FLATNESS AND PASSIVITY FROM A BOND GRAPH

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ABSTRACT

This work shows a first approximation of the use of bond graphs to determine flatness and passivity properties of physical systems. In the case of the flatness property, the topology of the bond graph was used directly in order to obtain paths in the bond graph which generated the equations of the linearized outputs. In the case of the passivity property, we tried to use the topology of the graph directly but not being successful, the scattering matrix of the system was used as an intermediate step.

1. INTRODUCTION

The main goal of control theory is to study the behavior of systems and their interaction with their environment. This study is donde using diverse techniques that deal with different issues of this interaction. Two important issues in the behavior of a system are its controllability and its stability. These two issues are closely related to properties that allow to detect them indirectly: flatness and passivity. These properties can also be used to design control strategies straightforwardly.

A bond graph is a graphic description that allows the modeling of dynamic physical systems. The method which generates a bond graph is based on the separation of components that exchange energy through connections called ports. The energy exchange is studied using to types of general variables: power variables (effort and flow) and energy variables (momentum an displacement). Power variables are related to the energy variables through integral relations.

Bond graphs have demonstrated to be very useful, not only for being a graphic description that permits to model dynamic physical systems properly, but also for being a tool that allows to analyze systems from a structural point of view [6], [23], [25], [28], [29], [30], [36] and generate the symbolic equations [5], [7], [15], [18]. Considering these two characteristics, it was thought that bond graphs could be adequate to determine the passivity and flatness of physical systems.

This paper is organized as follows. In section 2. the concept of flatness is explained and the methodology used in the search of the flatness property from a bond graph is shown. Section 3. describes the concept of passivity and its determination using the scattering matrix generated from a bond graph. Section 4. shows an example where flatness and passivity are determined for the same system. The conclusions of the work are shown in section 5..

2. FLATNESS

One way of analyzing physical systems is through the use of differential algebra [4], [26], [31]. This approach has permitted to establish the relationship between flat and controllable systems. The relation is based on the works about strong accesibility presented by Sussman and Jurdjevic [31] for dynamics of the form $\dot{x} = f(x, u)$, the proof made by Sontag [26] about the existence of control laws for strongly accessible systems and on the proofs by Coron [4] and Sontag [27] which show that such controllers are generic. Applying these concepts, Fliess et al. [9] introduced the concept of flat systems and showed that they are controllable by means of state variable feedback.

In spite of the existence of several methods to determine whether a system is controllable or not, these can not be applied to non-linear systems. Flatness gives us an answer to this particular problem, since a flat system is controllable whether it is linear or not. Besides, when flatness is determined, we obtain explicitly the possible control laws using state variable feedback.

Flat systems are equivalent to linear systems by means of a special type of feedback called endogenous, [9]. This type of feedback receives its name because it does not depend on exogenous variables, which are independent of the original variables of the system and its derivatives.New variables are made up, $\boldsymbol{y}$, that can be seen as a fictitious outputs, called *linearizing outputs* or *flat outputs*. There can be more than one combination of variables that satisfy flatness conditions, since the linearizing output is not unique. Isidori [13] shows the necessary and sufficient conditions for certain non-linear systems to be linearizable by means of state variable feedback.

Although flatness is not a generic property, many systems found in the engineering field are flat. Furthermore, the linearizing output usually has a well defined physical meaning [16].

Definition 2.1 *[9] A non-linear multivariable system of the form:*

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}) \qquad \boldsymbol{x} \in \Re^n \qquad \boldsymbol{u} \in \Re^m, \quad (1)$$

where $\boldsymbol{x}$ is the state vector and $\boldsymbol{u}$ is the output vector is differentially flat if there are m scalar functions differentially independent (linearizing or flat outputs) that depend on $\boldsymbol{x}$, $\boldsymbol{u}$ and a finite number α of its time derivatives,

$$y_j = h_j\left(\boldsymbol{x}, \boldsymbol{u}, \dot{\boldsymbol{u}}, \ldots, \boldsymbol{u}^{\alpha}\right), \qquad j = 1, \ldots, m \quad (2)$$

such that the inverse of the system (1) with $\boldsymbol{u}$ as input and $\boldsymbol{y} = (y_1, \ldots, y_m)$ as output, is independent of the state. This is equivalent to say that each variable of the system can be expressed as a differential function of the linearizing output $\boldsymbol{y}$:

$$\boldsymbol{u} = \eta\left(y, \dot{y}, \ldots, y^{\beta+1}\right) \quad (3)$$

$$\boldsymbol{x} = \phi\left(y, \dot{y}, \ldots, y^{\beta}\right) \quad (4)$$

At the present time, there is no effective method to determine the flatness of a system. Instead, the different variables of the system (1), or a function that depends on them, are checked to verify if they can be written in a way that satisfies the conditions imposed be equations (3) and (4).

