13th Triennial World Congress, San Francisco, USA

DYNAMICAL ADAPTIVE BACKSTEPPING CONTROL/OBSERVER DESIGN FOR UNCERTAIN NONLINEAR SYSTEMS

M. Rios-Bolívar *,[1] **A. S. I. Zinober** * **H. Sira-Ramírez** **

* *Applied Mathematics Section*
School of Mathematics and Statistics
The University of Sheffield, Sheffield S10 2TN, U.K.
e-mail: A.Zinober@sheffield.ac.uk
** *Departamento Sistemas de Control*
Universidad de Los Andes, Mérida 5101, Venezuela

Abstract. In this paper the design of observers for a class of nonlinear uncertain systems, transformable into the Generalized Observer Canonical Form, is addressed. A backstepping-like algorithm is used to design a dynamical adaptive control to ensure the output tracks a prescribed bounded reference signal, based on the estimation of the unmeasured states. The destabilizing effects of the observation errors are compensated via nonlinear damping terms.

Keywords. Adaptive control, Nonlinear systems, Observers, Tracking, Uncertainty

1. INTRODUCTION

In the last few years nonlinear adaptive control of plants having unknown parameters and known nonlinearities has attracted the attention of many researchers and practitioners. Important contributions to this area have been given in various control design algorithms based on the *backstepping* approach (see, for example, Kanellakopoulos *et al.*, 1991a,b,c; Kokotović 1991, Krstić *et al.* 1992), which were recently compiled in Krstić *et al.*, (1995). The adaptive backstepping algorithms were originally developed for a broad class of linearizable uncertain nonlinear systems and transformable into either *pure* or *strict* feedback forms (see Kokotović, 1991), guaranteeing global tracking and stabilization for plants in this latter class of systems. Then, Seto *et al.* (1994) extended the class of systems stabilizable via backstepping to nonlinear systems with a triangular structure.

More recently, a recursive procedure has been proposed by Sira-Ramírez *et al.* (1995), which implements the fundamental ideas related to the backstepping design with tuning functions, developed by Krstić *et al.* (1992), in combination with dynamical input-output linearization. The general algorithm was reported by Rios-Bolívar *et al* (1995b), and is applicable to a class of observable minimum-phase uncertain nonlinear systems. Two major advantages characterize this approach. Firstly, transformation of the controlled plant into triangular, pure or strict forms are not required. Secondly, dynamical adaptive controllers are obtained from this procedure because the control input and its derivatives are allowed to appear at intermediate steps of the algorithm. This aspect is particularly important when the approach is combined with discontinuous control schemes, such as Pulse Width Modulation (PWM) or Sliding Mode Control (SMC), because the resulting controllers achieve

[1] On leave from Departamento Sistemas de Control, ULA, Mérida 5101, Venezuela.

robust asymptotic stability with considerably reduced chattering (see Sira-Ramírez *et al.* 1995; Rios-Bolívar *et al.* 1995a).

In previous work considering dynamical adaptive backstepping control, full-state measurement has been assumed (Rios-Bolívar *et al.* 1995b). Here only the output is measured and an observer is designed to generate the unmeasured states. Then, dynamical adaptive backstepping is used to drive the output to track a desired bounded reference signal.

2. PROBLEM STATEMENT

The proposed algorithm in Rios-Bolívar *et al.* (1995b) involves nonlinear transformations of the controlled plant into tracking error coordinates depending on the control input and its derivatives. It is applicable to the class of observable minimum-phase nonlinear systems, dynamically input-output linearizable and with constant but unknown parameters, which can be represented by

$$\dot{x} = f_0(x) + g_0(x)u + \sum_{i=1}^{p} (\theta_i \gamma_i(x) + \theta_i \psi_i(x)u) \quad (1)$$
$$y = h(x)$$

The same problem has been solved by Krstić *et al.* (1995), using static controllers when system (1) is placed into either pure or strict feedback forms. Here the practical restriction of full-state measurement is relaxed, by assuming that only the output is measured, while the class of uncertain nonlinear systems corresponds to systems with only output-dependent nonlinearities multiplying the uncertain parameters. Firstly, an adaptive observer is designed for the system placed into the *Generalized Observer Canonical Form* (GOCF) (Keller and Fritz 1986), and then a modified version of the algorithm in Rios-Bolívar *et al.* (1995b), incorporating nonlinear damping terms, is used to design dynamical adaptive output tracking control.

