ON THE SLIDING MODE CONTROL OF DISCRETE-TIME NONLINEAR SYSTEMS BY OUTPUT FEEDBACK

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Abstract

In this artilce we propose an output feedback control scheme for the robust stabilization, through sliding mode creation, of nonlinear discrete-time systems described in Fliess' Generalized Observability Canonical Form. The results are applied to the regulation of a discretized model of a nonlinear DC motor.

1 Introduction

Sliding regimes in continuous-time systems have been the object of intensive investigations. A complete account and update of the theory can be found in specialized books published over the years by authors like Utkin, [1], Zinober [2], and many others. Special issues of journals such as the *International Journal of Control*, the *IEEE Transactions on Industrial Electronics* as well as *Electrotehniški Vestnik* and *Pure and Applied Mathematics* (PUMA), have also appeared in recent times, with state of the art contributions and numerous references.

For discrete-time systems, sliding motions were first studied in the work of Miloslavjevic in [3] in the context of sampled data linear systems. Later on Sarpturk *et al* [4] devoted studies for various classes of discrete-time linear systems. An interesting contribution with useful generalities is that of Drakunov and Utkin [5]. Contributions to the saturated output feedback sliding mode control of linear systems were also given by Magaña and Żak in [6]. More recently, the problem has been treated by Furuta [7]. For nonlinear systems the article of Sira-Ramírez [8] critically evaluates the proposed definitions of quasi-sliding motions. The output feedback regulation problem, by means of sliding modes, has been treated by El Khazali and DeCarlo in [9] for the case of multivariable, linear, time invariant systems. A recent book by Emelyanov *et al*, [10], thoroughly deals with the sliding mode control of discrete and digital systems.

In this article we propose an output feedback control scheme for the robust stabilization of discrete-time nonlinear sytems of general form. Our proposed scheme uses a sliding mode control approach for the regulation of an auxiliary output function which induces a desirable asymptotically stable linear closed loop dynamics on its zero level set. The nonlinear systems are assumed to be placeable in the discrete-time counterpart of Fliess' Generalized Observability Canonical Form (see [11]) and utilizes a dynamical observer for the estimation of the generalized phase variables.

Section 2 contains the general results of the article which closely follows the developments found, for the continuous time case, in Teel and Praly [12] and Isidori [13]. Section 3 applies the general results to the output feedback regulation of a discretized nonlinear DC motor model. Section 4 contains the conclusions and suggestions for further research.

2 Output Feedback Sliding Mode Control of Nonlinear Discrete-Time Systems

In this section we propose an output feedback regulation scheme for discrete-time nonlinear systems based on sliding motions to the zero level set, of an auxiliary output function comprised of "generalized phase" variables, reconstructed through a nonlinear observer with predominantly linear reconstruction error dynamics. The approach closely parallels, where possible, that already found in Isidori [13] for the continuous time case. Consider the system

$$\begin{aligned} x(k+1) &= F(x(k),u(k)) \ ; \ x(k) \in R^n, u(k) \in R, \forall k \\ y(k) &= h(x(k)) \ ; \ y(k) \in R \end{aligned} \quad (1)$$

Consider the following sequence of maps

$$\begin{aligned} \psi_0 \ : \ R^n &\mapsto R \\ x &\mapsto \psi_0(x) \end{aligned} \quad (2)$$

and

$$\begin{aligned} \psi_k \quad &: \quad R^n \times R^k \quad \mapsto R^n \\ (x, v_0, v_1, \ldots, v_{k-1}) \quad &\mapsto \psi_k(x, v_0, v_1, \ldots, v_{k-1}) \\ & 1 \le k \le n \end{aligned} \tag{3}$$

Defined in the following way

$$\begin{aligned} \psi_0(x) &= h(x) \\ \psi_1(x, v_0) &= h(F(x, v_0)) =: h \circ F(x, v_0) \\ &\vdots \\ \psi_j(x, v_0, \ldots, v_{j-1}) &= h \circ F^j(x, v_0, v_1, \ldots, v_{j-1}) \end{aligned} \tag{4}$$

These mappings evidently express the dependence of the output y at time $k+j$ on the state $x(k)$ and the control input sequence $u(k), u(k+1), \ldots, u(k+j-1)$.