This procedure does not always offer concrete results, since the writing of equations (3) and (4) is unknown and depends on the functions that describe the system. Even for linear systems it is difficult to obtain these equations, and the use of computer programs capable of symbolic calculation is needed. These difficulties made us think about using a graphical description such as the bond graph to determine if a system is flat.

Flatness from a Bond Graph

The methodology we developed was based on the bond graph's property to generate equations whenever certain paths in the graph are followed [5]. Since flatness involves the necessity to express a given variable as a function of on boldmath$x$, boldmath$u$ and a finite number α of its time derivatives, a path in the graph between the given initial variable and the candidate to linearizing output was sought.

One of the main ideas of the methodology developed is that the variables involved in the equation of the linearizing output are obtained using causal paths [2]. Two sets of variables were defined: the former (input vector S) groups the inputs of the system (the sources) and the state variables (displacements and momentum) and the latter (output vector L) groups the *candidate variables* to be linearizing or flat outputs. The causal paths between all the variables of the first set and a subset of the second set are sought. Lastly, the bond graph is run following the causal paths and the equations obtained are studied to determine whether the required conditions for the system to be flat are met. The methodology deals easily with simple systems whereas those that contain resistive fields introduce additional calculation problems. Energy storing fields do not pose this inconvenience since they contain starting or arriving points of the causal paths.

One of the advantages of the methodology developed is its recurrence, which permits to try all the possible alternatives when we analyze the variables that could be linearizing outputs. However, the methodology only works for those systems whose linearizing outputs are state variables. Furthermore, the non-linearities of the bond graph elements must be invertible, so that the relations between flow and effort and vice versa can be obtained explicitly. If this condition is not met, it would not be possible to obtain the flatness equations explicitly. For this reason, it is required that the constitutive equations of the fields can be written both in causal derivative form and causal integral form. For the moment, the methodology has only been applied to SISO systems.

The proposed methodology can not be applied when the linearizing output is a function that depends on two or more variables. Even though the algorithm was developed for linear and non-linear scalar systems, a formal proof of the conditions that non-linear scalar systems must satisfy in order to apply the method effectively has not been found yet. A detailed description of the methodology can be found in [19]

3. PASSIVITY

One of the most interesting concepts in control theory is passivity [8] [32] [33]. Initially, this concept appears in the context of electric networks where great efforts were made to give an exact definition of its meaning and its consequences in terms of time and energy [24]. In control theory context, it appears through Popov's works [21] [22], and its development can be seen through different works [3] [8] [12] [17] [32] [33] [34] [35]. The importance of passivity is expressed by two of its main characteristics: passive systems are stable and the interconnection of passive systems is passive.

Basically, the definition of passivity establishes that a system is passive if the energy supplied to the system plus the energy previously stored is greater than the final energy stored in the system. This means that some energy has been *dissipated*. In the following, the concept of passivity will be presented from an intuitive point of view, based on the energetic relations between systems. This concept is closely related to electric network theory.

The definitions given by Wyatt et al. [33] will be used, they deal with finite dimension time invariant systems described by their state equations. The inputs are applied and the outputs observed in the time interval $\Re^+ = [0, \infty)$, and the "initial state" means the state in $t = 0$. The voltages and currents will be referred as efforts e, and flows f, in order to generalize, and they will always have reference directions associated, so that instantaneous power going into the n-port is:

$$\sum_{j=1}^{K} e_j(t) f_j(t) = < e(t), f(t) > \qquad (5)$$

Generally, an n-port is represented by a state equation and two maps that give effort and flow of the ports as function of the input and the state variable, with a group of rules defining the class of inputs that can be applied.

Definition 3.1 *The power input function $P : \Sigma \times U \rightarrow \Re$ is defined by:*

$$p(\boldsymbol{x}, \boldsymbol{u}) \equiv < \boldsymbol{e}(\boldsymbol{x}, \boldsymbol{u}), \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) > \qquad (6)$$

where $\mathbf{x}$ is the state vector and $\mathbf{u}$ is the input vector.