3. OBSERVER DESIGN

An appealing problem in control theory is the design of state observers for nonlinear systems (see, for example, Krener and Respondek 1985). This problem becomes more difficult in the presence of uncertain parameters. Various contributions have been reported for uncertain systems. For instance Kanellakopoulos *et al.* (1991c) and Marino and Tomei (1991) have proposed the use of stable filters, or a combination of stable filters and an observer derived from a filtered transformation, when system (1) is transformable into the *output feedback form.* Marino and Tomei (1991) have developed state observers and output-feedback control for systems transformable into the *adaptive observer form.*

The problem of designing observers for nonlinear systems, involving the control input and its derivatives, has been studied by Keller and Fritz (1986), and a solution has been proposed, provided that the system can be placed into the GOCF. However, the results were obtained for nonlinear systems with no uncertainties. In the next two subsections the method proposed by Keller and Fritz is presented and, it is shown that under certain conditions an adaptive observer can be synthesized from the GOCF for uncertain systems. The method considered here avoids the use of filters.

3.1 *Observer design for nonlinear systems with derivatives of the input*

Consider the nonlinear system with no uncertainties

$$\begin{aligned} \dot{x} &= f_0(x) + g_0(x)u \\ y &= h(x) \end{aligned} \quad (2)$$

where $x \in \Re^n$ is the state, $u \in \Re$ the control input, $y \in \Re$ the output. It is assumed that (2) is a minimum-phase system with a well-defined relative degree ρ, i.e. $1 \leq \rho \leq n$. We also assume that (2) satisfies the observability condition

$$\operatorname{rank} \frac{\partial [y, \dot{y}, \ldots, y^{(n-1)}]}{\partial x} = n \quad (3)$$

Then (2) can be transformed into the *Generalized Observability Canonical Form* (GOBCF)

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= f(z, u, \dot{u}, \ldots, u^{(n-\rho)}) \\ y &= z_1 \end{aligned} \quad (4)$$

This GOBCF was initially obtained by Zeitz (1984) for time-variant nonlinear systems, and then by Fliess (1990) from a differential algebraic viewpoint. A solution to the problem of designing state observers for systems transformable into GOBCF, when the full state is not measured, has been given by Keller and Fritz (1986) using the following *Generalized Observer Canonical Form* (GOCF)

$$\dot{\xi}_1 = -\alpha_0(y, u, \dot{u}, \dots, u^{(n-\rho)}) \quad (5)$$
$$\dot{\xi}_i = \xi_{i-1} - \alpha_{i-1}(y, u, \dot{u}, \dots, u^{(n-\rho-i+1)}) \qquad i = 2, \dots, n$$
$$y_\xi = \xi_n = c(y)$$

where y is the output of (2). The GOCF can be obtained from (4) if the scalar function $f(z, u, \dot{u}, \dots, u^{(n-\rho)})$ fulfills a special structural condition derived from the following *Generalized Characteristic Equation* (GCE)

$$d^n c(y) + \sum_{i=1}^{n} d^{n-1}\alpha_{n-i}(y, u, \dots, u^{(k-\rho)}) = 0 \quad (6)$$

with the differential operator d defined as

$$d^k\nu(x,u) = \left[\frac{\partial d^{k-1}}{\partial x}\nu(x,u)\right]^T (f_0(x) + g_0(x)u) + \left[\frac{\partial d^{k-1}}{\partial u}\nu(x,u)\right]^T \dot{u} \quad (7)$$