Indeed, the previous definitions imply that,

$$y(k+j) = \psi_j(x(k), u(k), u(k+1), \ldots, u(k+j-1)) \tag{5}$$

Define a mapping Φ by stacking all the previous defined maps from $j = 0$ to $n-1$

$$\begin{aligned} \Phi \quad &: \quad R^n \times R^{n-1} \quad \mapsto R^n \\ & (x, v) \quad \mapsto w = \Phi(x, v) \end{aligned} \tag{6}$$

in which

$$v = \begin{bmatrix} v_0 \\ v_1 \\ \cdots \\ v_{n-2} \end{bmatrix} \quad ; \quad \Phi(x, v) = \begin{bmatrix} \psi_0(x) \\ \psi_1(x, v_0) \\ \cdots \\ \psi_{n-1}(x, v_0, \ldots, v_{n-2}) \end{bmatrix} \tag{7}$$

By the previous construction, one evidently has

$$\begin{aligned} \Phi \quad : \quad & (x(k), u(k), u(k+1), \ldots, u(k+n-2)) \mapsto \\ & (y(k), y(k+1), \ldots, y(k+n-1)) \end{aligned} \tag{8}$$

We assume that for some $(x^0, v^0) \in R^n \times R^{n-1}$

$$\text{rank} \left(\frac{\partial \Phi}{\partial x} \right) (x^0, v^0) = n \tag{9}$$

By virtue of the Implicit Function theorem, this assumption implies that there exists a neighborhood $\mathcal{U}^0$ of x^0 in R^n, a neighborhood $W^0 \times V^0$ in $R^n \times R^{n-1}$ (where $w^0 = \Phi(x^0, v^0)$), and a unique mapping

$$\begin{aligned} \Psi \quad &: \quad W^0 \times V^0 \quad \mapsto \mathcal{U}^0 \\ & (w, v) \quad \mapsto x = \Psi(w, v) \end{aligned} \tag{10}$$

such that

$$w = \Phi(\Psi(w, v), v) \tag{11}$$

for all $(w, v) \in W^0 \times V^0$. It is then possible to conclude that if at some time k^0,

$$\begin{aligned} x^0 &= x(k^0) \quad \text{and} \\ v^0 &= \text{col}\,(u(k^0), u(k^0+1), \ldots, u(k^0+n-2)) \end{aligned} \tag{12}$$

are such that the rank condition (9) holds valid, then the mapping Ψ can be used to compute $x(k^0)$.

$$x(k^0) = \Psi(w(k^0), v(k^0)) \tag{13}$$

at time k^0, with

$$w(k^0) = \text{col}\,(y(k^0), \ldots, y(k^0+n-1))$$

$$v(k^0) = \text{col}\,(u(k^0), \ldots, u(k^0+n-2))$$

In order to be able to use the above mappings on any sequence of sampling instants other than the instant k^0, one must impose a stronger hypothesis on the previous "observability" condition of the system which is valid not only for time k^0, but for all times and states.

We say that the system (1) is *uniformly observable* if the following conditions are satisfied

1. the mapping

$$\begin{aligned} H \quad &: \quad R^n \times R^{n-1} \quad \mapsto R^n \\ & (x, v_0, v_1, \ldots, v_{n-2}) \quad \mapsto \\ \text{col}\,(h(x), & h \circ F(x, v_0), \ldots, h \circ F^{n-1}(x, v_0, v_1, \ldots, v_{n-2})) \end{aligned} \tag{14}$$

   is a global diffeomorphism of the state space, for all possible sequences, $\{v_0, v_1, \ldots, v_{n-2}\} \in \mathcal{U} \times \mathcal{U} \times \ldots \times \mathcal{U}$.

2. The rank condition:

$$\text{rank} \left(\frac{\partial \Phi}{\partial x} \right) (x, v) = n \tag{15}$$

   holds valid for each $(x, v) \in R^n \times R^{n-1}$.

If a system is uniformly observable, the mapping Ψ is globally defined.

