

## PASSIVITY-BASED REGULATION OF A CLASS OF MULTIVARIABLE DC-TO-DC POWER CONVERTERS<sup>1</sup>

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**Abstract.** A passivity-based adaptive Pulse-Width-Modulation (PWM) feedback regulation scheme is proposed for the stabilization of a new class of multivariable dc-to-dc power converters, constituted by a cascaded arrangement of “boost” converters. These are assumed to be loaded by an unknown, but constant, resistive element. The performance of the proposed dynamical regulators is tested through computer simulations which include stochastic perturbation inputs.

**Keywords.** DC-to-DC Power Converters, Adaptive Control, Passivity.

### 1. INTRODUCTION

Feedback regulation of dc-to-dc power supplies has been extensively treated in the literature. Conference proceedings, such as the IEEE Power Electronics Specialist Conference (PESC) Records, a growing list of textbooks and an edited collection of research articles, Bose (1992), reflect both the theoretical and practical importance of this field.

A Lyapunov-based adaptive control of approximately linearized dc-to-dc power supplies has been treated by Sanders and Verghese (1992). A full adaptive nonlinear state feedback linearization viewpoint was proposed by Sira-Ramírez *et al* (1993). A feedback design technique that suitably combines input-output linearization

and the adaptive backstepping design procedure, was recently presented by Sira-Ramírez *et al* (1995).

In this work, a passivity-based controller design methodology for the adaptive regulation of a new class of switch-mode PWM regulated dc-to-dc power converters is proposed. We specifically treat a multivariable version of the “boost” (or, “step up”) converter, consisting of several cascaded “boost” converters. In fact, all results pertaining the adaptive feedback control of the single, traditional, “boost” converter can be immediately derived, from our results, as a particular case.

Section 2 presents some generalities about the average PWM models of multivariable “boost” converters. In Section 3 a passivity-based adaptive feedback regulation scheme is derived. Section 4 contains simulation results while Section 5 contains the conclusions and suggestions for further research in this area.

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## 2. A MULTIVARIABLE "BOOST" CONVERTER CIRCUIT

Consider a cascade connection of  $n$  "boost" dc-to-dc power converters, as shown in Figure 1. The multivariable switch-regulated model of such a composite converter is given by.

$$\begin{aligned} \dot{x}_1 &= -(1-u_1)\frac{1}{L_1}x_2 + \frac{E}{L_1} \\ \dot{x}_2 &= (1-u_1)\frac{1}{C_1}x_1 - \frac{x_3}{C_1} \\ \dot{x}_3 &= -(1-u_2)\frac{1}{L_2}x_4 + \frac{x_2}{L_2} \\ \dot{x}_4 &= (1-u_2)\frac{1}{C_2}x_3 - \frac{x_5}{C_2} \\ &\dots \\ \dot{x}_{2n-1} &= -(1-u_{n-1})\frac{1}{L_n}x_{2n} + \frac{x_{2n-2}}{L_n} \\ \dot{x}_{2n} &= (1-u_n)\frac{1}{C_n}x_{2n-1} - \frac{x_{2n}}{R_L C_n} \end{aligned} \quad (1)$$

where  $x_i$ ;  $i = 1, 2, \dots, 2n-1$  represent the inductor currents,  $x_j$ ;  $j = 2, 4, \dots, 2n$ , are the capacitor voltages. The variables  $u_i$ ;  $i = 1, \dots, n$  denote the switch position functions.

For simplicity, we assume a multivariable PWM feedback regulation policy for the switch position functions  $u_i$ ;  $i = 1, \dots, n$  characterized by *synchronous samplings*,

$$\begin{aligned} u_i &= \begin{cases} 1 & \text{for } t_k \leq t < t_k + \mu_i(t_k)T \\ 0 & \text{for } t_k + \mu_i(t_k)T \leq t < t_k + T \end{cases} \\ i &= 1, 2, \dots, n. \\ t_{k+1} &= t_k + T \quad ; \quad k = 0, 1, \dots \end{aligned} \quad (2)$$

where  $\mu_i$ ;  $i = 1, 2, \dots, n$  are the duty ratios.  $T$  is the common sampling interval and  $t_k$  represents the synchronized sampling instants for all the converters. The duty ratio functions  $\mu_i(\cdot)$  take values in the closed interval  $[0, 1]$  of the real line.

