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Passivity Based Regulation of Nonlinear Continuos Processes [1]

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Abstract

Through a process of "feedback passivitization", a large class of nonlinear monovariable systems may benefit from systematic controller design techniques available for passive systems. An illustrative design example from the continuously stirred tank reactor control area is presented which include computer simulations.

Keywords : Passivity. Nonlinear Systems. Feedback Passivization.

1 Introduction

Passivity-based controllers have been traditionally applied to the class of lagrangian systems [3]. In particular, the approach has been applied to mechanical systems (such as robots [4], etc.), electro-mechanical systems (such as induction motors) and to purely electrical systems (such as dc-to-dc power converters [5]). We show that nonlinear systems such as those describing chemical, biological and level control processes, may benefit from a systematic feedback controller design procedure already available for nonlinear passive systems. As a first step, it is shown that a large class of nonlinear systems with *nonzero constant equilibrium* states, are "passivifiable" by means of a suitable state-dependent input coordinate transformation. The crucial requirement for passivization through feedback is that the system must have a storage function which is locally strict relative degree one in a region containing the equilibrium state. Although this result is independent of the minimum or nonminimum phase character of the system, and, also, independent of the output relative degree, passivization of nonminimum phase systems will result in an unfeaseable growth of the state coordinate transformation when the system motions are sustained at the required equilibrium point. Hence, in connection with stabilizing designs, our technique should only be applied to minimum phase systems.

In this article, the geometric features of passivization are studied in connection with the system's defining vector fields and their relation to a family of smooth manifolds representing constant values of the storage function. A decomposition of the system's drift vector field is proposed. We show that the "passivifying" input coordinate transformation renders lossless (i.e., invariant), with respect to the storage function, the nondissipative component of such a drift vector field. The proposed decomposition is in a loose sense "canonical" and it is intimately related to the traditional passivity-based feedback regulation design, carried out through energy storage function modification and feedback damping injection possibilities [3].

In Section 2 we review some concepts about dissipative, lossless and passive systems. We also present in this section a feedback passivization technique and the geometric aspects of the proposed feedback passivization scheme. Section 3 presents a general state space "canonical form" for passive nonlinear systems. This section also revisits the "energy modification plus damping injection" controller design methodology in light of the proposed passivity canonical form. Section 4 is devoted to show an application of this technique in the chemical control systems area. Section 5 contains the conclusions and suggestions for futher research in this field.

2 A geometric Approach to Passivity-Based Regulation

We consider affine nonlinear systems described by

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \qquad (1)$$

where $x \in \mathcal{X} \subset R^n$ is the state vector, $u \in \mathcal{U} \subset R$ is the control input and the scalar function $y \in \mathcal{Y} \subset R$ is

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the output function of the system. The region $\mathcal{X} \subset R^n$ is the *operating region* of the system. The *supply rate* function is defined as a function $s : \mathcal{U} \times \mathcal{Y} \mapsto R$.

Assumptions 2.1

- *The vector fields $f(x)$ and $g(x)$ are smooth vector fields on $\mathcal{X}$.*
- *There exists an isolated* nonzero *state of interest, $x = x_e \in \mathcal{X}$, where $f(x_e) + g(x_e)\bar{u} = 0$, for some nonzero constant $\bar{u}$.*
- *There exists an* energy storage function, $V : \mathcal{X} \mapsto R^+$ *which may be zero* outside *of $\mathcal{X}$ (at the origin, for instance), associated with system (1).*

2.1 Review of passive systems theory

We review some of the basic definitions about dissipative, lossless and passive systems. The reader is referred to [1], [8] and [9] for additional details.

