An experimental comparison of several non linear controllers for power converters

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Abstract - In this paper we present an experimental comparative study of five controllers for boost dc-to-dc converters recently reported in the control literature. They all enjoy some provable stability properties, which are briefly recalled in the paper. To carry out the experiments we constructed a low cost electronic card, which captures the essential features of a commercial product, but with all the sensors required to monitor the behaviour of the system. The algorithms are compared with respect to ease of implementation, in particular their sensitivity to the tuning parameters, and closed-loop performance. The latter is evaluated with the standard criteria of steady-state and transient behaviour, and disturbance attenuation. Mortavated by the experimental evidence we propose several modifications to the basic schemes, for some of them we establish some new theoretical results.

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I. INTRODUCTION

The regulation of switched power converters is a very active area of research both in power electronics and automatic control theory. A typical example of this kind of devices, that captures the essence of the problem, is the boost DC-DC converter. This circuit is described by a bilinear second order model with a binary input. The control task is further complicated by the fact that, with respect to the output to be regulated, the model is nonminimum phase.

This paper summarizes the results of an experimental comparison of nonlinear control algorithms on a DC-DC power converter of the boost type. We compare five algorithms, including the linear design, with respect to their ease of implementation and their closed-loop performance. For all these algorithms local asymptotic stability of the desired equilibrium is insured. The motivation of the present study is not to illustrate the validity of these theoretical results, but to test their performance when confronted with situations not predicted by the theory. The behaviour of the schemes is compared with the following basic criteria: transient and steady state response to steps and sinusoidal references, attenuation of disturbances in the power supply and sensitivity to unknown loads. Particular emphasis is placed throughout the paper on the flexibility provided by the tuning parameters to shape the responses. Eventhough this issue is not always appreciated in theoretical studies, we have found it of prime importance in experimentation.

The rest of the paper is organized as follows: in section 2 we introduce the model of the boost circuit. In section 3 we present the five differents strategies to be studied. Their adaptive versions, when applicable, are given in section 4. In section 5 we give a brief description of the experimental setup. In section 6 we present the experimental results. We finish this article with our conclusions in section 7.

II. SWITCH-REGULATED BOOST CONVERTER

A. Exact and averaged model

Throughout the paper we consider the switch-regulated "boost" converter circuit of figure 1.

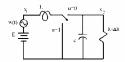


Fig. 1: Switch-regulated Boost circuit

The differential equations describing the circuit are given by

$$\dot{x}_1 = -(1-u)\frac{1}{L}x_2 + \frac{E+\omega}{L}
\dot{x}_2 = (1-u)\frac{1}{C}x_1 - \frac{1}{(R+\Delta R)C}x_2$$
(1)

where x_1 , x_2 are, respectively, the input inductor current and the output capacitor voltage variables; E>0 is the nominal constant value of the external voltage source and ω is an unknown (time-varying) disturbance, which satisfies $|\omega| < E$; R is the nominal constant value of the output resistance and ΔR reflects the parametric uncertainty; u, which takes values in the discrete set $\{0,1\}$, denotes the switch position function, and acts as a control input. The regulated output is x_2 which should be driven to some constant desired value $V_d > E$.

The control laws that we consider are classified into two groups depending on whether they generate directly the switching signal u, or they require an auxiliary Pulse Width Modulation (PWM) circuit to determine the switch position.

While the first class of controllers directly proceeds from the exact (hybrid) description (1), the control algorithms that use a PWM policy are designed based on an approximate (continuous-time) averaged model [8], which is given by

$$\dot{z}_1 = -(1-\mu) \frac{1}{L} z_2 + \frac{E+\omega}{L}
\dot{z}_2 = (1-\mu) \frac{1}{C} z_1 - \frac{1}{(R+\Delta R)C} z_2$$
(2)

where we denote by z_1 and z_2 the average input current and the average output capacitor voltage, respectively. As discussed in [8] this model accurately describes the behaviour of the converter provided the switching is sufficiently fast and the capacitor voltage is bounded away from zero, i.e., $x_2 \ge \epsilon > 0$. Notice that the only difference between the two models is that now μ is a continuous, and not a binary, signal.