The average PWM model of the cascaded set of "boost" converters, shown in Figure 1, is given by,

$$\begin{aligned} \dot{z}_1 &= -(1-\mu_1)\frac{1}{L_1}z_2 + \frac{E}{L_1} \\ \dot{z}_2 &= (1-\mu_1)\frac{1}{C_1}z_1 - \frac{z_3}{C_1} \\ \dot{z}_3 &= -(1-\mu_2)\frac{1}{L_2}z_4 + \frac{z_2}{L_2} \\ \dot{z}_4 &= (1-\mu_2)\frac{1}{C_2}z_3 - \frac{z_5}{C_2} \\ &\dots \end{aligned}$$

$$\begin{aligned} \dot{z}_{2n-1} &= -(1-\mu_n)\frac{1}{L_n}z_{2n} + \frac{z_{2n-2}}{L_n} \\ \dot{z}_{2n} &= (1-\mu_n)\frac{1}{C_n}z_{2n-1} - \frac{z_{2n}}{R_L C_n} \end{aligned} \quad (3)$$

Let  $v$  denote the vector

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \dots \\ v_n \end{bmatrix} = \begin{bmatrix} 1-\mu_1 \\ 1-\mu_2 \\ \vdots \\ 1-\mu_n \end{bmatrix} \quad (4)$$

We rewrite the average system (3) in matrix form as follows,

$$\mathcal{D}_{MB}\dot{z} + (\mathcal{J}_{MB}(v) + \mathcal{R}_{MB})z = \mathcal{E}_{MB} \quad (5)$$

where  $z^T = [z_1, z_2, \dots, z_{2n}] \in \mathbb{R}^{2n}$  with  $z_i$ ;  $i = 1, 3, 5, \dots, 2n-1$  representing the average inductor currents. The variables  $z_j$ ;  $j = 2, 4, \dots, 2n$ , are the average capacitor voltages, while  $\mu_i \in [0, 1]$   $i = 1, \dots, n$ , are the duty ratio functions associated with the PWM operation of the regulating switches,  $u_i$ ;  $i = 1, 2, \dots, n$ .

The matrices in equation (5) are given by

$$\begin{aligned} \mathcal{D}_{MB} &= \text{diag} \{L_1, C_1, L_2, C_2, \dots, L_n, C_n\} \quad ; \\ \mathcal{R}_{MB} &= \text{diag} \{0, 0, \dots, 0, 1/R_L\} \\ \mathcal{E}_{MB}^T &= [E, 0, \dots, 0] \\ \mathcal{J}_{MB}(v) &= \begin{bmatrix} 0 & v_1 & 0 & 0 & \dots & 0 & 0 \\ -v_1 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 0 & v_2 & \dots & 0 & 0 \\ 0 & 0 & -v_2 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & v_n \\ 0 & 0 & 0 & 0 & \dots & -v_n & 0 \end{bmatrix} \end{aligned} \quad (6)$$

Note that for all  $v_i$ ;  $i = 1, 2, \dots, n$ , the matrix  $\mathcal{J}_{MB}(v)$  satisfies:  $\mathcal{J}_{MB}(v) + \mathcal{J}_{MB}^T(v) = 0$ .