For a second order system with relative degree $\rho = 1$, the GCE is reduced to

$$d^2c(y) + d\alpha_1(y,u) + \alpha_0(y,u,\dot{u}) = 0 \quad (8)$$

and applying the differential operator d to the system transformed in the GOBCF (4)

$$\frac{d^2c(y)}{dy^2}z_2^2 + \frac{dc(y)}{dy}f(z,u,\dot{u}) + \frac{\partial\alpha_1(y,u)}{\partial y}z_2 + \frac{\partial\alpha_1(y,u)}{\partial u}\dot{u} + \alpha_0(y,u,\dot{u}) = 0 \quad (9)$$

The structural condition

$$f(z,u,\dot{u}) = k_2(z_1)z_2^2 + k_1(z_1,u)z_2 + k_0(z_1,u,\dot{u}) \quad (10)$$

results. If this condition is fulfilled, the functions k_0, k_1 and k_2 are known and the three unknown functions $c(y), \alpha_1$ and α_0 can be determined from the three partial differential equations

$$-k_2(z_1)\frac{dc(y)}{dy} = \frac{d^2c(y)}{dy^2}$$
$$-k_1(z_1,u)\frac{dc(y)}{dy} = \frac{\partial\alpha_1(y,u)}{\partial y} \quad (11)$$
$$-k_0(z_1,u,\dot{u})\frac{dc(y)}{dy} = \frac{\partial\alpha_1(y,u)}{\partial u}\dot{u} + \alpha_0(y,u,\dot{u})$$

By rewriting the GOCF (5) as follows

$$\dot{\xi} = A\xi + \alpha(y, u, \dots, u^{(n-\rho)})$$
$$y_\xi = c^T\xi \quad (12)$$

with

$$A = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \quad ; \quad \alpha(\cdot) = \begin{bmatrix} -\alpha_0(\cdot) \\ \vdots \\ -\alpha_{n-1}(\cdot) \end{bmatrix} \quad (13)$$
$$c^T = [0, \dots, 0, 1]$$

an observer can be readily obtained as

$$\dot{\hat{\xi}} = A\hat{\xi} + \alpha(y, u, \dots, u^{(n-\rho)}) + K(\xi_n - \hat{\xi}_n)$$
$$y = c^T\hat{\xi} \quad (14)$$

with $K = [k_1, \dots, k_n]^T$ a vector of positive gains. Thus, the observer error $e = \xi - \hat{\xi}$ exhibits the exponentially stable dynamics

$$\dot{e} = A_0 e \quad (15)$$

with

$$A_0 = (A - Kc^T) = \begin{bmatrix} 0 & 0 & \dots & 0 & -k_1 \\ 1 & 0 & \dots & 0 & -k_2 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -k_n \end{bmatrix} \quad (16)$$

3.2 *An example*

Consider the second order nonlinear system

$$\dot{x}_1 = -x_1 + x_2^2 + u$$
$$\dot{x}_2 = x_1x_2 + u \quad (17)$$
$$y = x_2$$

The observability condition (10) is satisfied if $x_2 \neq 0$. Therefore only equilibrium points different from the origin can be considered for this example. The control-dependent coordinate trasnformation

$$z_1 = y = x_2$$
$$z_2 = \dot{y} = x_1x_2 + u \quad (18)$$

places (17) in the GOBCF

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = \left(\frac{1}{z_1}\right)z_2^2 - \left(1 + \frac{u}{z_1}\right)z_2 + (1+z_1)u + z_1^3 + \dot{u} = f(z,u,\dot{u}) \quad (19)$$
$$y = z_1$$

Note $f(z,u,\dot{u})$ satisfies the structural condition (10) and the functions k_0, k_1 and k_2 are identified as

$$k_2(z_1) = \frac{1}{z_1} \quad ; \quad k_1(z_1,u) = -\left(1+\frac{u}{z_1}\right) \qquad (20)$$
$$k_0(z_1,u,\dot{u}) = (1+z_1)u + z_1^3 + \dot{u}$$