A matrix representation of the averaged model (2) is,

$$\mathcal{D}\dot{z} - (1 - \mu)\mathcal{J}z + \mathcal{R}z = \mathcal{E} \tag{3}$$

where, $\mathcal{D} = diag\{L, C\}$, $\mathcal{R} = diag\{0, \frac{1}{R + \Delta R}\}$, $\mathcal{E} = [E + \omega, 0]^T$ and

$$\mathcal{J} = \left[\begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right]$$

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B. Control properties

Control of the boost converter is a challenging problem because, besides being a bilinear system, -with a binary input in its exact description, or a saturated one in the averaged model-, it is also a non-minimum phase system with respect to z_2 [8]. The existence of unstable zero dynamics puts a hard constraint on the achievable performance [2], a fact which is well known in the power electronics community [4]. It also considerably complicates the task of designing a nonlinear controller, since must existing techniques rely (in one way or another) on stable invertibility of the plant.

Fortunately, the non-minimum phase obstacle can be overcomed noting that, with respect to the inductor current, it is a minimum-phase system, and that there is a one-to-one correspondence between the output voltage and the inductor current equilibria. The voltage is then indirectly controlled via regulation of the current. An important drawback of this approach, that was observed in the experiments, is the high sensitivity to the circuit parameters, in particular to the load resistance. Another fundamental property of the system that is exploited by some of the controllers is that (as expected) the circuit defines a passive mapping from the input voltage E to the inductor current x_1 . Roughly speaking, this feature is used in passivity-based schemes to design a controller that preserves passivity in closed-loop.

III. CONTROL LAWS

As explained above they are divided in continuous and switched laws depending on whether they use or not an auxiliary PWM circuit to generate the control signal. This means also that for the control design, they use the continuous (averaged) or the switched (exact) model. As we will se, in the absence of external disturbances and parameter uncertainty, i.e., when $\omega \equiv 0, \Delta R \equiv 0$, they all achieve (local) asymptotic stabilization.

A. Continuous control laws

Linear Averaged Controller (LAC)

This controller is based on the linearization of the averaged model (2) around an equilibrium point (see [11] for more details). Given a desired output volage $\overline{z}_2 = V_d$ the corresponding \overline{z}_1 and $\overline{\mu}$ can be uniquely obtained as

$$\overline{z}_1 = \frac{V_d^2}{RE}$$
 ; $\overline{z}_2 = V_d$; $\overline{\mu} = 1 - \frac{E}{Vd}$

Defining the following error variables

$$\tilde{z}_1 = z_1 - \overline{z}_1$$
 ; $\tilde{z}_2 = z_2 - \overline{z}_2$; $\tilde{\mu} = \mu - \overline{\mu}$

we write the linearized model as

$$\dot{\tilde{z}} = \begin{bmatrix} 0 & -\frac{E}{V_q L} \\ \frac{E}{V_d C} & -\frac{RC}{RC} \end{bmatrix} \tilde{z} + \begin{bmatrix} \frac{V_d}{L} \\ \frac{V_d}{REC} \end{bmatrix} \tilde{\mu} = A\tilde{z} + B\tilde{\mu} \quad (4)$$

Some simple calculations show that the pair (A, B) is controllable. Hence, the poles of (A-BK) can be located arbitrarily with a suitable choice of the state feedback gains $K = [k_1 \ k_2]$. Feedback Linearizing Controller (FLC)

In [6] a nonlinear (static state feedback) controller that linearizes the input-output behaviour of the system, with output the circuit total energy, $H = \frac{1}{2}z^TDz$, was proposed as follows.