The state equilibrium point of the average model (3), for constant duty ratios  $\mu_i = U_i \in (0, 1)$ ;  $i = 1, 2, \dots, n$ , is readily obtained as,

$$\begin{aligned} \bar{z}_1 &= \frac{\bar{z}_3}{1-U_1} \quad ; \quad \bar{z}_3 = \frac{\bar{z}_5}{1-U_2} \quad \dots \quad \bar{z}_{2n-1} = \frac{\bar{z}_{2n}}{R_L(1-U_n)} \\ \bar{z}_2 &= \frac{E}{1-U_1} \quad ; \quad \bar{z}_4 = \frac{\bar{z}_2}{1-U_2} \quad \dots \quad \bar{z}_{2n} = \frac{\bar{z}_{2n-2}}{1-U_n} \end{aligned} \quad (7)$$

It is clear that the average output voltage  $\bar{z}_{2n}$  significantly magnifies the value of the external input source  $E$ .

The steady state values of the average inductor currents and the average capacitor voltage are then related by,

$$\begin{aligned} \bar{z}_1 &= \frac{\bar{z}_{2n}^2}{ER_L} ; \quad \bar{z}_3 = \frac{\bar{z}_{2n}^2}{\bar{z}_2 R_L} ; \quad \bar{z}_5 = \frac{\bar{z}_{2n}^2}{\bar{z}_4 R_L} ; \quad \dots ; \\ \bar{z}_{2n-1} &= \frac{\bar{z}_{2n}^2}{\bar{z}_{2n-2} R_L} ; \end{aligned} \quad (8)$$

Given a desired value,  $V_d$ , for the average output voltage  $\bar{z}_{2n}$ , one must follow the procedure described next, in order to obtain a set of steady state desired inductor currents,

- Establish an arbitrary growing sequence of desired steady state average capacitor voltages,

$$E < \bar{z}_2 < \bar{z}_4 < \dots < \bar{z}_{2n-2} < \bar{z}_{2n} = V_d \quad (9)$$

- From the second row of expressions in (7) obtain the corresponding required steady state duty ratios,  $U_1, \dots, U_n$ .
- With  $\bar{z}_1$  known from the first equality of (8), substitute the determined duty ratio values,  $U_i$ ;  $i = 1, \dots, n$ , into the first row of expressions in (7) in order to determine a set of desired steady state values of the inductor currents,  $\bar{z}_3, \bar{z}_5, \dots, \bar{z}_{2n-1}$ .

### 3. PASSIVITY-BASED ADAPTIVE CONTROLLER DESIGN FOR BOOST CONVERTER CIRCUITS

The following proposition summarizes the properties of a passivity-based nonlinear adaptive dynamical controller for the cascaded “boost” converter that, indirectly, regulates the average output capacitor voltage towards a desired value  $V_d$ . This is achieved by regulating the average inductor currents towards a given set of desired steady state values, uniquely determined from the arbitrarily chosen sequence of growing steady state capacitor voltages (9).

**Proposition 1** Consider the averaged dynamics (3) of the multivariable cascaded “boost” converter circuit, where  $C_i, L_i > 0$ ;  $i = 1, 2, \dots, n$ , and  $E > 0$  are known constants representing the capacitances, inductances and voltage of the external source, respectively. The constant parameter,  $R_L > 0$ , is the unknown load charge resistance.

Define an adaptive nonlinear dynamic state feedback controller as

$$\begin{aligned} \mu_1 &= 1 - \frac{1}{z_{2d}} \left[ E + R_1 \left( z_1 - \hat{\theta} \frac{V_d^2}{E} \right) \right. \\ &\quad \left. + L_1 \frac{V_d^2}{E} z_{(2n)d} (z_{(2n)} - z_{(2n)d}) \right] \end{aligned}$$