Definition 2.2 *[8] System (1) is said to be* dissipative *with respect to the supply rate $s(u, y)$ if there exists a storage function $V : \mathcal{X}$ and for all $t_1 \geq t_0$, and all input functions u, the following relation holds*

$$V(x(t_1)) - V(x(t_0)) \leq \int_{t_0}^{t_1} s(u(t), y(t))dt \qquad (2)$$

with $x(t_0) = x_0$ and $x(t_1)$ is the state resulting, at time t_1, from the solution of system (1) taking as initial condition x_0 and as control input the function $u(t)$. If V is differentiable with respect to time t for all $x \in \mathcal{X}$ and $u \in \mathcal{U}$, then the inequality (2) is equivalent to ([9]):

$$\dot{V} \ \leq \ s(u(t), y(t)) \qquad (3)$$

The system is lossless *if the inequalities (2), or (3), are, in fact, equalities.*

Definition 2.3 *[8] System (1) is* passive *if it is dissipative with respect to the supply rate $s(u, y) = uy$. The system is* strictly input passive *if there exists $\delta > 0$ such that the system is dissipative with respect to $s(u, y) = uy - \delta u^2$. The system is* strictly output passive *if there exist an $\gamma > 0$ such that the system is dissipative with respect to $s(u, y) = uy - \gamma y^2$.*

The following definition and result constitute a generalization of the classical Kalman-Yakubovich-Popov property and of its associated lemma for positive real linear systems [1].

Definition 2.4 *A system (1) has the Kalman-Yakubovich-Popov (KYP) property if there exists a continuously differentiable nonnegative function $V : \mathcal{X} \mapsto R$, with $V(0) = 0$, such that*

$$L_f V(x) \leq 0$$
$$L_g V(x) = h(x)$$

for all $x \in \mathcal{X}$.

The following result follows directly from the definition of passivity and the fact that $V(x)$ is a relative degree one function of the system.

Proposition 2.5 *[1] A system which has the KYP property is passive with storage function V. Conversely a passive system having a continuously differentiable storage function has the KYP property.*

2.2 Feedback Passivization

We are going to consider systems in which the drift vector field $f(x)$ has a natural decomposition.

Definition 2.6 *The drift vector field $f(x)$ of (1) has a* natural decomposition with respect to the storage function V, *whenever $f(x)$ can be expressed as the sum of three components*

$$f(x) = f_d(x) + f_{nd}(x) + f_I(x)$$

such that,

$$\begin{aligned} L_{f_d}V(x) \ &\leq \ 0 \ ; \ \forall\, x \in \mathcal{X} \\ L_{f_{nd}}V(x) \ &\left\{ \begin{array}{l} \text{is, either sign -- undefined in } \mathcal{X} \\ \text{or, else, it is nonnegative in } \mathcal{X} \end{array} \right. \\ L_{f_I}V(x) \ &= \ 0 \ ; \ \forall\, x \in \mathcal{X} \end{aligned}$$

where $f_d(x)$ is the dissipative component *of $f(x)$, $f_{nd}(x)$ is the* non-dissipative component *of $f(x)$ and $f_I(x)$ is the* invariant component *of $f(x)$.*

We shall be considering means of rendering a system of the form (1) passive, or at least "lossless", by means of state feedback. We therefore introduce a definition of "passivifiable" system in the following terms:

Definition 2.7 *System (1) is said to be* "passivifiable" *with respect to the storage function V if there exists a* regular affine feedback law *of the form*

$$u = \alpha(x) + \beta(x)v \ ; \ \alpha(x) \in R \ ; \ \beta(x) \in R \qquad (4)$$

where $\beta(x)$ is a nonzero scalar function in $\mathcal{X}$, and such that the closed loop system (1)-(4) becomes passive with new scalar control input v.

The following proposition presents the feedback passivization technique:

Proposition 2.8 *System (1) is locally strictly output* passivifiable *with respect to the storage function V, by means of affine feedback of the form (4) if and only if*

$$L_g V(x) \neq 0 \, \forall \, x \in \mathcal{X}$$

The affine feedback law, or state dependent input coordinate transformation, that achieves strict output passivization is given by the expression

$$u = \frac{h(x)}{L_g V(x)} v - \frac{L_{f_{nd}} V(x)}{L_g V(x)} - \gamma \frac{h^2(x)}{L_g V} \qquad (5)$$

where γ is an arbitrary strictly positive scalar.