$$\mu = \frac{1}{\left(\frac{E}{L} + \frac{2}{RC}z_1\right)z_2} \left\{ \left(\frac{2}{R^2C} - \frac{a_1}{R} + \frac{a_2C}{2}\right)z_2^2 + \left(a_1E + \frac{a_2L}{2}z_1\right)z_1 + \frac{E^2}{L} - a_2H_d \right\}$$
 (5)

where $a_1, a_2 > 0$ are the design parameters, and

$$H_d := \frac{V_d^2}{2} (C + \frac{L}{R^2 E^2} V_d^2). \tag{6}$$

More precisely, it is shown in [6], that the converters total energy, H, evaluated along the trajectories of the closed-loop system (2), (5), (6) satisfies the linear equation

$$\ddot{H} + a_1 \dot{H} + a_2 H = a_2 H_d \tag{7}$$

Notice that H_d is the energy level required to ensure that as $H \to H_d$ we have $z_2 \to V_d$, as desired. Since the dynamics is now linear, the convergence rate can be fixed arbitrarily with a suitable choice of the controller parameters a_1 , a_2 . Passivity-Based Controller (PBC)

In [8] the following (nonlinear dynamic) controller that preserves passivity of the closed loop was proposed

$$\mu = -\frac{1}{z_{2d}} \left[E + R_1 \left(z_1 - \frac{V_d^2}{RE} \right) \right]$$
 (8)

where the controller dynamics is given by

$$\dot{z}_{2d} = -\frac{1}{RC} \left\{ z_{2d} - \frac{V_d^2}{E z_{2d}} \left[E + R_1 \left(z_1 - \frac{V_d^2}{RE} \right) \right] \right\}, \ z_{2d}(0) > 0$$
(9)

where $R_1 > 0$ is a design parameter.

The system (2) in closed loop with the controller (8), (9) is described by

$$\mathcal{D}\dot{\tilde{z}} - (1 - \mu)\mathcal{J}\tilde{z} + \mathcal{R}_d\tilde{z} = 0 \tag{10}$$

where $\tilde{z} = z - [\frac{V_d^2}{RE}, z_{2d}]^T$ and $\mathcal{R}_d = diag\{R_1, \frac{1}{R}\}$ Looking now at the quadratic function $V_d := \frac{1}{2}\tilde{z}^T\mathcal{D}\tilde{z}$, whose derivative satisfies

$$\dot{V}_d = -\tilde{z} \begin{bmatrix} R_1 & 0 \\ 0 & \frac{1}{R} \end{bmatrix} \tilde{z} \le -\alpha V_d, \quad \alpha := \frac{\min(R_1, \frac{1}{R})}{\max(L, C)} > 0$$
(11)

we see that R_1 injects the damping required for asymptotic stability, and that the convergence rate of \tilde{z} to zero is improved by pumping up R_1 .

B. Switched control laws

Sliding Mode Controller (SMC)

In [7] the following indirect sliding mode controller is proposed for the exact model (1)

$$u = 0.5 \left[1 - sgn(s) \right] = 0.5 \left[1 - sgn(x_1 - V_d^2/RE) \right]$$
 (12)

It can be shown that this switching policy locally creates a stable sliding regime on the line s=0 with ideal sliding dynamics characterized by

$$\overline{x}_1 = \frac{V_d^2}{RE} \; ; \; \dot{\overline{x}}_2 = -\frac{1}{RC} \left[\overline{x}_2 - \frac{V_d^2}{\overline{x}_2} \right] \; ; \; u_{eq} = 1 - \frac{E}{\overline{x}_2}$$
 (13)

Notice that, similarly to PBC, it is the open-loop time constant that regulates this dynamics. Furthermore, we will show in our experiments that this remarkably simple approach is, unfortunately, extremely sensitive to parameter uncertainty and noise. Finally, as usual with sliding mode strategies, the energy consumption is very high.