$$\begin{aligned} \mu_2 &= 1 - \frac{1}{z_{4d}} \left[ z_{2d} + R_3 \left( z_3 - \hat{\theta} \frac{V_d^2}{\bar{z}_2} \right) \right. \\ &\quad \left. + L_2 \frac{V_d^2}{\bar{z}_2} z_{(2n)d} (z_{(2n)} - z_{(2n)d}) \right] \\ &\dots \\ \mu_n &= 1 - \frac{1}{z_{(2n)d}} \left[ z_{(2n-2)d} \right. \\ &\quad \left. + R_{2n-1} \left( z_{2n-1} - \hat{\theta} \frac{V_d^2}{\bar{z}_{2n-2}} \right) \right. \\ &\quad \left. + L_n \frac{V_d^2}{\bar{z}_{2n-2}} z_{(2n)d} (z_{(2n)} - z_{(2n)d}) \right] \\ \dot{z}_{2d} &= -\frac{\hat{\theta}}{C_1} \left\{ 1 - \frac{\bar{z}_2}{E z_{2d}} \left[ E + R_1 \left( z_1 - \hat{\theta} \frac{V_d^2}{E} \right) \right. \right. \\ &\quad \left. \left. + L_1 \frac{V_d^2}{E} z_{(2n)d} (z_{(2n)} - z_{(2n)d}) \right] \right\} \frac{V_d^2}{\bar{z}_2} \\ &\quad + \frac{1}{R_2 C_1} (z_2 - z_{2d}) \\ \dot{z}_{4d} &= -\frac{\hat{\theta}}{C_2} \left\{ 1 - \frac{\bar{z}_4}{z_2 z_{4d}} \left[ z_{2d} + R_3 \left( z_3 - \hat{\theta} \frac{V_d^2}{\bar{z}_2} \right) \right. \right. \\ &\quad \left. \left. + L_2 \frac{V_d^2}{\bar{z}_2} z_{(2n)d} (z_{(2n)} - z_{(2n)d}) \right] \right\} \frac{V_d^2}{\bar{z}_4} \\ &\quad + \frac{1}{R_4 C_2} (z_4 - z_{4d}) \\ &\dots \\ \dot{z}_{(2n)d} &= -\frac{\hat{\theta}}{C_n} \left\{ z_{(2n)d} - \frac{1}{z_{(2n)d}} \left[ z_{(2n-2)d} \right. \right. \\ &\quad \left. \left. + R_{2n-1} \left( z_{2n-1} - \hat{\theta} \frac{V_d^2}{\bar{z}_{2n-2}} \right) \right. \right. \\ &\quad \left. \left. + L_n \frac{V_d^2}{\bar{z}_{2n-2}} z_{(2n)d} (z_{(2n)} - z_{(2n)d}) \right] \right\} \frac{V_d^2}{\bar{z}_{2n-2}} \\ \dot{\hat{\theta}} &= -z_{(2n)d} (z_{2n} - z_{(2n)d}) \end{aligned} \quad (10)$$

where the dynamical controller initial conditions are chosen so that,  $z_{(2i)d}(0) > 0$ ;  $i = 1, 2, \dots, n$ , and  $\hat{\theta}(0) > 0$ .  $V_d$ , the constant reference value for the output voltage  $z_{(2n)d}$ , is chosen to be a strictly positive quantity. The set of constants,  $\bar{z}_2, \bar{z}_4, \dots, \bar{z}_{2n-2}, \bar{z}_{2n}$ , satisfy the restriction

$$E < \bar{z}_2 < \bar{z}_4 < \dots < \bar{z}_{2n-2} < \bar{z}_{2n} = V_d$$

but they are, otherwise, completely arbitrary. The scalar variable  $\hat{\theta}$  denotes the estimate of  $\frac{1}{R_L}$ . The parameters  $R_1, R_2, \dots, R_{2n-1}$  are designer chosen constants with the only restriction of being strictly positive. Under these conditions, it is always possible to choose the initial state of the controller  $z_{(2j)d}$   $j = 1, 2, \dots, n$  and  $\hat{\theta}(0)$ , such that the closed loop system (3), (10) has an equilibrium point given by,