The resulting closed loop system given by (1)-(5) satisfies the KYP property as it may be easily verified. The proof of this result is given in [6].

2.3 A Geometric Interpretation of Passivization by Feedback

Suppose a system of the form (1), with a natural decomposition of $f(x)$, is passivifiable, i.e., $L_g V(x) \neq 0 \; \forall \; x \in \mathcal{X}$ with control input (5). The closed loop system (1)-(5) is given by

$$\begin{aligned} \dot{x} &= f_d(x) + f_I(x) + \left[I - g(x) \frac{\partial V(x)/\partial x}{L_g V(x)} \right] f_{nd}(x) \\ &\quad + \frac{h}{L_g V} g(x) v - \gamma \frac{h^2(x)}{L_g V} g(x) \qquad (6) \end{aligned}$$

The first summand is, according to its definition, a naturally dissipative term. The second and third summands are the *workless* terms or invariant terms, the fourth summand is the power adquisition term responsible for the "supply rate" in terms of the new control input and, finally, the fifth summand is an artificially induced dissipation term making use of nonlinear (quadratic) output feedback.

The geometric interpretation of the several components in the transformed equation (6) is shown in Fig. 1. Note that the matrix

$$M(x) = \left[I - g(x) \frac{\partial V(x)/\partial x}{L_g V(x)} \right]$$

is a *projection operator* onto the tangent space to the level surface $V(x) =$ *constant*, along the distribution span $\{g\}$. This projection operator "hides" all destabilizing components of $f_{nd}(x)$ by making the vector $M(x) f_{nd}(x)$ tangent to the level surfaces of constant stored energy, i.e., to the family of sets (or *foliation*) $\{ \, x$ s.t. V(x) = constant $\}$. Thus, any unstable behaviour contained in $f_{nd}(x)$ does not increment, nor

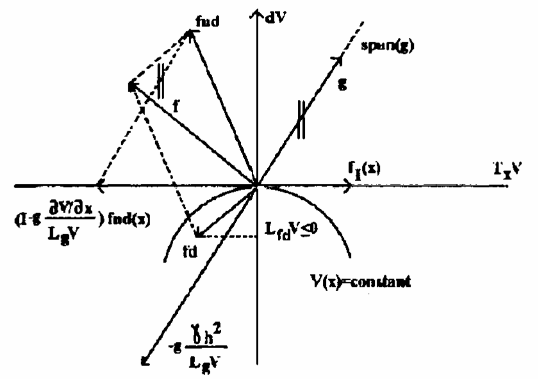

Figure 1: A geometric interpretation of passivization.

diminishes, the value of the energy function $V(x)$ along the controlled trajectories of the transformed system. However, it should be made clear that passivization of non-minimum phase systems would be achieved at the expense of possibly unbounded (i.e., unfeasible) feedback control actions. The *requirement* of a minimum phase system for passivization is, hence, natural and convenient.

It is easy to verify that $M(x)$ satisfies the following properties which are characteristic of projection operators onto tangent spaces, along the span of a given vector g :

$$M(x) g(x) = 0 \;\; \forall \, x \in \mathcal{X}$$

$$\tfrac{\partial V}{\partial x} M(x) = 0 \;\; \forall \, x \in \mathcal{X}$$

$$M^2(x) = M(x) \;\; \forall \, x \in \mathcal{X}$$

3 Passivity-Based Feedback Controller Design

We review a systematic procedure for the synthesis of passivity based feedback controllers. This procedure, based on storage function modification and damping injection through feedback, has been extensively used in the area of mechanical, electromechanical and electric systems. The reader may find further details in [5] and in the references therein.