Assuming the system is uniformly observable and considering the function $\psi_n(x, v_0, \ldots, v_{n-1})$. which, by construction, is such that

$$y(k+n) = \psi_n(x(k), u(k), \ldots, u(k+n-1)) \tag{16}$$

Using $\Psi(w, v)$, whose existence is attributed to the implicit function theorem, we define the system

$$\begin{aligned} w_0(k+1) &= w_1(k) \\ w_1(k+1) &= w_2(k) \\ &\vdots \\ w_{n-2}(k+1) &= w_{n-1}(k) \\ w_{n-1}(k+1) &= \psi_n(\Psi(w(k), v(k)), v_0(k), \ldots, v_{n-1}(k)) \end{aligned} \tag{17}$$

It should be clear that if

$$\begin{aligned} v_i(k) &= u(k+i) \quad \forall\, k \ge 0 \quad \text{and} \quad 0 \le i \le n-1 \\ w_i(0) &= y(i) \end{aligned} \tag{18}$$

then

$$w_i(k) = y(k+i) \quad \forall\, k \quad \text{and} \quad 1 \le i \le n-1 \tag{19}$$

In other words, if the initial state and the inputs of (17) are appropriately set, the various components of the state of this system reproduce the output of (1) and its next $n-1$ values.

Consider the system (17) and let $s(k)$ stand for the following auxiliary output, acting as a sliding surface coordinate,

$$s(k) = \alpha_0 w_0(k) + \alpha_1 w_1(k) + \ldots + \alpha_{n-2} w_{n-2}(k) + w_{n-1}(k) \tag{20}$$

with coefficients properly chosen so that the polynomial

$$\lambda^{n-1} + \alpha_{n-2}\lambda^{n-2} + \cdots + \alpha_0 \tag{21}$$

has all its roots inside the unit disk of the complex plane.

If we impose on $s(k)$, the dynamics

$$s(k+1) = \zeta(s(k)) \tag{22}$$

where the function ζ is given by (see figure 1)

$$\zeta(s) = \begin{cases} b \,\text{sign}\, s & \text{for } |s| \ge a+r \\ \frac{b}{r}(s-a) & \text{for } a < |s| < a+r \\ 0 & \text{for } |s| \le a \end{cases} \tag{23}$$

then, evidently, $s(k) \to 0$, in a finite number of steps depending only on the verification of the relation $a+r > b$ between a, r and b and irrespectively of the value of the inital condition $z(0)$.

The controller that drives $s(k)$ to zero is given by the following *implicit* dynamics

$$\psi_n(\Psi(w,v), v_0(k), \ldots, v_{n-1}(k)) = \zeta\left(\sum_{i=0}^{n-1} \alpha_i w_i(k)\right) - \sum_{i=0}^{n-2} \alpha_i w_{i+1}(k) \tag{24}$$

Under the assumption that $\partial\psi_n/\partial v_{n-1} \neq 0$, and by virtue of the implicit function theorem, a possible, locally valid, *explicit* expression for v_{n-1}, can be obtained from the previous relation, as

$$v_{n-1} = \Xi\left(\Psi(w(k), v(k)), v_0(k), \ldots, v_{n-2}(k)\right)$$

If we now let $v_j(k) = v_{j-1}(k+1)$ for $j = 0, \ldots, n-1$ and $v_0(k) = u(k)$ we obtain the following dynamical controller

$$\begin{aligned} v_0(k+1) &= v_1(k) \\ v_1(k+1) &= v_2(k) \\ &\vdots \\ v_{n-3}(k+1) &= v_{n-2}(k) \\ v_{n-2}(k+1) &= \\ &\Xi(\Psi(w(k), v(k)), v_0(k), v_1(k), \ldots, v_{n-2}(k)) \\ u(k) &= v_0(k) \end{aligned} \tag{25}$$

This controller, locally, asymptotically stabilizes the system (1) when written in the form

$$\begin{aligned} v_0(k+1) &= v_1(k) \\ v_1(k+1) &= v_2(k) \\ &\vdots \\ v_{n-3}(k+1) &= v_{n-2}(k) \\ v_{n-2}(k+1) &= \\ &\Xi(x(k), v_0(k), v_1(k), \ldots, v_{n-2}(k)) \\ u(k) &= v_0(k) \end{aligned} \tag{26}$$

