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Regulation of Non-minimum Phase Outputs in a PVTOL Aircraft [1]

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Abstract

The regulation of the non-minimum phase outputs of a Planar Vertical Take-Off and Landing (PVTOL) aircraft, is approached using the differential parameterization provided by the system's flatness property. The parametrization allows to establish an open loop relationship between the regulated outputs equilibria and the flat outputs equilibria. An indirect stabilization of the aircraft's nonminimum phase center of gravity position coordinate outputs can then be achieved in terms of a corresponding trajectory tracking task for the differentially flat outputs, represented in this case by the coordinates of an equivalent Huygens center of oscillation. A suitable trajectory tracking task is then specified for the set of flat outputs which is solved by means of a dynamical linearizing state feedback controller.

1 Introduction

The regulation of nonminimum phase outputs represents an interesting problem which has received sustained attention in the past. Different regulation schemes are available from the existing literature. One of such schemes, which seems to be the most popular, was proposed in Benvenuti *et al* in [1]. The method consists in an indirect regulation scheme of the nonminimum phase output by regulating instead a judiciously chosen minimum phase output. This scheme has been extensively exploited in the output voltage stabilization of dc-to-dc power converters and also in the angular position stabilization of flexible joint robotic manipulators (see Sira-Ramírez and Lischinsky-Arenas [4], and Sira-Ramírez *et àl* [3]). A second approach consists in approximating the nonminimum phase system by means of a minimum phase system (see Hauser *et al* [2] and the references therein). A different scheme which is based on a piecewise dynamical unstable controller design, combined with the possibilities of controller state resetting, has also been proposed for nonminimum phase systems in the context of power electronics systems (see Llanes-Santiago and Sira-Ramírez [5]).

Differential flatness, introduced in recent articles [6]-[7], is a far reaching ***structural*** system property which can be related to many feedback controller desgin techniques (backstepping, passivity, dynamical feedback linearization, etc). Roughly speaking, a multivariable nonlinear system is ***flat*** if there exists a certain vector of independent functions, called the ***flat outputs***, of the same dimension as the vector of control inputs, which are ***differential functions*** of the ***state*** of the system (i.e., these outputs are a function of the state variables and also of a finite number of their time derivatives), with the additional property that, every system variable, i.e., states, original outputs and also the inputs, can, in turn, be expressed as differential functions of the flat outputs. Many systems in practise are differentially flat (See a recent tutorial on the subject [11]).

PVTOL aircraft systems have been the object of study by many researchers. The modeling aspects have been thoroughly treated in an industry report [8] from where most of the recent theoretical studies have derived. An exact linearization solution to the VTOL position transfer problem has been given by Hauser *et al* in [2] using an approximation of the nonminimum phase system by regarding it to be a ***slightly nonminimum phase*** system. The PVTOL aircraft dynamics have been shown to be ***differentially flat*** in the work of Martin [9]. The regulation aspects of the nonminimum phase outputs of the PVTOL aircraft system have been studied in Martin, Devasia and Paden [10] where flatness is exploited in a scheme using inverse trajectory feedforward

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in combination with a state tracker while guaranteeing a bounded zero dynamics (see also [11]).

In this article, we approach the regulation of the PVTOL aircraft center of gravity position coordinates, which are known to be nonminimum phase outputs ([2]), via an indirect feedback regulation scheme involving the flat outputs which are indeed minimum phase outputs. The flat outputs, as already shown in [10], of the PVTOL example are represented by the equivalent of the Huygens center of oscillation coordinates when the PVTOL aircraft dynamics is regarded as a virtual pendulum model. Our approach while also exploiting flatness differs from those presented in [10], and [11], in which we base our considerations on the differential parametrization naturally provided by the flatness property. This differential parametrization is sufficiently rich to establish not only important structural properties of flat systems (see [12]) but also, to establish the needed open loop static relationships between the nonminimum phase outputs equilibria and the flat outputs equilibria. These relationships allow us to define an alternative, but equivalent, equilibrium transfer problem for the PVTOL center of gravity coordinates in terms of the minimum phase flat output vector coordinates. The required dynamical feedback controller is then obtained by solving a suitable trajectory tracking problem, with linear tracking error dynamics, which is defined in terms of a planned trajectory linking the flat coordinates equilibria involved in the equilibrium position transfer task.

In section 2 we present the model of the aircraft and proceed to obtain the dynamical feedback controller which regulates a transfer of the nonminimum phase outputs between two given constant equilibrium points. Section 3 presents the simulation results and Section 4 is devoted to present some conclusions and suggestions for further research.