3.1 A Canonical Form for Passive Systems

Suppose that system (1) is passivifiable and $f(x)$ has a natural decomposition. Suppose, furthermore, that $V(x)$ is given in its simplest form

$$V(x) = \frac{1}{2} x^T x$$

Assuming that $L_g V(x) = x^T g(x) \neq 0$ in the operating region $\mathcal{X}$ of the state space. The expression (6) may always be rewritten in the following form

$$\begin{aligned} \dot{x} &= -\mathcal{R}(x)x - \mathcal{J}(x)x + \mathcal{M}(x)v \\ y &= h(x) \end{aligned} \quad (7)$$

where,

$$\begin{aligned} f_d(x) - \gamma \frac{h^2(x)}{x^T g(x)} g(x) &= -\mathcal{R}(x)x \\ f_I(x) + \left[I - g(x) \frac{x^T}{x^T g(x)} \right] f_{nd}(x) &= -\mathcal{J}(x)x \\ \frac{h}{x^T g(x)} g(x) &= \mathcal{M}(x) \end{aligned}$$

with $\mathcal{R}(x)$ being a positive semidefinite matrix in $\mathcal{X}$, and $\mathcal{J}(x)$ being an anti-symmetric matrix.

3.2 Feedback Controller Design via Energy Modification and Damping Injection

Consider the following *modified storage function*

$$V_d(x, x_d) = \frac{1}{2}(x - x_d)^T (x - x_d)$$

where x_d is an auxiliary state vector to be defined later. Along the solutions of the system (7), the function $V_d(x, x_d)$ exhibits the following time derivative

$$\dot{V}_d(x, x_d) = (x - x_d)^T \left[-\mathcal{R}(x)x - \mathcal{J}(x)x + \mathcal{M}(x)v - \dot{x}_d \right]$$

Adding a *damping injection* term of the form $-\mathcal{R}_{di}(x)x$, so that $\mathcal{R}_m(x) = \mathcal{R}(x) + \mathcal{R}_{di}(x)$ is a positive definite matrix for all $x \in \mathcal{X}$, one obtains

$$\begin{aligned} \dot{V}_d(x, x_d) = \; & (x - x_d)^T \left[-\left(\mathcal{R}(x) + \mathcal{R}_{di}(x)\right)(x - x_d) \right. \\ & -\mathcal{J}(x)(x - x_d) - \dot{x}_d - \mathcal{R}(x)x_d \\ & \left. -\mathcal{J}(x)x_d + \mathcal{R}_{di}(x)(x - x_d) + \mathcal{M}(x)v \right] \end{aligned}$$

Letting the auxiliary vector $x_d(t)$ satisfies the following system of differential equations

$$\dot{x}_d = -\mathcal{R}(x)x_d - \mathcal{J}(x)x_d + \mathcal{R}_{di}(x)(x - x_d) + \mathcal{M}(x)v \quad (8)$$

then it results

$$\begin{aligned} \dot{V}_d(x, x_d) &= -(x - x_d)^T \mathcal{R}_m(x)(x - x_d) \\ &\leq -\frac{a}{b}(x - x_d)^T (x - x_d) \leq 0 \end{aligned}$$

where, in terms of the minimum and maximum eigenvalues $(\lambda_{\min}, \lambda_{\max})$ of $R_m(x)$, a and b are given by,

$$\begin{aligned} a &= \inf_{x \in \mathcal{X}} \lambda_{\min}(R_m(x)) > 0 \\ b &= \sup_{x \in \mathcal{X}} \lambda_{\max}(R_m(x)) > 0 \end{aligned}$$

It follows that the vector $x(t)$ exponentially asymptotically converges towards the auxiliary vector trajectory $x_d(t)$. Typically, one sets for a particular component of the vector x_d a desired constant equilibrium value. This is made in correspondance with the component value in the equilibrium state x_e of the original state vector. The objective of such a particularization is to obtain a feedback expression for the external control input v in terms of the available state vector x, as well as the rest of the auxiliary variables in the vector x_d. The differential equations defining the remaining auxiliary variables in x_d, are to be regarded as *state components* of a *dynamical feedback compensator* [5].