$$\begin{aligned}
& (z_1, z_2, z_3, z_4, \dots, z_{2n-1}, z_{2n}, z_{2d}, z_{4d}, \\
& \dots, z_{(2n-2)d}, z_{(2n)d}, \hat{\theta}) \\
& = \left( \frac{V_d^2}{R_L E}, \bar{z}_2, \frac{V_d^2}{R_L \bar{z}_2}, \bar{z}_4, \dots, \right. \\
& \quad \left. \frac{V_d^2}{R_L \bar{z}_{2n-2}}, V_d, \bar{z}_2, \bar{z}_4, \dots, \bar{z}_{2n-2} V_d, \frac{1}{R_L} \right)
\end{aligned} \tag{11}$$

which is asymptotically stable.

### Proof

It can be verified, by direct substitution, that (11) represents an equilibrium point for the closed loop system.

Define

$$\begin{aligned}
z_{1d} &= \hat{\theta} \frac{V_d^2}{E} ; \quad z_{3d} = \hat{\theta} \frac{V_d^2}{\bar{z}_2} ; \quad \dots ; \\
z_{(2n-1)d} &= \hat{\theta} \frac{V_d^2}{\bar{z}_{2n-2}}
\end{aligned} \tag{12}$$

Note that  $z_{jd}$  ;  $j = 1, 3, \dots, 2n-1$  and  $z_j$  ;  $j = 1, 3, \dots, 2n-1$  coincide at the equilibrium point. Let,  $z - z_d$ , stand for the error vector, denoted by  $\tilde{z}$ . In terms of the error signals, (5) is rewritten as,

$$\mathcal{D}_{MB} \dot{\tilde{z}} + (\mathcal{J}_{MB}(v) + \mathcal{R}_{MBd}) \tilde{z} = \psi \tag{13}$$

where,

$$\begin{aligned}
\psi &= \mathcal{E}_{MB} - \left\{ \mathcal{D}_{MB} \dot{z}_d + (\mathcal{J}_{MB}(v) + \mathcal{R}_{MB}) z_d \right. \\
&\quad \left. - \begin{bmatrix} R_1 \tilde{z}_1 \\ \tilde{z}_2 / R_2 \\ R_3 \tilde{z}_3 \\ \dots \\ R_{2n-1} \tilde{z}_{2n-1} \\ 0 \end{bmatrix} \right\}
\end{aligned} \tag{14}$$

and  $\mathcal{R}_{MBd}$  is a positive definite matrix given by,

$$\begin{aligned}
\mathcal{R}_{MBd} &= \mathcal{R}_{MB} \\
&+ \text{diag}\{R_1, \frac{1}{R_2}, R_3, \dots, \frac{1}{R_{2n-2}}, R_{2n-1}, 0\} \\
&; \quad R_i > 0 ; i = 1, 2, \dots, 2n-1
\end{aligned} \tag{15}$$

This term is included to achieve the damping injection that enforces asymptotic stability.

Expression (14) is explicitly written as

$$\psi_1 = -L_1 \dot{z}_{1d} - (1 - \mu_1) z_{2d} + E + R_1 \tilde{z}_1$$

$$\begin{aligned}
\psi_2 &= -C_1 \dot{z}_{2d} + (1 - \mu_1) z_{1d} - z_{3d} + \frac{1}{R_2} \tilde{z}_2 \\
&\dots \\
\psi_{2n-1} &= -L_n \dot{z}_{(2n-1)d} - (1 - \mu_n) z_{(2n)d} + z_{(2n-2)d} \\
&\quad + R_{2n-1} \tilde{z}_{2n-1} \\
\psi_{2n} &= -C_n \dot{z}_{(2n)d} + (1 - \mu_n) z_{(2n-1)d} - \frac{1}{R_L} \tilde{z}_{(2n)d}
\end{aligned} \tag{16}$$

Using (10) and (13) one has  $\psi_i = 0$  ;  $i = 1, 2, \dots, 2n-1$  and  $\psi_{2n} = \tilde{\theta} z_{(2n)d}$ , where  $\tilde{\theta} = \hat{\theta} - \frac{1}{R_L}$ .