Sliding Mode plus Passivity Based Controller (SM+PBC)

To try to reduce the energy consumption in the latter scheme we proposed in [9] to combine sliding modes with passivity. The switching policy is now given by

$$u = 0.5[1 - sgn(s)] = 0.5[1 - sgn(x_{1d} - V_d^2/RE)]$$
 (14)

with the controller dynamics given by

$$\dot{x}_{1d} = -\frac{1}{L}(1-u)x_{2d} + \frac{R_1}{L}(x_1 - x_{1d}) + \frac{E}{L}$$

$$\dot{x}_{2d} = \frac{1}{C}(1-u)x_{1d} - \frac{1}{RC}x_{2d}$$
(15)

which is a copy of the open-loop dynamics to which we have added damping via R_1 .

For this new control policy we can also prove that it locally creates a sliding regime on the switching line s=0. Moreover, $(x_1,x_2) \rightarrow (x_{1d},x_{2d}) \rightarrow (V_d^2/RE,V_d)$.

Consistent with the statement above, our experiments will show that the as the initial conditions for the controller are chosen "closer" to the desired equilibrium state of the controlled plant, the state responses of the plant become smoother with slightly larger settling times but with a much better behaved transient shape. Unfortunately, it suffers from the same drawback as PBC of providing no freedom to shape the response of the output voltage.

IV. MODIFIED CONTROL LAWS

All the indirect control strategies presented above are clearly very sensitive to parameter uncertainty, in particular load resistance changes. To overcome this drawback we have tried two different modifications to the control laws, the simple addition of an integral loop around the voltage error, and the incorporation of an adaptation mechanism.

A. Adaptive schemes

Adaptive PBC

In [10] we proposed the following adaptive version of the PBC

$$\mu = 1 - \frac{1}{z_{2d}} \left[E + R_1 \left(z_1 - \hat{\theta} \frac{V_d^2}{E} \right) + L \frac{V_d^2}{E} \gamma z_{2d} (z_2 - z_{2d}) \right]$$
(16)

with controller auxiliar dynamics given by

$$\dot{z}_{2d} = -\frac{\hat{\theta}}{C} \left\{ z_{2d} - \frac{V_d^2}{z_{2d}E} \left[E + R_1 \left(z_1 - \hat{\theta} \frac{V_d^2}{E} \right) + L \frac{V_d^2}{E} \gamma z_{2d} (z_2 - z_{2d}) \right] \right\}$$
(17)

being the parameter $\hat{\theta}$ the estimate of $\frac{1}{R}$, which is updated with the following adaptive law

$$\hat{\hat{\theta}} = -\gamma z_{2d} (z_2 - z_{2d}) \tag{18}$$

 $\gamma>0$ is a designer chosen constant. It is shown in [10] that the system (2) in closed loop with the controller (16) has an equilibrium point given by, $(z_1,z_2,z_{2d},\hat{\theta})=(V_d^2/RE,V_d,V_d,1/R)$, which is asymptotically stable. Adaptive SMC

To add adaptation to the basic SMC we propose to modify the switching line as $s = x_1 - \hat{\theta} V_d^2 / E$, with the parameter $\hat{\theta}$ estimated by

$$\dot{\hat{\theta}} = -\gamma V_d \left(x_2 - V_d \right), \quad \gamma < \frac{E^2}{V_d^4 L}$$
 (19)

The adaptation law (19) was motivated by the form of the corresponding adaptive version for the case PBC where z_{2d} has been substituted by V_d .