Then, solving the partial differential equations (11)

$$c(y) = c(z_1) = \ln z_1 \qquad (21)$$
$$\alpha_1(\cdot) = \ln z_1 - \frac{u}{z_1} \quad ; \quad \alpha_0(\cdot) = -\frac{u}{z_1} - u - z_1^2$$

Therefore, the coordinate transformation

$$\xi = \Phi(x) = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} \ln(x_2) + x_1 \\ \ln(x_2) \end{bmatrix} \qquad (22)$$

places the system (17) into the GOCF

$$\begin{aligned} \dot{\xi}_1 &= \frac{u}{y} + y^2 + u \\ \dot{\xi}_2 &= \xi_1 - \ln y + \frac{u}{y} \\ y_\xi &= \xi_2 \end{aligned} \qquad (23)$$

and, finally, the observer is

$$\begin{aligned} \dot{\hat{\xi}}_1 &= \frac{u}{y} + y^2 + u + k_1(\xi_2 - \hat{\xi}_2) \\ \dot{\hat{\xi}}_2 &= \hat{\xi}_1 - \ln y + \frac{u}{y} + k_2(\xi_2 - \hat{\xi}_2) \\ y_\xi &= \hat{\xi}_2 \end{aligned} \qquad (24)$$

This method is proper when the system order is low. However, with increasing order the structural conditions become stronger (Keller and Fritz 1986).

3.3 *Observer design for a class of uncertain nonlinear systems*

Consider now the case with parametric uncertainties, under the condition that the nonlinearities multiplying the uncertain parameter depend only on the output variable

$$\begin{aligned} \dot{x} &= f_0(x) + g_0(x)u + \sum_{i=1}^{p} \theta_i \gamma_i(y) \\ y &= h(x) \end{aligned} \qquad (25)$$

If the output dependence of these nonlinearities is invariant to the nonlinear transformation $\xi = \Phi(x)$, placing (25) into the GOCF, the system (25) is transformable into the system

$$\begin{aligned} \dot{\xi} &= A\xi + \alpha(y,u,\ldots,u^{(n-\rho)}) + \sum_{i=1}^{p} \theta_i \psi_i(y) \\ y &= c^T \xi \end{aligned} \qquad (26)$$

with A, c and α defined as in (13) and $\psi_i : \Re \rightarrow \Re^n$ smooth functions for $i = 1, \ldots, p$. Therefore, the following system

$$\begin{aligned} \dot{\hat{\xi}} &= A\hat{\xi} + \alpha(y,u,\ldots,u^{(n-\rho)}) + K(\xi_n - \hat{\xi}_n) + \Psi(y)^T \hat{\theta} \\ y &= c^T \hat{\xi} \end{aligned} \qquad (27)$$

with

$$\Psi(y)^T = \begin{bmatrix} \psi_{1,1}(y) & \cdots & \psi_{p,1}(y) \\ \vdots & & \vdots \\ \psi_{1,n}(y) & \cdots & \psi_{p,n}(y) \end{bmatrix} \qquad (28)$$

is an observer for (26) and the error system yields

$$\dot{e} = A_0 e + \Psi(y)^T \tilde{\theta} \qquad (29)$$

where $A_0 = A - Kc^T$, $e = \xi - \hat{\xi}$ and $\tilde{\theta} = \theta - \hat{\theta}$. This error system possesses a strict passivity property from the input $\tilde{\theta}$ to the output $\Psi(y)e$, i.e. the nonlinear operator $\tilde{\theta} \rightarrow \Psi(y)e$ is strictly passive (see Krstić *et al.* 1995 for details). Therefore, the following parameter estimate

$$\dot{\hat{\theta}} = \tau_0 = \Gamma \psi_n e_n \qquad (30)$$

will be used as the first tuning function.