2 A Planar Vertical Take-Off and Landing Aircraft Stabilization Example

2.1 Description of the System

In Hauser et al [2] (see also [10]), the following model is proposed for the simplified description of the dynamics of a planar vertical take-off and landing (PVTOL) aircraft (see Figure 1)

$$\begin{aligned} \ddot{x} &= -u_1 \sin\theta + \epsilon u_2 \cos\theta \\ \ddot{z} &= u_1 \cos\theta + \epsilon u_2 \sin\theta - g \\ \ddot{\theta} &= u_2 \end{aligned} \tag{1}$$

where x and z are the horizontal and vertical coordinates of the center of gravity of the aircraft, respectively measured along an orthonormal set of fixed horizontal and vertical coordinates. The angle θ is the aircraft's longitudinal axis angular rotation as measured with respect to the fixed horizontal coordinate axis. The controls u_1 and u_2 represent normalized quantities related to the vertical thrust and the angular rolling torque applied around the longitudinal axis of the aircraft respectively. The constant g is the gravity acceleration and ϵ is a fixed constant related to the geometry of the aircraft.

The system outputs x and z are known to be nonminimum phase. Indeed, if x and z are held constant by means of a suitable control action, then, in particular, $\ddot{x} = 0$ and $\ddot{z} = 0$. Using the system dynamics (1) one readily obtains the required control inputs as

$$u_1 = g\cos\theta \quad ; \quad u_2 = \frac{g}{\epsilon}\sin\theta$$

The corresponding *zero dynamics* is then represented by the following autonomous differential equation for the angular position of the aircraft,

$$\ddot{\theta} = \frac{g}{\epsilon}\sin\theta \tag{2}$$

The dynamics (2) exhibits an unstable (saddle) equilibrium point at the origin $\theta = 0$, $\dot{\theta} = 0$ and a center around $\theta = \pi$, $\dot{\theta} = 0$. For initial conditions with zero angular velocity, the periodic nature of the solutions of 2, imply a "rocking" motion of the aircraft around its longitudinal axis. For zero initial conditions of the roll angle and nonzero initial angular velocity, the system (2) is unstable and hence, as time increases, the aircraft rotates about its longitudinal axis while its center of gravity remains fixed at a constant position in the x-z plane (see Figure 2).

2.2 A transfer problem for the non-minimum phase outputs

It is desired to transfer, in a finite amount of time $\Delta T > 0$, the aircraft position in the x-z plane, from a given fixed initial position, specified by a given set of constant horizontal and constant vertical coordinate values, $\overline{x}_{in}$ and $\overline{z}_{in}$, towards a second constant position represented by the set of coordinates $\overline{x}_f$ and $\overline{z}_f$ with the angular coordinate θ changing from an initial value $\overline{\theta}_{in} = 0$ towards a final value $\overline{\theta}_f = 0$. In [10] the same problem is solved by constructing a bounded trajectory for the internal dynamics, represented by the angular displacement θ, on the basis of the solutions of a sequence of linear ordinary differential equations with suitable initial conditions. This trajectory is in turn translated into a state space trajectory which is then tracked in a conventional manner.

2.3 A differential parametrization of the dynamics

It has been shown in [10] and also in [11] that the PVTOL model is differentially flat, with flat output given by the horizontal and vertical coordinates (F, L) of the *Huygens center of oscillation* when the aircraft dynamics is re-interpreted as the dynamics of a pendulum of length ϵ. Such outputs are given by,

$$F = x - \epsilon \sin\theta \quad ; \quad L = z + \epsilon\cos\theta \tag{3}$$

By considerations related to obtaining a singularity free structure at infinity of the position coordinate outputs which coincides with the structure at infinity of the tangent system as well as conditions for a well defined zero dynamics, it has been shown in [10] that the PVTOL aircraft system model requires a second order dynamical extension on the control input u_1. Instead of taking u_1 and $\dot{u}_1$ as additional state variables, the following auxiliary variable $\varsigma = u_1 - \epsilon\left(\dot{\theta}\right)^2$ is introduced as a new state variable. The following set of relations is then obtained,

$$\begin{array}{rcl} F &=& x - \epsilon\sin\theta \\ \dot{F} &=& \dot{x} - \epsilon\dot{\theta}\cos\theta \\ \ddot{F} &=& -\varsigma\sin\theta \\ F^{(3)} &=& -\dot{\varsigma}\sin\theta - \varsigma\dot{\theta}\cos\theta \\ F^{(4)} &=& -\ddot{\varsigma}\sin\theta - \dot{\varsigma}\dot{\theta}\cos\theta - u_2\varsigma\cos\theta \\ && -\dot{\theta}\dot{\varsigma}\cos\theta + \varsigma\left(\dot{\theta}\right)^2\sin\theta \end{array} \tag{4}$$

$$\begin{array}{rcl} L &=& z + \epsilon\cos\theta \\ \dot{L} &=& \dot{z} - \epsilon\dot{\theta}\sin\theta \\ \ddot{L} &=& \varsigma\cos\theta - g \\ L^{(3)} &=& \dot{\varsigma}\cos\theta - \varsigma\dot{\theta}\sin\theta \\ L^{(4)} &=& \ddot{\varsigma}\cos\theta - \dot{\varsigma}\dot{\theta}\sin\theta - u_2\varsigma\sin\theta \\ && -\dot{\theta}\dot{\varsigma}\sin\theta - \varsigma\left(\dot{\theta}\right)^2\cos\theta \end{array} \tag{5}$$