This result, simply follows from the fact that the above controller imposes a linear, reduced order, closed loop dynamics on the generalized canonical form of the system (17) with $x = \Psi(w, v)$. Indeed, it is easy to see that when $s(k) = 0$ for all k, then the closed loop system is described by

$$\begin{aligned} w_0(k+1) &= w_1(k) \\ w_1(k+1) &= w_2(k) \\ &\vdots \\ w_{n-2}(k+1) &= -\alpha_0 w_0(k) - \alpha_1 w_1(k) - \cdots \\ &\quad -\alpha_{n-2} w_{n-2}(k) \end{aligned} \tag{27}$$

Of course, the above stabilization is possible, provided the generalized phase variables are properly set (or predicted) at the values:

$$w_i(k) = y(k+i) \quad \forall\, k \;\; ; \;\; \forall\, i$$

In order to guarantee that the states w of the system are eventually appropriately set and thus avoiding unnecessary predictions of the output variable, other than those arising from an initial state setting procedure, we propose a dynamical nonlinear observer of the form

$$\begin{bmatrix} \eta_0(k+1) \\ \eta_1(k+1) \\ \vdots \\ \eta_{n-2}(k+1) \\ \eta_{n-1}(k+1) \end{bmatrix} = \begin{bmatrix} \eta_1(k) \\ \eta_2(k) \\ \vdots \\ \eta_{n-1}(k) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \psi_n(\Psi(\eta(k), w(k)), v_0(k), \ldots, v_{n-1}(k)) \end{bmatrix} + \begin{bmatrix} Lc_{n-1} \\ L^2 c_{n-2} \\ \vdots \\ L^n c_0 \end{bmatrix} (y(k) - \eta_0(k)) \tag{28}$$

in which $|L| < 1$ is a constant to be determined and the constant coefficients $c_0, c_1, \ldots, c_{n-1}$ are chosen so that the polynomial

$$p(\lambda) = \lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_1\lambda + c_0$$

has all its roots in the interior of the unit disk in the complex plane.

One next defines the generalized phase variables estimation error vector as

$$\begin{aligned} e_0 &= L^{n-1}(\psi_0(x) - \eta_0) \\ e_i &= L^{n-i-1}(\psi_i(x, v_0, \ldots, v_{i-1}) - \eta_i) \quad ; \\ &\quad 1 \le i \le n-1 \end{aligned} \tag{29}$$

Letting also $\theta = \text{col}\,(\hat{x}, v_0, v_1, \ldots, v_{n-1})$, then the closed loop system with $\hat{x} = \Psi(\eta, v)$ is described by

$$\begin{aligned} \theta(k+1) &= \phi_1(\theta(k), e(k)) \\ e(k+1) &= LAe(k) + \phi_2(\theta(k), e(k)) \end{aligned} \tag{30}$$

with

$$A = \begin{bmatrix} -c_{n-1} & 1 & \cdots & 0 \\ -c_{n-2} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -c_1 & 0 & \cdots & 1 \\ -c_0 & 0 & \cdots & 0 \end{bmatrix}$$

1. If L is sufficiently small, all eigenvalues of the matrix

   $$LA + \left[\frac{\partial \phi_2}{\partial e}\right](0,0)$$

   are located inside the unit disk.

2. The function $\phi_2(\theta, e)$ is such that $\phi_2(\theta, 0) = 0$.

3. The function $\phi_1(\theta, e)$ is such that the equilibrium $\theta = 0$ of

   $$\theta(k+1) = \phi_1(\theta(k), 0)$$

   is globally asymptotically stable.

3 Applications to a DC Motor Example

Consider the following simplified (singularly perturbed) nonlinear model of a DC motor (see [13]),

$$\begin{aligned} \dot{x}_2 &= -\frac{R_r}{L_r}x_2 + \frac{V_r}{L_r} - \frac{KL_s}{L_r R_s}x_3 u \\ \dot{x}_3 &= -\frac{F}{J}x_3 + \frac{KL_s}{JR_s}x_2 u \\ y &= x_3 \end{aligned} \tag{31}$$

where the state variable x_3 represents the motor shaft angular velocity and x_2 is the rotor current. The control variable u is the stator voltage.