4. DYNAMICAL BACKSTEPPING CONTROL DESIGN

In this section a modified version of the algorithm proposed by Rios-Bolívar *et al.* (1995b) is used to design a dynamical adaptive tracking control. The differences are due to the incorporation of an adaptive observer to estimate the unmeasured states, and damping nonlinear terms to compensate the destabilizing effects of the observation errors (similarly to those used by Krstić *et al.* 1995). Here, for reasons of limited space, we describe the method only for a second order system with relative degree one and in the GOCF

$$\begin{aligned} \dot{\xi}_1 &= -\alpha_0(\xi_2,u,\dot{u}) + \varphi_1(\xi_2)\theta \\ \dot{\xi}_2 &= \xi_1 - \alpha_1(\xi_2,u) + \varphi_2(\xi_2)\theta \\ y_\xi &= \xi_2 \end{aligned} \qquad (31)$$

By using the observer (27) and considering the parameter estimate

$$\dot{\hat{\theta}} = \tau_0 = \Gamma \varphi_2 e_2 \qquad (32)$$

as our first tuning function, the derivative of the tracking error $z_1 = \xi_2 - \xi_r$ is given by

$$\dot{z}_1 = \hat{\xi}_1 - \alpha_1(\xi_2, u) + \varphi_2(\xi_2)\hat{\theta} - \dot{\xi}_r + \varphi(\xi_2)\tilde{\theta} + e_1 \quad (33)$$

Note that ξ_r is the transformation of the bounded desired reference signal y_r into the ξ coordinates. Using the Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}\Gamma^{-1}\tilde{\theta} \quad (34)$$

with $\Gamma = \Gamma^T > 0$, the time derivative

$$\dot{V}_1 = z_1\left[\hat{\xi}_1 - \alpha_1(\xi_2, u) + \varphi_2(\xi_2)\hat{\theta} - \dot{\xi}_r\right] + z_1 e_1 + \tilde{\theta}\Gamma^{-1}\left(-\dot{\hat{\theta}} + \tau_0 + \Gamma z_1\varphi_2\right) \quad (35)$$

is obtained. Taking the second tuning function as follows

$$\dot{\hat{\theta}} = \tau_1 = \tau_0 + \Gamma z_1\varphi_2 = \Gamma\varphi_2(e_2 + z_1) \quad (36)$$

and defining our second error coordinate as

$$z_2 = \hat{\xi}_1 - \alpha_1(\xi_2, u) + \varphi(\xi_2)\hat{\theta} - \dot{\xi}_r + (c_1 + d_1)z_1 \quad (37)$$

with c_1 and d_1 positive design parameters, $\dot{V}_1$ becomes

$$\dot{V}_1 = -(c_1 + d_1)z_1^2 + z_1 z_2 + z_1 e_1 + \tilde{\theta}\Gamma^{-1}(-\dot{\hat{\theta}} + \tau_1) \quad (38)$$

The time derivative of z_2 is

$$\dot{z}_2 = -\alpha_0 + \varphi_1\hat{\theta} + \omega(z_2 - (c_1 + d_1)z_1 + \dot{\xi}_r + \varphi_2\tilde{\theta}) - \frac{\partial\alpha_1}{\partial u}\dot{u} + \varphi_2\dot{\hat{\theta}} - \ddot{\xi}_r - (c_1 + d_1)\dot{\xi}_r + \omega e_1 \quad (39)$$

with

$$\omega = c_1 + d_1 - \frac{\partial\alpha_1}{\partial\xi_2} + \theta\frac{\partial\varphi_2}{\partial\xi_2} \quad (40)$$

Augmenting the Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_2^2 \quad (41)$$

the time derivative is

$$\dot{V}_2 = -c_1 z_1^2 - d_1 z_1^2 + z_1 e_1 + \tilde{\theta}\Gamma^{-1}(-\dot{\hat{\theta}} + \tau_1 + \Gamma z_2\varphi_2\omega) + z_2\Big[z_1 - \alpha_0 + \varphi_1\hat{\theta} + \omega(z_2 - (c_1 + d_1)z_1 + \dot{\xi}_r) - \frac{\partial\alpha_1}{\partial u}\dot{u} + \varphi_2\dot{\hat{\theta}} - \ddot{\xi}_r - (c_1 + d_1)\dot{\xi}_r\Big] + z_2\omega e_1 \quad (42)$$