4 Example

Consider the following process model that describes a first-order reversible reaction $A \longleftrightarrow B$ that occurs in a constant-volume continuously stirred tank reactor (CSTR), taken from [2], shown in Figure 2.

$$\begin{aligned} \dot{x}_1 &= \frac{q}{V}(C_{Ai} - x_1) - k_1(x_3)x_1 + k_2(x_3)x_2 \\ \dot{x}_2 &= \frac{q}{V}(C_{Bi} - x_2) + k_1(x_3)x_1 - k_2(x_3)x_2 \\ \dot{x}_3 &= \frac{q}{V}(u - x_3) + \frac{(-\Delta H)}{\rho C_p}\left[k_1(x_3)x_1 - k_2(x_3)x_2\right] \\ y &= x_2 \end{aligned} \quad (9)$$

with

$$k_i(x_3) = C_i \exp\left(-\frac{E_i}{Rx_3}\right) \quad ; \quad i = 1, 2.$$

The variable x_1 is the effluent concentration of A, x_2 is the effluent concentration of B, x_3 is the reactor temperature, u is the input variable and it represents the inlet temperature. C_{Ai} is the inlet concentration of A, C_{Bi} is the inlet concentration of B, q is the inlet flow rate, V is the reactor volume, C_1 is the preexponential factor for forward reaction, C_2 is the preexponential factor for reverse reaction, E_1 is the activation energy for forward reaction, E_2 is the activation energy for reverse reaction, $-\Delta H$ is the heat of reaction, ρ is the density, C_p is the heat capacity, R is the gas constant.

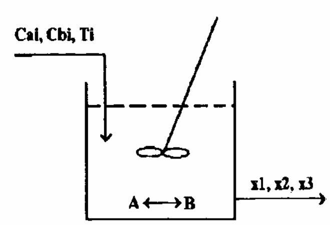

Figure 2: Continuously Stirred Tank Reactor System.

The system output has relative degree equals to two and the operating region (9) is given by points strictly located in the first orthant of R^3, where the concentrations and the temperatures are all positive. In other words,

$$\mathcal{X} = \left\{x = (x_1, x_2, x_3)^T \in R^3, \ \text{s.t. } x_i > 0; \ i = 1,2,3\right\}$$

The vector fields $f(x)$ and $g(x)$ are readily found to be

$$\begin{aligned} f(x) &= \begin{bmatrix} \frac{q}{V}(C_{Ai} - x_1) - k_1(x_3)x_1 + k_2(x_3)x_2 \\ \frac{q}{V}(C_{Bi} - x_1) + k_1(x_3)x_1 - k_2(x_3)x_2 \\ -\frac{q}{V}x_3 + \frac{(-\Delta H)}{\rho C_p}[k_1(x_3)x_1 - k_2(x_3)x_2] \end{bmatrix} \\ g(x) &= \begin{bmatrix} 0 \\ 0 \\ \frac{q}{V} \end{bmatrix} \end{aligned}$$

Consider the following energy storage function,

$$V = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right)$$

The condition, $L_g V \neq 0$, results, in this case, in

$$L_g V = \frac{q}{V}x_3 \neq 0$$

which is satisfied over the operating region $\mathcal{X}$.