Adaptive SM+PBC

An adaptive version for the SM+PBC can be similarly obtained considering the same switching line as above, but using the estimator

$$\hat{\theta} = -\gamma x_{2d} \left(x_2 - x_{2d} \right) \tag{20}$$

The controller auxiliary dynamics is modified accordingly to

$$\dot{x}_{1d} = -\frac{1}{L}(1-u)x_{2d} + \frac{R_1}{L}(x_1 - x_{1d}) + \frac{E}{L}$$

$$\dot{x}_{2d} = \frac{1}{C}(1-u)x_{1d} - \frac{\hat{\theta}}{C}x_{2d}$$
(21)

B. Adding an integral term

In all our experiments we have observed the presence of steady state errors in the output which is further accentuated when we perturb the load resistance. Experimentally we have seen that adding an integral term to the control law we could compensate this error or make it negligible, such an integral term can be computed in the following way

$$-K_i \int_0^t [x_2(s) - V_d] ds$$
; $K_i > 0$

Note that this term is continuous so we can add it only to the duty ratio $\mu(t)$ in the laws LAC, FLC and PBC.

V. EXPERIMENTAL CONFIGURATION

In fig. 2 we show the main card which is formed by a boost circuit, a pulse width modulation circuit (PWM), and some signal conditioners, receiving control signals from a D/A converter of a DSpace card placed in a PC. This design is very close to that of [3]. The values of the boost circuit elements

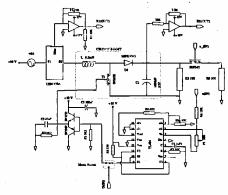


Fig. 2: Boost circuit card

are capacitor: $1000\mu F$, inductor: 170mH, resistor: $100~\Omega$ and power supply: 10~Volt. In this card we have the choice between controlling the boost circuit by means of a PWM generated signal or by directly introducing a switching signal coming from the DSpace card. Interesting features of the card are the possibility to connect or disconnect another resistive charge in parallel to the output and the possibility to introduce disurbance signals from a signal generator or even from the PC.

VI. EXPERIMENTAL RESULTS

Unless indicated otherwise, in all the experiments we consider as desired output voltage the value $V_d=20$ Volts. This corresponds to a desired inductor current $V_d^2/(RE)=0.4$ Amp. and to an equivalent duty ratio $\bar{\mu}=1-E/V_d=0.5$. Due to space limitations we present only the responses to step references and robustness to load uncertainty. Responses to sinusoidal references and results of disturbance attenuation can be found in the complete version of the paper.

A. Response to output voltage references

Step references

In fig. 3 we show the typical behaviour of the open loop system introducing a step in the duty ratio μ of 0.5. As we can see, the behaviour of the output voltage x_2 is quite good, it's fast and not to oscillatory, but the current in the inductance x_1 has a very large overshoot that exceeds the limit of current available in the power source for a considerable time. This behaviour is not desirable because it could trigger the security elements that would disable the power source. We also observe that there exists a small undershoot due to the non-minimum phase characteristic of the system. In fig 4 we show

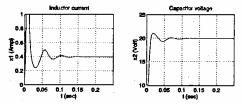


Fig. 3: Open loop step response

a family of step responses of the system under LAC for different locations of the closed-lop poles of the linearized system. As expected, for faster poles we obtained faster responses. However, due to the presence of nonlinearities, specially the saturation of the inductor current, the time responses do not correspond to the pole placement proposed. In particular, the response of x_2 is quite sluggish, and we can not obtain oscillatory responses that are predicted by the linear approximation theory. The former can be explained via a simple root locus analysis which reveals that for large k_1 one pole approaches -20 while the other goes to $-\infty$, so the time response is dominated by the slowest pole. We also observed that for relatively small gains a significant steady state error appears, while the undershoot amplitude increases for faster responses. In fig.