Finally, the actual update law for the unknown parameters yields

$$\dot{\hat{\theta}} = \tau_2 = \tau_1 + \Gamma z_2\varphi_2\omega = \Gamma\varphi_2(e_2 + z_1 + z_2\omega) \quad (43)$$

and the dynamical adaptive control is

$$-d_2\omega^2 z_2 - c_2 z_2 = -\alpha_0 + \varphi_1\hat{\theta} + \omega(z_2 - (c_1 + d_1)z_1 + \dot{\xi}_r) z_1 - \frac{\partial\alpha_1}{\partial u}\dot{u} + \varphi_2\tau_2 - \ddot{\xi}_r - (c_1 + d_1)\dot{\xi}_r \quad (44)$$

where $-d_2\omega^2 z_2$ is the second nonlinear damping term to compensate the destabilizing effect of the observation error e_1.

5. EXAMPLE

The following example illustrates the use of this algorithm. Consider

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2^2 + u + \theta x_2^4 \\ \dot{x}_2 &= x_1 x_2 + u + \theta x_2^2 \\ y &= x_2 \end{aligned} \quad (45)$$

Note that the nominal system ($\theta = 0$) coincides with the system (17). Therefore, applying the transformation (22) on (45) yields

$$\begin{aligned} \dot{\xi}_1 &= \frac{u}{y} + y^2 + u + \theta(y + y^4) \\ \dot{\xi}_2 &= \xi_1 - \ln y + \frac{u}{y} + \theta y \\ y_\xi &= \xi_2 \end{aligned} \quad (46)$$

or, fully transformed into the ξ coordinates

$$\begin{aligned} \dot{\xi}_1 &= u\exp(-\xi_2) + \exp(2\xi_2) + u + \theta(\exp(\xi_2) + \exp(4\xi_2)) \\ \dot{\xi}_2 &= \xi_1 - \xi_2 + u\exp(-\xi_2) + \theta\exp(\xi_2) \\ y_\xi &= \xi_2 \end{aligned} \quad (47)$$

which is obviously in the GOCF (31) with

$$\begin{aligned} \alpha_0(\xi_2, u) &= -u\exp(-\xi_2) - \exp(2\xi_2) - u \\ \alpha_1(\xi_2, u) &= \xi_2 - u\exp(-\xi_2) \\ \varphi_1(\xi_2) &= \exp(\xi_2) + \exp(4\xi_2) \\ \varphi_2(\xi_2) &= \exp(\xi_2) \end{aligned} \quad (48)$$

Computer simulations were obtained to illustrate the tracking performance of the designed dynamical adaptive backstepping control/observer. Figure 1 shows the tracking performance for a smooth transition of x_2 between two equilibrium points.

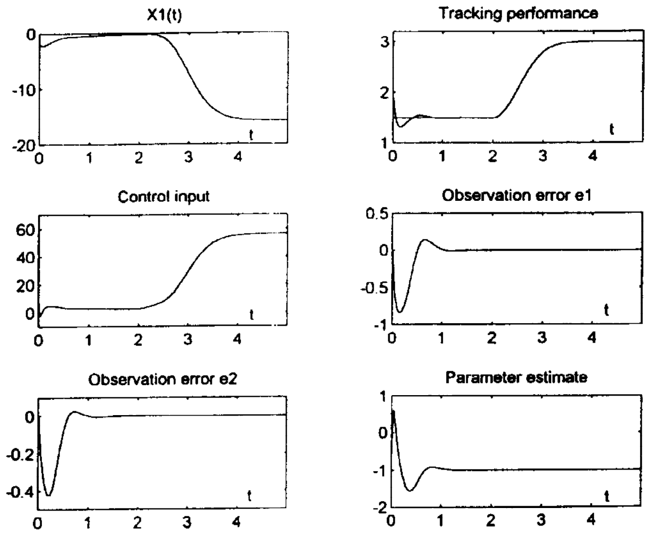

Fig. 1. Response of x_1, tracking performance, control input, observation errors and parameter estimate