A first order discretization of the above model (31) leads to

$$\begin{aligned} x_2(k+1) &= a_{11}x_2(k) + a_{12}x_3(k)u(k) + a_{13} \\ x_3(k+1) &= a_{21}x_3(k) + a_{22}x_2(k)u(k) \\ y(k) &= x_3(k) \end{aligned} \tag{32}$$

where

$$a_{11} = (1 + \tau b_1) \ ; \ a_{12} = \tau b_3 \ ;$$
$$a_{13} = \tau b_2 \ ; \ a_{21} = (1 + \tau b_4) \ ; \ a_{22} = \tau b_5$$

with

$$b_1 = -\frac{R_r}{L_r} \ ; \ b_2 = \frac{V_r}{L_r} \ ; \ b_3 = -\frac{KL_s}{L_r R_s} \ ; \ b_4 = -\frac{F}{J} \ ;$$
$$b_5 = \frac{KL_s}{JR_s}$$

and $\tau = 0.01s$ is the sampling interval.

In order to transform the system into the discrete-time counterpart of *Fliess' Generalized Observability Canonical* form (see [11]), we take the following *input dependent* generalized phase error variables as states of the transformed system

$$\begin{aligned} w_0(k) &= \psi_0(x(k)) = x_3(k) - y^*(k) \\ w_1(k) &= \psi_1(x(k), u(k)) = a_{21}x_3(k) + \\ &\quad a_{22}x_2(k)u(k) - y^*(k+1) \end{aligned} \tag{33}$$

The corresponding inverse transformation is given by

$$\begin{aligned} x_2(k) &= \frac{w_1(k) - a_{21}(w_0(k) + y^*(k)) + y^*(k+1)}{a_{22}u(k)} \\ x_3(k) &= w_0(k) + y^*(k) \end{aligned} \tag{34}$$

The transformed system then reads

$$\begin{aligned} w_0(k+1) &= w_1(k) \\ w_1(k+1) &= \\ &\psi_2(w_0(k), w_1(k), u(k), u(k+1), \\ &\quad y^*(k), y^*(k+1), y^*(k+2)) \end{aligned} \tag{35}$$

with

$$\begin{aligned} &\psi_2(w_0(k), w_1(k), u(k), u(k+1), y^*(k), \\ &\qquad y^*(k+1), y^*(k+2)) = \\ &a_{21}(w_1(k) + y^*(k+1)) + a_{11}\frac{u(k+1)}{u(k)}[w_1(k) \\ &-a_{21}(w_0(k) + y^*(k)) + y^*(k+1)] \\ &+a_{22}a_{12}(w_0(k) + y^*(k))u(k)u(k+1) + \\ &a_{22}a_{13}u(k+1) - y^*(k+2) \end{aligned} \tag{36}$$

The sliding surface expression $s(k)$, in terms of the transformed coordinates, is defined as

$$s(k) = \alpha_0 w_0(k) + w_1(k) \tag{37}$$

We impose on the coordinate s, the following first order slding dynamics

$$s(k+1) = \zeta\Big(s(k)\Big)$$

with the function ζ as defined in (23) in the previous section.

From this last condition we can find an (explicit) expression for the dynamical feedback controller synthesizing the required input that drives s to zero in a finite number of steps.

$$u(k+1) = u(k)\frac{n(k)}{d(x)} \tag{38}$$

where

$$\begin{aligned} n(k) &= [\zeta(s(k)) - \alpha_1 w_1(k) - \\ &\quad a_{21}(w_1(k) + y^*(k+1)) + y^*(k+2)] \end{aligned}$$

$$\begin{aligned} d(k) &= a_{11}\left(w_1(k) + y^*(k+1) - a_{21}(w_0(k) + y*(k))\right) \\ &\quad + a_{22}a_{12}\left(w_0(k) + y^*(k)\right)u^2(k) + a_{22}a_{13}u(k) \end{aligned} \tag{39}$$

In order to implement the above controller, the generalized phase variable $w_1(k)$ has to be estimated, while $w_0(k) = x_3(k) - y^*(k)$ is assumed to be measurable. We could, thus, propose a *reduced order* observer for the estimation of $w_1(k)$ in the control law. However, in order not to unnecessarily cloud the develpments we consider, just for simplicity, a full order oberver of the form

$$\begin{aligned} \eta_0(k+1) &= \eta_1(k) + Lc_0\left(w_0(k) - \eta_0(k)\right) \\ \eta_1(k+1) &= \\ \psi_2(\eta_0(k), \eta_1(k), & u(k), u(k+1), y^*(k), \\ & y^*(k+1), y^*(k+2)) \\ & + L^2 c_1\left(w_0(k) - \eta_0(k)\right) \end{aligned} \tag{40}$$

with the function ψ_2 as given in (36), but with the new arguments replacing the original ones.