Definition 3.2 *A selection of input and output variables $\boldsymbol{u}$ and $\boldsymbol{y}$ for an n-port is called a hybrid pair if $\boldsymbol{u}$ and $\boldsymbol{y}$ are n-dimensional and for each $K \in \{1, \ldots, n\}$, $u_k = e_k$ and $y_k = f_k$ or $u_k = f_k$ and $y_k = e_k$, where $u_k = e_k$ and $y_k = f_k$ denote the $k-th$ component of u and y respectively, and e_k and f_k denote the $k-th$ effort and flow port, respectively.*

Definition 3.3 *Given an n-port $\mathcal{N}$ with state representation S, the available energy $E_A : \Sigma \rightarrow \Re^+ \cup \{+\infty\}$ is defined by*

$$E_A \equiv \sup_{x \rightarrow} \left\{ \int_0^T - < e(t), f(t) > dt \right\} \qquad \forall T \geq 0 \qquad (7)$$

where the notation $\sup_{x \rightarrow}$, means that the supreme is taken for all admissible pair $\{e(\cdot), f(\cdot)\}$ and $T \geq 0$ with the initial state x fixed.

Since we are supposing that $t \rightarrow < e(\cdot), f(\cdot) >$ is locally L^1, the integral (7) always exists and is finite [33]. This means that the available energy in a particular state x is the maximum energy that can be extracted from the system when its initial state is x.

Definition 3.4 *an n-port $\mathcal{N}$ with state representation S is passive, if for every x that belongs to the space state S, $E_A(x) < +\infty$. If this is not satisfied, $\mathcal{N}$ is active.*

As can be seen in the previous definitions, passivity is defined in terms of a bounding condition of its available energy (Definition 3.3). Analogously, it can be said that a system is passive if the amount of energy that can be supplied through its ports is a finite quantity (Definition 3.4). This means that if the available energy function of a system is determined, then it is possible to study its passivity.

The determination of passivity using the available energy function can result in a complex process disregarding the case studied. For this reason an alternate way was thought that allows to determine passivity using a bond graph.

Passivity from a Bond Graph

As in the case of flatness, we tried to use the topology of the bond graph to detect the passivity of a system. It was found that it is not possible to determine the passivity by only studying the graph's topology, the elements description must be included. This is due to the fact that passivity is a property that depends on the input and output of the system, so it is necessary to consider both descriptions. Even though the bond graph contains all this information, a direct path could not be found to reach the proposed goal.

Based on this previous result, alternative ways were explored to reach the main objective. This lead us to the selection of the scattering matrix [3] as an intermediate step between the bond graph and the determination of the passivity. The reason for this choice is that the scattering matrix describes in one piece of information both the elements and its interconnection.

In the following, the relationship between passivity and the scattering formalism will be presented. For this, we consider a n-port where the average power P_j that goes into the port j is described by the expression [11]:

$$P_j = |w_{ij}|^2 - |w_{rj}|^2 \qquad (8)$$

where w_{ij} and w_{rj} are the incident and reflected wave variables respectively [20]. Then, the average power absorbed by the n-port is:

$$\sum_{j=1}^{n} P_j = \sum_{j=1}^{n} |w_{ij}|^2 - |w_{rj}|^2 = W_i{}^+ W_i - W_r{}^+ W_r \quad (9)$$

where W_i and W_r are the incident and reflected wave matrix respectibly.

It is known that:

$$W_r = S W_i \quad (10)$$

where S is the scattering matrix.

Relation 10 is substituted in equation (9), to obtain:

$$\sum_{j=1}^{n} P_j = W_i{}^+ (I - S^+ S) W_i \quad (11)$$

where I is the identity $n\,x\,n$ matrix and the superindex $^+$ denotes the complex conjugated transpose matrix.

Theorem 3.1 *Let S be the dispersion matrix of a n-port $\mathcal{N}$. The n-port $\mathcal{N}$ is passive, if and only if:*

$$I - S^+ S \geq 0 \quad (12)$$

where 0 is a $n\,x\,n$ null matrix. Furthermore, if the dispersion matrix is unitary, i.e:

$$S^+ S = I \quad (13)$$

then the n-port $\mathcal{N}$ is lossless.