6. CONCLUSIONS

An alternative method for the design of observers for a class of nonlinear uncertain systems, transformable into the GOCF, was studied. A backstepping-like algorithm yields a dynamical adaptive control driving the output to track a prescribed bounded reference signal. The control design algorithm is based on the estimation of the unmeasured states and incorporates nonlinear damping terms to compensate the destabilizing effects of the observation errors. Computer simulations were carried out for a second order nonlinear system.

7. REFERENCES

Fliess, M. (1990). Generalized Controller Canonical Forms for Linear and Nonlinear Dynamics. *IEEE Trans. on Automatic Control*, **AC-35**, 994–1001.

Kanellakopoulos, I., P. V. Kokotović and A. S. Morse (1991a). Systematic Design of Adaptive Controllers for Feedback Linearizable Systems. *IEEE Transactions on Automatic Control*, **AC-36**, 1241–1253.

Kanellakopoulos, I, P. V. Kokotović and A. S. Morse (1991b). A toolkit for nonlinear feedback designs", *Systems and Control Letters*, **18**, 83–92.

Kanellakopoulos, I., P. V. Kokotović and A. S. Morse (1991c). Adaptive Output-Feedback Control of a Class of Nonlinear Systems. *Proc. 30th IEEE CDC*, Brighton (UK), **2**, 1082–1087.

Keller H. and H. Fritz (1986). Design of Nonlinear Observers by a Two-Step-Transformation. In: *Algebraic and Geometric Methods in Nonlinear Control Theory*, (M. Fliess and M. Hazewinkel (Ed.)), 89–98, D. Reidel Publishing Company.

Kokotović, P. V. (1991). The joy of feedback: nonlinear and adaptive. 1991 Bode Prize Lecture, *Control Systems Magazine*, **12**, 7–17.

Krener, A. J. and W. Respondek (1985). Nonlinear Observers with Linearizable Error Dynamics. *SIAM J. Control and Optimization*, **23**, 197–216.

Krstić, M., I. Kanellakopoulos and P.V. Kokotović (1992). Adaptive Nonlinear Control without Overparametrization. *Systems and Control Letters*, **19**, 177–185.

Krstić, M., I. Kanellakopoulos and P.V. Kokotović (1995). *Nonlinear and Adaptive Control.* John Wiley & Sons, New York.

Marino R. and P. Tomei (1991). Global Adaptive Observers and Output-Feedback Stabilization for a Class of Nonlinear Systems. In: *Foundations of Adaptive Control*, (P.V. Kokotović (Ed.)), 455–493, Springer-Verlag, Berlin.

Rios-Bolívar, M., A. S. I. Zinober, and H. Sira-Ramírez (1995a). Sliding Mode Output Tracking via Backstepping for Uncertain Nonlinear Systems. *Proc. European Control Conference*, **1**, 699–704.

Rios-Bolívar, M., H. Sira-Ramírez and A. S. I. Zinober (1995b). Output Tracking Control via Adaptive Input-Output Linearization: A Backstepping Approach. *Proc. 34th IEEE CDC*, **2**, 1579–1584.

Seto, D., A. M. Annaswamy and J. Baillieul (1994). Adaptive Control of Nonlinear Systems with a Triangular Structure. *IEEE Trans. on Automatic Control*, **AC-39**, 1411–1427.

Sira-Ramírez, H., M. Ríos-Bolívar and A. S. I. Zinober (1995). Adaptive Input-Output Linearization for PWM Regulation of DC-to-DC Power Converters. *Proc. American Control Conference*, **1**, 81–85.

Zeitz, M. (1984). Observability Canonical (phase-variable) Form for Non-linear Time-variable Systems. *Int. J. Systems Sci.*, **15**, 949–958.