3.1 Simulation results

Simulations were carried out for the DC motor example with the following parameters with the following numerical values for the parameters

$$R_s = 10 \ ; \ R_r = 3 \ ; \ K = 1.5 \ ; \ F = 0.001$$

$$L_s = 0.1 \ ; \ \mathrm{L}_r = 0.1 \ ; \ J = 0.5 \ ; \ V_r = 150$$

As design parameters for the proposed controller, we took the corresponding numerical values as

$$\alpha_0 = -0.7 \ ; \ a = 2 \ ; \ r = 2 \ ; \ b = 1.5$$

while for the observer design, the parameters were set to

$$L = 0.3 \ ; \ c_0 = 0.2 \ ; \ c_1 = 0.9$$

The sampling interval was chosen to be $\tau = 0.01$ s.

As a reference signal, y^*, for the state variable x_3, we chose a constant value of 40 rad/s. Figure 2 shows the state responses of the closed loop output feedback controlled system. The trajectories are seen to converge to their corresponding equilibrium values. The figure also shows the behaviour of the control input variable $u(k)$ and the sliding surface coordinate evolution, $s(k)$, converging to zero in just two steps. The evolutions of the observation error vector components are also depicted in this figure.

4 Conclusions

In this article a general output feedback control scheme has been proposed for the sliding mode regulation and tracking of nonlinear discrete time systems of general form. The result parallels a recently introduced output feedback control scheme for the semi-global stabilization of nonlinear continuous time systems (see [12]). Aside from minor technical considerations the scheme can also be used in discrete-time cases, as shown here.

As an illustrative example, the case of an approximately discretized model of a nonlinear DC motor, was presented in the context of an angular velocity tracking problem. The simulation results of the dynamical output feedback sliding mode controller were highly encouraging.

5 Figures

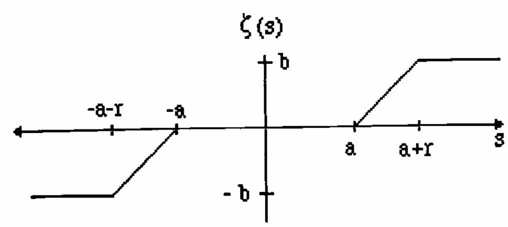

Figure 1: Function ζ

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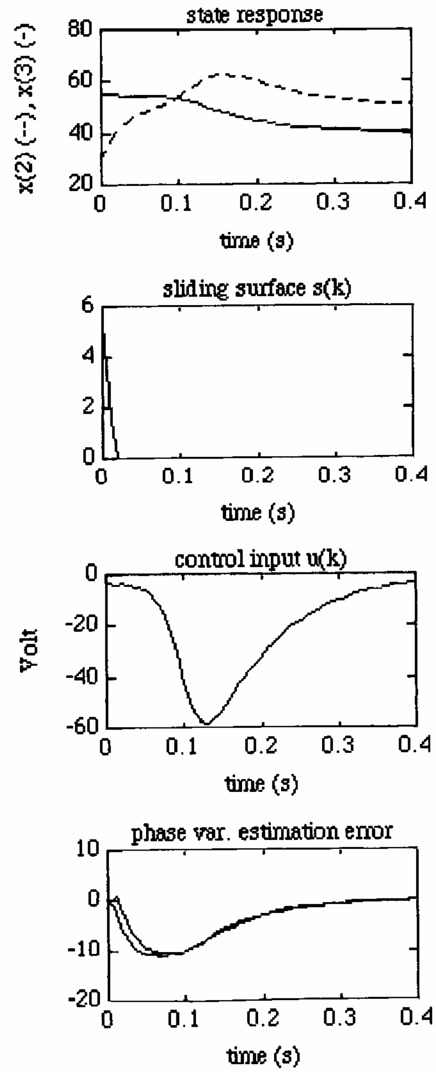

Figure 2: Sliding mode output feedback controlled responses of DC motor.

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