The resulting stabilization error system is then given by the following perturbed dynamics,

$$\mathcal{D}_{MB} \dot{\tilde{z}} + \mathcal{J}_{MB}(v) \tilde{z} + \mathcal{R}_{MBd} \tilde{z} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ \tilde{\theta} z_{(2n)d} \end{bmatrix} \tag{17}$$

Using as a Lyapunov function candidate the total energy of the stabilization error system plus the “energy” associated with the parameter estimation error,

$$H_d(t) = \frac{1}{2} \left[ \tilde{z}^T \mathcal{D}_{MB} \tilde{z} + \tilde{\theta}^2 \right] \tag{18}$$

one obtains that, along the trajectories of (17), the following relation is satisfied,

$$\dot{H}_d(t) = -\tilde{z}^T \mathcal{R}_{MBd} \tilde{z} + \tilde{\theta} \left[ \dot{\tilde{\theta}} + z_{(2n)d} (z_{2n} - z_{(2n)d}) \right] \tag{19}$$

Using the last equation in (10) and the fact that  $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$  one obtains

$$\dot{H}_d(t) = -\tilde{z}^T \mathcal{R}_{MBd} \tilde{z} \leq -\frac{\alpha}{\beta} H_d \tag{20}$$

where  $\alpha$  and  $\beta$  may be taken to be

$$\begin{aligned}
\alpha &= \min \left\{ R_1, \frac{1}{R_2}, \dots, R_{2n-1}, \frac{1}{R_L} \right\} \\
\beta &= \max \{ L_1, C_1, \dots, L_n, C_n \}
\end{aligned}$$

One concludes that  $\tilde{z}$  and  $\tilde{\theta}$  are bounded and that  $\tilde{z}$  is square integrable. To actually show that  $\tilde{z} \rightarrow 0$  asymptotically, it must be verified that  $\tilde{z}$  is uniformly continuous. For this, it suffices to show that  $\dot{\tilde{z}}$  is bounded. From the perturbed error dynamics (17), and the boundedness of  $\tilde{\theta}$  and  $\tilde{z}$ , it follows that  $\dot{\tilde{z}}$  is bounded if, and only if,  $z_{(2n)d}$  is bounded. A proof of the fact that  $z_{(2n)d}$  is

bounded follows from standard Lyapunov stability arguments.

Since  $\bar{z}_i$ ;  $i = 1, 2, \dots, n$  tend to zero asymptotically, the converter capacitor voltages,  $z_{2j}$   $j = 1, \dots, n$ , converge, respectively, to the values,  $\bar{z}_{2j}$   $j = 1, \dots, n$ , with,  $\bar{z}_{2n} = V_d$ . According to (8), this implies that the converter inductor currents,  $z_{2i+1}$   $i = 0, 1, \dots, n-1$ , converge, respectively, towards the uniquely determined corresponding equilibria, namely,  $(V_d^2/\bar{z}_{2i}) R_L$   $i = 0, 1, \dots, n-1$ , which, in turn, must coincide with the equilibrium point of  $z_{(2i+1)d}$  for each  $i$ . This implies then that, necessarily,  $\hat{\theta} \rightarrow 1/R_L$ .

The stability of the dynamics for  $z_{jd}$ ;  $j = 2, 4, \dots, 2n$ , given in (10), is, ultimately, determined by the following “zero dynamics”.