Together with the previous result, we found that Kamel and Dauphin-Tanguy [14] and Amara and Scavarda [1] developed different methods to obtain the scattering matrix from the bond graph model. We choose Kamel and Dauphin-Tanguy's method because of its simplicity, it is base on identification of series and parallel structures (0 and 1 junctions with their related R,C, and I elements associated) and finally, wave matrices [20] are used to obtain the final scattering matrix of the interconnected structures.

The methoology we propose is applicable only to linear systems, since it is based on the study of the scattering matrix and can only be used to study the passive relationships between the effort and flow variables defined in the each of the ports of the n-port . A detailed description od the method can be found in [10]

4. EXAMPLES

The following two examples will show how the flatness and passivity of a system is determined from a bond graph. The system is shown in figure 1 and its corresponding bond graph is shown in figure 2. The numerical values of the parameters are the following: $R_2 = \frac{1}{3}\,\Omega$, $R_5 = \frac{4}{3}\,\Omega$, and $I_4 = \frac{4}{9}\,H$.

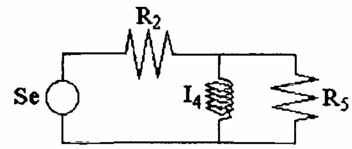

Figure 1: RL Circuit

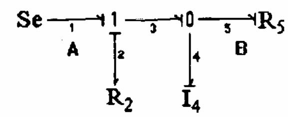

Figure 2: Bond Graph of RL Circuit

Flatness

In this case, the input vector S is defined as $S = [e_1,\ p_4]$, whereas the output vector L is defined as $L = [p_4]$.

Variable p_4, the only candidate to be flat output, is checked to see if it is the *real* flat output. We want to see if the variables of the input vector S (e_1 and p_4 can be expressed as a function of p_4. starting with e_1, a causal path is found between e_1 and p_4, 1–3–4. Following this path, and using the relations of the elements found, we obtain the following equations:

$$\begin{aligned}
e_1 &= e_2 + e_3 && (14)\\
e_1 &= R_2 f_2 + e_4 && (15)\\
e_1 &= R_2 f_3 + \dot{p}_4 && (16)\\
e_1 &= R_2 (f_4 + f_5) + \dot{p}_4 && (17)\\
e_1 &= \left(\frac{R_2}{I_4}\right) p_4 + \left(\frac{R_2}{R_5}\right) e_5 + \dot{p}_4 && (18)\\
e_1 &= \left(\frac{R_2}{I_4}\right) p_4 + \left(\frac{R_2}{R_5}\right) e_4 + \dot{p}_4 && (19)\\
e_1 &= \left(\frac{R_2}{I_4}\right) p_4 + \left(1 + \frac{R_2}{R_5}\right) \dot{p}_4 && (20)
\end{aligned}$$

We see that e_1 is already expressed as a function of p_4 and its first derivative.

The last variable of the input vector (p_4) is tested:

$$p_4 = p_4 \quad (21)$$

In this particular case, the procedure ends in the first step because the starting variable of the path is the candidate to flat output. Equations (20) and (21) show all the elements of vector S as a function of variable p_4 and its first derivative. Then, p_4 is the linearizing output of this system.

Passivity

The study of passivity of the system shown in figure 1 will be done through the input port, bond number 1. In this case we will determine if the system is passive with respect to the energy supplied through the effort source. Applying the chosen method [14], we obtain the following scattering parameter:

$$S(s) = \frac{1}{2}\frac{s-3}{2s+3} \quad (22)$$

As our system only has one port, the condition of passivity is reduced to verify that the module of the scattering parameter is bounded by unity for all frequency w:

$$|S(jw)| = \sqrt{\frac{1}{2}\frac{1}{4w^2+9}\sqrt{(2w^2-9)^1+81w^2}} \quad (23)$$

It can be determined that this module is always less than unity, so this system is passive. We could verify this result through the study of the real positiveness of the impedance function of the system [3]. This function is:

$$Z(s) = \frac{5s+3}{3s+9} \quad (24)$$

and it can be seen clearly that it is real positive.

5. CONCLUSION

This work shows the use of the bond graph to determine the controllability and the stability of a system through the properties of flatness and passivity respectively.

In the case of flatness, the graph was successfully used to determine this propriety. Causal paths between the flat variable and the state and input variables generate equations that express the latter as a function of the former and a finite number of its time derivatives.

In the case of passivity, the difficulty to find a straight way to determine it from the bond graph's topology obliged us to find an alternative through the use of the scattering matrix. Even though this is not a direct method, it provides a tool to reach the proposed goal.

The methods developed can be coded in a programming language.

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