system with transformed input has the more suggestive form:

$$\mathcal{D}\dot{x} + \mathcal{J}(x)\, x + \mathcal{R}(x)\, x = \mathcal{M} v \quad (3.7)$$

where, $x^T = [x_1 \quad x_2]$, and

$$\mathcal{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \;;\; \mathcal{J}(x) = \begin{bmatrix} 0 & \frac{y}{x_1+Q} \\ -\frac{x_2}{x_1+Q} & 0 \end{bmatrix}$$
$$\mathcal{R}(x) = \begin{bmatrix} \frac{y}{x_1+Q} & 0 \\ 0 & b \end{bmatrix} \;;\; \mathcal{M} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad (3.8)$$

where $\mathcal{J}^T(x) + \mathcal{J}(x) = 0$, and $\mathcal{R}(x) = \mathcal{R}^T(x) > 0$, as it may be easily verified.

3.1 Passivity based controller design

Consider the modified energy function V_d, defined with the aid of an auxiliary state vector $x_d = [x_{1d} \quad x_{2d}]^T$, representing the desired state vector, to be determined later.

Let V_d be given by

$$\begin{aligned} V_d(x, x_d) &= \frac{1}{2}(x - x_d)^T \mathcal{D}(x - x_d) \\ &= \frac{1}{2}\left[(x_1 - x_{1d})^2 + (x_2 - x_{2d})^2\right] \end{aligned} \qquad (3.9)$$

From the results of the previous section, the following set of auxiliary controlled differential equations yield $\dot{V}_d(x, x_d)$ negative definite.

$$\begin{aligned} \dot{x}_{1d} &= v - \frac{x_2}{x_1+Q}\, x_{1d} - \frac{x_2}{x_1+Q} x_{2d} + R_1(x_1 - x_{1d}) \\ \dot{x}_{2d} &= -b x_{2d} + \frac{x_2}{x_1+Q}\, x_{1d} + R_2(x_2 - x_{2d}) \end{aligned} \qquad (3.10)$$

where R_1 and R_2 are the diagonal components of the positive definite matrix $\mathcal{R}_{di}(x)$ which, for simplicity, it will be taken to be a constant matrix, $\mathcal{R}_{di}(x) = \mathcal{R}_{di} = \text{diag}[R_1 \quad R_2]$.

Letting $x_{1d} = \overline{x}_1 = constant$, one obtains the following dynamical controller expression, where x_{2d} has been substituted by the controller state variable ξ,

$$\begin{aligned} v &= \frac{x_2(\xi + \overline{x}_1)}{x_1 + Q} - R_1(x_1 - \overline{x}_1) \\ \dot{\xi} &= \frac{x_2}{x_1+Q}\, \overline{x}_1 - b\xi + R_2(x_2 - \xi) \end{aligned} \qquad (3.11)$$

In original control variables the controller takes the form

$$\begin{aligned} u &= -P - a x_2 + \frac{x_2(\xi + \overline{x}_1 - x_2)}{x_1 + Q} - R_1(x - \overline{x}) \\ \dot{\xi} &= \frac{x_2}{x_1+Q}\, \overline{x}_1 - b\xi + R_2\,(x_2 - \xi) \end{aligned} \qquad (3.12)$$

Figure 2 shows the closed loop behaviour of the hemostat system with nice stabilization features and a substantially reduced settling time when compared with the open loop output response, which is of about 50 time units.

4 Conclusions

In this article a simple but powerful geometric interpretation has been given to the possibilities of passivifying an arbitrary nonlinear system by means of partial state feedback or, more properly, by means of a state-dependent input coordinate transformation. Passivity based controllers have been mainly applied to the class of lagrangian systems, especially, to mechanical (such as robots) and electro-mechanical systems (induction motors, power converters, etc.). The results here proposed apply to any system provided with a positive energy storage (i.e., Lyapunov) function. To emphasize this point, the results were applied to a biological process control problem. in these class of nonlinear systems the concept of "energy" is not as clear cut as in the area of mechanical, electrical or electromechanical systems.