$$\begin{aligned} \dot{z}_{2d} &= -\frac{\hat{\theta}}{C_1} \left\{ 1 - \frac{\bar{z}_2}{z_{2d}} \right\} \frac{V_d^2}{\bar{z}_2} \\ \dot{z}_{4d} &= -\frac{\hat{\theta}}{C_2} \left\{ 1 - \frac{\bar{z}_4 z_{2d}}{z_{4d} \bar{z}_2} \right\} \frac{V_d^2}{\bar{z}_4} \\ &\dots \\ \dot{z}_{(2n-2)d} &= -\frac{\hat{\theta}}{C_{n-1}} \left\{ 1 - \frac{\bar{z}_{2n-2} z_{(2n-4)d}}{z_{(2n-2)d} \bar{z}_{2n-4}} \right\} \frac{V_d^2}{\bar{z}_{2n-2}} \\ \dot{z}_{(2n)d} &= -\frac{\hat{\theta}}{C_n} \left\{ z_{(2n)d} - \frac{V_d^2}{z_{(2n)d}} \frac{z_{(2n-2)d}}{\bar{z}_{2n-2}} \right\} \end{aligned} \quad (21)$$

which has an asymptotically stable equilibrium point at

$$(z_{2d}, z_{4d}, \dots, z_{(2n-2)d}, z_{(2n)d}) = (\bar{z}_2, \bar{z}_4, \dots, \bar{z}_{(2n-2)}, V_d) \quad (22)$$

for all initial conditions satisfying  $z_{(2j)d}(0) > 0$ ;  $j = 1, 2, \dots, n$ .

#### 4. SIMULATION RESULTS

Simulations of the closed loop behaviour of a three stage cascaded average “boost” converter and the passivity based indirect adaptive feedback controller were performed on the following perturbed version of the “boost” converter circuit,

$$\begin{aligned} \dot{z}_1 &= -(1 - \mu_1) \frac{1}{L_1} z_2 + \frac{E + \eta}{L_1} \\ \dot{z}_2 &= (1 - \mu_1) \frac{1}{C_1} z_1 - \frac{z_3}{C_1} \\ \dot{z}_3 &= -(1 - \mu_2) \frac{1}{L_2} z_4 + \frac{z_2}{L_2} \end{aligned}$$

$$\begin{aligned} \dot{z}_4 &= (1 - \mu_2) \frac{1}{C_2} z_3 - \frac{z_5}{C_2} \\ \dot{z}_5 &= -(1 - \mu_3) \frac{1}{L_3} z_6 + \frac{z_4}{L_3} \\ \dot{z}_6 &= (1 - \mu_3) \frac{1}{C_3} z_5 - \frac{z_6}{R_L C_3} \end{aligned} \quad (23)$$

where  $\eta$  represents an external stochastic perturbation input affecting the system behaviour directly through the external voltage source value. Note that this perturbation input is of the “unmatched” type. i.e., it enters the system equations through an input channel vector field, given by  $[1/L_1, 0, 0, 0, 0, 0]^T$  which is *not* in the range space of the control input channel, given by the vector field,

$$\left[ \frac{1}{L_1} z_2, -\frac{1}{C_1} z_2, \frac{1}{L_2} z_4, -\frac{1}{C_2} z_3, \frac{1}{L_3} z_6, -\frac{1}{C_3} z_5 \right]^T$$

. The magnitude of the noise was chosen to be, approximately, 10 % of the value of  $E$ . The circuit parameter values were taken to be the following “typical” values,

$$\begin{aligned} C_1 = C_2 = C_3 &= 20 \mu\text{F} ; R_L = 30 \Omega ; \\ L_1 = L_2 = L_3 &= 20 \text{mH} ; E = 15 \text{V} \end{aligned}$$

We let the desired average output voltage be  $\bar{z}_6 = V_d = 200$  Volts, and arbitrarily chose  $\bar{z}_4 = 100$ ;  $\bar{z}_2 = 50$ . These values corresponded to steady state duty ratios:  $U_1 = 0.7$ ;  $U_2 = 0.5$ ;  $U_3 = 0.5$ . The nominal value of  $\bar{z}_1$ , corresponding to  $V_d$ ,  $R_L$  and  $E$  is  $\bar{z}_1 = 88.88$  Amp. The steady state duty ratio values and the value of  $\bar{z}_1$  yielded the following desired steady state inductor currents  $\bar{z}_3 = 26.66$  Amp. and  $\bar{z}_5 = 13.33$  Amp. Figure 2 shows the closed loop state trajectories corresponding to the feasible adaptive duty ratio synthesizer derived for the three-stage multivariable “boost” converter. This figure also presents the trajectories of the duty ratio functions, the trajectory of the parameter estimation values and a realization of the computer generated stochastic perturbation signal  $\eta$ .