The system is, then, clearly passivifiable with storage function $V(x)$. The time derivative of V along the regulated evolution of the system satisfies the following inequality,

$$\begin{aligned} \dot{V} \ &\leq \ \frac{q}{V}(C_{Ai}x_1 + C_{Bi}x_2 + ux_3) + k_1(x_3)x_1 \\ &\quad \times \left(x_2 + \frac{(-\Delta H)}{\rho C_p}x_3\right) + k_2(x_3)x_1x_2 \quad (10) \end{aligned}$$

The decomposition of $f(x)$ in $\mathcal{X}$ into dissipative and non-dissipative components is clearly given by

$$\begin{aligned} f_d(x) &= \begin{bmatrix} -\left(\frac{q}{V} + k_1(x_3)\right)x_1 \\ -\left(\frac{q}{V} + k_2(x_3)\right)x_2 \\ -\frac{q}{V}x_3 - \frac{(-\Delta H)}{\rho C_p}k_2(x_3)x_2 \end{bmatrix} \\ f_{nd}(x) &= \begin{bmatrix} \frac{q}{V}C_{Ai} + k_2(x_3)x_2 \\ \frac{q}{V}C_{Bi} + k_1(x_3)x_1 \\ \frac{(-\Delta H)}{\rho C_p}k_1(x_3)x_1 \end{bmatrix} \end{aligned}$$

Define the following state-dependent input coordinate transformation,

$$\begin{aligned} u &= \frac{q}{Vx_3}\Big[x_2 v - \frac{q}{V}(C_{Ai}x_1 - C_{Bi}x_2) - k_1(x_3)x_1 \\ &\quad -k_2(x_3)x_2)x_1x_2 - \frac{(-\Delta H)}{\rho C_p}k_1(x_3)x_1x_3 \\ &\quad -\gamma x_2^2\Big] \qquad (11) \end{aligned}$$

where γ is an arbitrary strictly positive scalar.

Transformation (11) results in a passive system operator relating the new input v and the output variable x_2.

The partially closed loop system may be placed in the form (7):

$$\dot{x} = -\mathcal{J}(x)x - \mathcal{R}(x)x + \mathcal{M}(x)v$$

where $x^T = [x_1 \quad x_2 \quad x_3]$ and

$$\begin{aligned} -\mathcal{J}(x) &= \begin{bmatrix} 0 & 0 & \mathcal{J}_{13} \\ 0 & 0 & \mathcal{J}_{23} \\ -\mathcal{J}_{13} & -\mathcal{J}_{23} & 0 \end{bmatrix} \\ -\mathcal{R}(x) &= \text{diag}\ \{-\left(\tfrac{q}{V} + k_1(x_3)\right), -\left(\tfrac{q}{V} + k_2(x_3)\right), \\ &\quad -\tfrac{(-\Delta H)}{\rho C_p}k_2(x_3)\tfrac{x_2}{x_3} - \tfrac{q}{V} - \gamma\tfrac{x_2^2}{x_3^2}\} \\ \mathcal{M}(x) &= \begin{bmatrix} 0 \\ 0 \\ \frac{x_2}{x_3} \end{bmatrix} \end{aligned}$$

where

$$\begin{aligned} \mathcal{J}_{13} &= \left(\frac{q}{V}C_{Ai} + k_2(x_3)x_2\right)\frac{1}{x_3} \\ \mathcal{J}_{23} &= \left(\frac{q}{V}C_{Bi} + k_1(x_3)x_1\right)\frac{1}{x_3} \end{aligned}$$

A dynamical passivity-based feedback controller for the system may be obtained by using the synthesis procedure outlined in Section 3.1.

Letting $x_{3d} = \bar{x}_3 =$ *constant*, one obtains the following dynamical controller expression, where x_{1d} and x_{2d} have been substituted by the controller state variable ξ_1 and ξ_2, respectively.