Extension of the above results to multi-input systems is straightforward with a similar geometric interpretation.

References

[1] J. C. Willems, *The Analysis of Feedback Systems*, Cambridge, MA. MIT Press, 1971.

[2] D. Hill and P. Moylan, "The stability of nonlinear dissipative systems", *IEEE Transactions on Automatic Control*, Vol. 21, pp. 708-711, 1976.

[3] D. Hill and P. Moylan, "Stability results for nonlinear feedback systems", *Automatica*, Vol. 13, pp. 377-382, 1977.

[4] C.I. Byrnes, A. Isidori and J. Willems, "Passiviy, Feedback Equivalence and the Global Stabilization of Minimum Phase Nonlinear Systems", *IEEE Transactions on Automatic Control*, Vol. AC-36. No. 11, pp. 1228-1240. 1991.

[5] W. Lin, "Global asymptotic stabilization of general nonlinear systems with stable free dynamics via passivity and bounded feedback", *Automatica*, Vol. 32, No. 6, pp. 915-924, 1996.

[6] P. V. Kokotovic and H. Sussmann, "A positive real condition for the stabilization on nonlinear systems" *Systems and Control Letters*, Vol. 13, pp. 1235-134, 1989.

[7] H. Sira-Ramírez, R. Ortega, M. García-Esteban and R. Pérez-Moreno, "Passivity-Based Controllers for the Stabilization of DC-to-DC Power Converters", *Automatica*, (accepted for publication, to appear).

[8] A. van der Schaft, *L_2-Gain and Passivity Techniques in Nonlinear Control*, Lecture Notes in Control and Information Sciences, Vol. 218, Springer-Verlag: London, 1996.

[9] A. G. Buivolova and V. B. Kolmanovskii, "Time Optimal Control of a Hemostat", *Automation and Remote Control* Vol. 55, No. 11, pp. 1585-1591, 1994.

Acknowledgements This research was supported by the Consejo de Desarrollo Científico, Humanístico and Tecnológico of the Universidad de Los Andes under Research Grant I-456-94, and by the Consejo Nacional de Investigaciones Científicas, CONICIT, under Research Grant S1-95-000886.

Figures

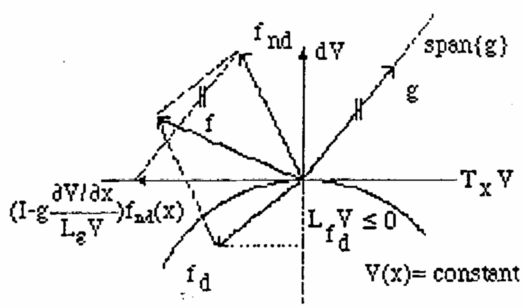

Figure 1: A geometric interpretation of passivization

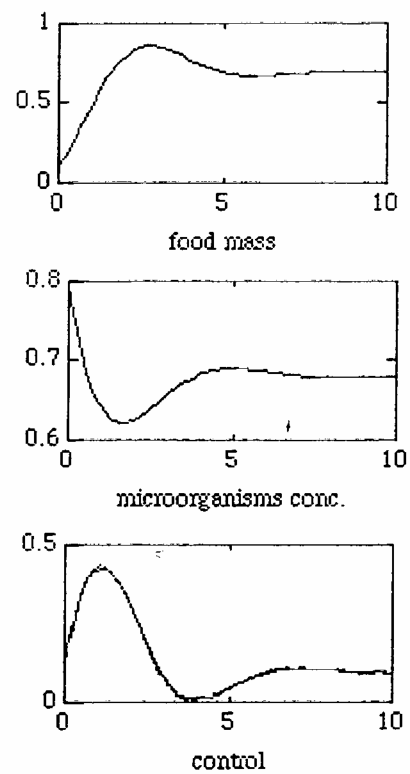

Figure 2: Simulations results of the passivity-based regulated hemostat system.