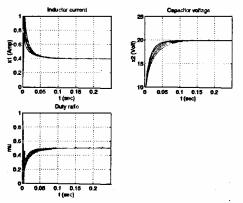


Fig. 4: Step responses for LAC

5 typical responses of the system under FLC for different values of a_1 and a_2 are presented. These corresponds to different pole locations of the closed-loop system described in the coordinates $[H, \dot{H}]$. Again, faster responses in x_2 are obtained with faster poles, which yields in it's turn higger peaks in x_1 , being the speed of convergence of x_2 limited by the saturation. In fig. 6 we show the responses of the PBC for different values of R_1 . From the plots we see that R_1 , which is the sole design parameter, affects only the behaviour of x_1 , while x_2 remains almost invariant. Actually what happens is that for small damping x_1 is very slow with a large overshoot, as we

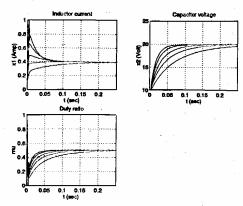


Fig. 5: Step responses for FLC

increase the damping x_1 converges faster—with no overshoot, finally at very large values it exhibits a fast peaking. As explained in section 3.1 damping determines the speed of convergence of x_2 towards z_{2d} , the wiggled responses in fig. 6 corresponding to small damping. However, z_{2d} remains essentially invariant to R_1 . In fig. 7 we present the response of the

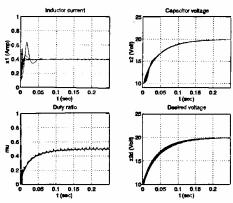


Fig. 6: Step responses for PBC

system with SMC. As we see, the sliding regime is reached almost instantaneously, thus x_1 reaches its desired value very fast. From the equations describing the sliding dynamics (13) we know that the response of x_2 is governed by the natural time constant $\frac{2}{RG}$, which makes slow this response. In fig. 8

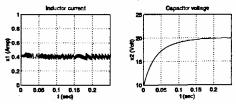


Fig. 7: Step response for SMC

we present the response of SM+PBC for different values of the design parameter R_1 . Again, as in PBC, x_2 and x_{2d} remain almost invariant. Only x_1 and x_{1d} change, both exhibit a peak during the transient that becomes higger for bigger R_1 . Our conclusion of this subsection is that only LAC and FLC provide some flexibility to shape the step response. Two important advantages of FLC over LAC is that it achieves the same convergence rates with smaller inductor currents—hence

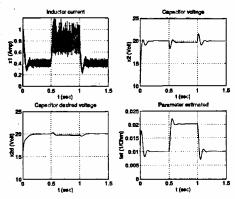


Fig. 13: Response to disturbance in the output resistance for adaptive SM+PBC

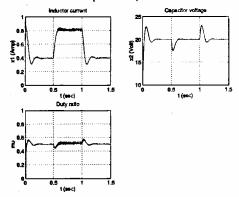


Fig. 14: Response to a disturbance in the output resistance for LAC + Integral term

- The main drawback of PBC, which is shared also by SMC and SM+PBC, is the inability to shape the output response, which evolves according to the open-loop dynamics. This, of course, stems from the fact that we cannot inject damping to the voltage subsystem without nonlinearity cancellation. On the other hand, PBC achieved a better disturbance attenuation, hence it may be a viable candidate for applications where rise time is not of prime concern. We should stress that, as shown in motor control, this is not a limitation intrinsic to passivity-based designs, rather it pertains to our ability to inject (pervasive) damping to the controlled variable.
- SMC and SM+PBC proved very robust to source disturbances but highly sensitive to parameter uncertainty. The latter could be alleviated incorporating a novel adaptation mechanism. The lack of flexibility of SMC is somehow alleviated in SM+PBC, at least to shape the disturbance attenuation characteristic. Unfortunately, both schemes suffer from the main drawback mentioned above.

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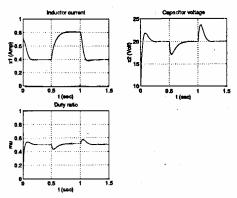


Fig. 15: Response to a disturbance in the output resistance for FLC + Integral term

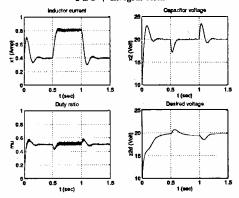


Fig. 16: Response to a disturbance in the output resistance for PBC + Integral term

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