$$\begin{aligned} \dot{\xi}_1 &= \frac{q}{V}\left(C_{Ai}\frac{\bar{x}_3}{x_3} - \xi_1\right) - k_1(x_3)\xi_1 + k_2(x_3)x_2\frac{\bar{x}_3}{x_3} \\ &\quad +R_1(x_1 - \xi_1) \\ \dot{\xi}_2 &= \frac{q}{V}\left(C_{Bi}\frac{\bar{x}_3}{x_3} - \xi_2\right) + k_1(x_3)\frac{x_1\bar{x}_3}{x_3} - k_2(x_3)\xi_2 \\ &\quad +R_2(x_2 - \xi_2) \\ v &= \frac{q}{Vx_2}(C_{Ai}\xi_1 + C_{Bi}\xi_2) + k_1(x_3)\frac{x_1\xi_2}{x_2} + k_2(x_3) \\ &\quad \times\xi_1 + \gamma\frac{x_2\bar{x}_3}{x_3} + \frac{q}{V}\frac{x_3}{x_2}\bar{x}_3 + \frac{(-\Delta H)}{\rho C_p}k_2(x_3)\bar{x}_3 \\ &\quad -R_3(x_3 - \bar{x}_3)\frac{x_3}{x_2} \end{aligned}$$

We take, after [2], the following system parameters for the simulation of the controlled stirred-tank reactor: $q = 1$ L/s, $C_{Ai} = 1$ mol/L, $C_{Bi} = 0$ mol/L, $V = 60$ L, $C_1 = 5 \times 10^3$ s^{-1}, $C_2 = 1 \times 10^6$ s^{-1}, $E_1 = 1 \times 10^4$ cal/mol, $E_2 = 1.5 \times 10^4$ cal/mol, $-\Delta H = 5000$ cal/mol, $\rho = 1$ kg/L, $C_p = 1000$ cal/kg·K, $R = 1.987$ cal/mol·K.

The required equilibrium point for x_3 was set to be $\bar{x}_3 = 394.4$ K, this corresponds with the steady state values: $\bar{x}_1 = 0.6$ mol/K, $\bar{x}_2 = 0.4$ mol/K and $\bar{u} = 392.4$ K.

Figure 3 shows the closed loop response of the stirred-tank reactor system, controller state and the synthesized control input.

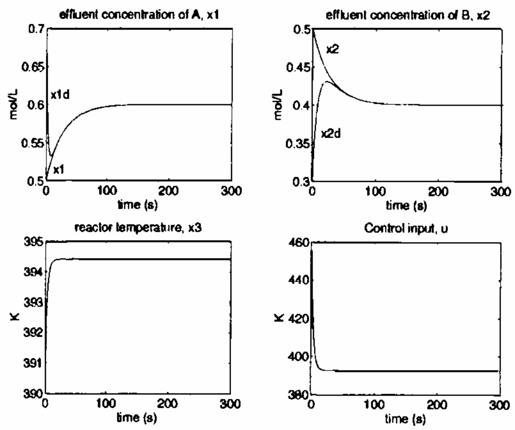

Figure 3: Simulations results of the passivity-based regulated CSTR system.

5 Conclusions

In this article, we have proposed a passivity-based approach for the regulation of a large class of continuous processes, specially chemical systems control processes. A geometric interpretation was given to the possibilities of "passivifying", by means of afine feedback, an arbitrary nonlinear system describing a continuous process. For monovariable systems, passivization is achievable by means of control input space coordinates transformations, provided the energy storage function of the system is strictly relative degree one in the region of interest. This requirement does not seem to be very stringent, for a large class of nonlinear monovariable systems describing common industrial continuous processes.

The use of these general energy concepts, in the passivity-based regulation scheme presented, may prove to be highly beneficial in the control of chemical processes. The results here proposed apply to *any* linear, or nonlinear system, independently of its output relative degree and of its minimum or nonminimum phase character. Of course, in the case of nonminimum phase systems, any stabilizing controller will result in unfeasible control actions, whether unstable or unbounded, as the desired equilibrium state is sustained. In general terms, the possibilities of "passivization" of nonlinear systems by means of regular affine feedback have been shown to be equally valid for monovariable and multivariable cases [7].

Passivity-based regulation, as presented here, can be easily compared against exact linearization techniques in terms of the controller complexity and other practically oriented criteria.

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