The simulations show that the proposed controller achieves the desired indirect stabilization of the output voltage around the desired equilibrium value while exhibiting a high degree of robustness with respect to the external stochastic perturbation input.

#### 5. CONCLUSIONS

In this article an adaptive passivity-based regulation scheme has been developed for the on-line feedback specification of the stabilizing duty ratio function in multivariable, and traditional, versions of dc-to-dc power

converters of the “boost” type. The proposed approach is based on a combination of closed loop energy shaping and appropriate stabilizing damping injections, accomplished through dynamical state feedback. The proposed technique, which uses the total energy of the system as a Lyapunov function, was shown to easily accommodate for parametric uncertainties at the load. The more difficult case of uncertainties in the circuit parameter values remains to be explored.

## 6. REFERENCES

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FIGURES

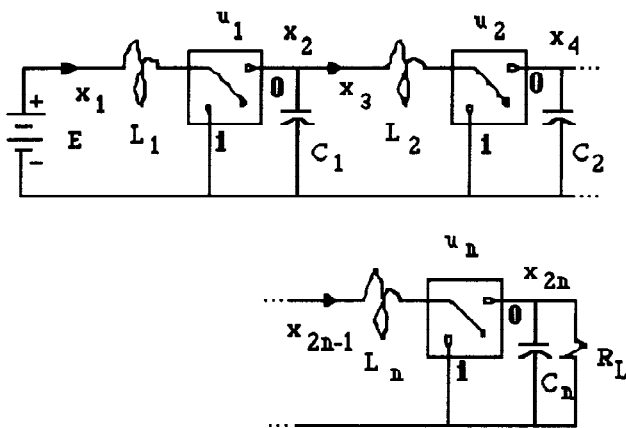


Fig. 1. Multivariable Cascaded “Boost” Converter.

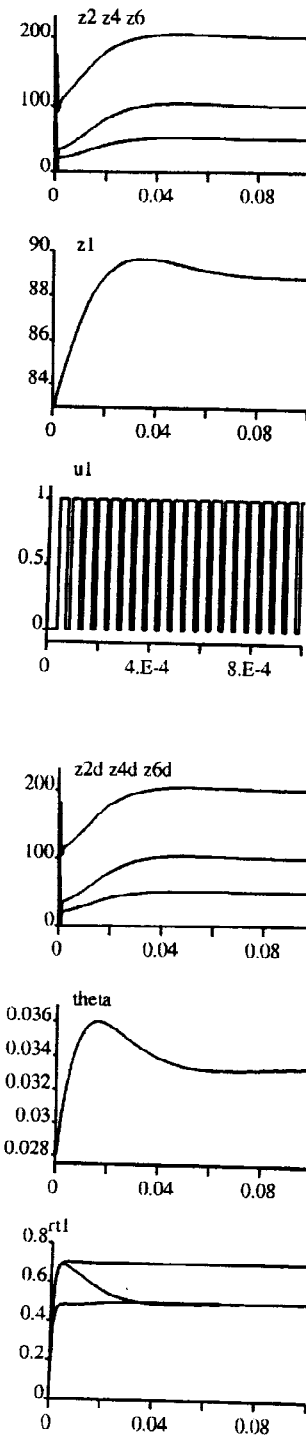


Fig. 2. Simulation Results for Performance Evaluation of Passivity-Based Adaptive Controller in a Perturbed Average Multivariable “Boost” Converter.