It can be shown, after some algebraic manipulations, that all the state variables in the system x,$\dot{x}$, z, $\dot{z}$, θ, $\dot{\theta}$, ς and $\dot{\varsigma}$, are expressible as *differential functions* of F and L, i.e.,

$$\begin{array}{rcl} x &=& F + \epsilon\dfrac{\ddot{F}}{\sqrt{\left(\ddot{F}\right)^2 + \left(\ddot{L}+g\right)^2}} \\ \dot{x} &=& \dot{F} + \epsilon\dfrac{F^{(3)}}{\left[\left(\ddot{F}\right)^2 + \left(\ddot{L}+g\right)^2\right]^{1/2}} \\ && -\dfrac{\ddot{F}\left[\ddot{F}F^{(3)} + \left(\ddot{L}+g\right)L^{(3)}\right]}{\left[\left(\ddot{F}\right)^2 + \left(\ddot{L}+g\right)^2\right]^{3/2}} \\ z &=& L - \epsilon\dfrac{\left(\ddot{L}+g\right)}{\sqrt{\left(\ddot{F}\right)^2 + \left(\ddot{L}+g\right)^2}} \\ \dot{z} &=& \dot{L} - \epsilon\dfrac{L^{(3)}}{\left[\left(\ddot{F}\right)^2 + \left(\ddot{L}+g\right)^2\right]^{1/2}} \\ && +\dfrac{\left(\ddot{L}+g\right)\left[\ddot{F}F^{(3)} + \left(\ddot{L}+g\right)L^{(3)}\right]}{\left[\left(\ddot{F}\right)^2 + \left(\ddot{L}+g\right)^2\right]^{3/2}} \\ \theta &=& \arctan\left(\dfrac{\ddot{F}}{\ddot{L}+g}\right) \\ \dot{\theta} &=& \dfrac{F^{(3)}\left(\ddot{L}+g\right) - \ddot{F}L^{(3)}}{\left(\ddot{F}\right)^2 + \left(\ddot{L}+g\right)^2} \\ \varsigma &=& -\dfrac{\left(\ddot{F}\right)^2}{\left(\ddot{F}\right)^2 + \left(\ddot{L}+g\right)^2} \\ \dot{\varsigma} &=& 2\dfrac{\ddot{F}F^{(3)}\left(\ddot{L}+g\right)^2 - \left(\ddot{F}\right)^2\left(\ddot{L}+g\right)L^{(3)}}{\left[\left(\ddot{F}\right)^2 + \left(\ddot{L}+g\right)^2\right]^2} \end{array} \tag{6}$$

Letting

$$F^{(4)} = v_1 \quad ; \quad L^{(4)} = v_2 \tag{7}$$

we obtain the following expressions for the original control inputs u_1 and u_2 as well as a second order differential equation describing the states $(\varsigma, \dot{\varsigma})$, corresponding to the second order extension of the control input variable u_1,

$$\begin{array}{rcl} u_1 &=& \varsigma + \epsilon\left(\dot{\theta}\right)^2 \\ u_2 &=& \dfrac{1}{\varsigma}\left(-v_1\cos\theta - v_2\sin\theta - 2\dot{\varsigma}\dot{\theta}\right) \\ \ddot{\varsigma} &=& -v_1\sin\theta + v_2\cos\theta + \varsigma\left(\dot{\theta}\right)^2 \end{array} \tag{8}$$

2.3.1 A static relationship between the equilibria: The differential parametrization (6) and the expressions (7) and (8) allow us to obtain a static *open loop* relationship linking the nonminimun phase planar position coordinates equilibria to the corresponding flat outputs equilibria. Indeed, letting $x = \overline{x} = constant$, $\dot{\overline{x}} = \ddot{\overline{x}} = \cdots = 0$ and $z = \overline{z} = constant$, $\dot{\overline{z}} = \ddot{\overline{z}} = \ldots = 0$ with $\theta = \overline{\theta} = \dot{\overline{\theta}} = \ldots = 0$ in the previous expressions, one readily obtains,

$$\begin{array}{rclcrcl} \overline{F} &=& \overline{x} && \overline{L} &=& \overline{z} + \epsilon \\ \dot{\overline{F}} &=& 0 && \dot{\overline{L}} &=& 0 \\ \ddot{\overline{F}} &=& 0 && \ddot{\overline{L}} &=& \overline{\varsigma} - g \\ F^{(3)} &=& 0 && L^{(3)} &=& \dot{\overline{\varsigma}} \\ F^{(4)} &=& -\overline{u}_2\overline{\varsigma} && L^{(4)} &=& \ddot{\overline{\varsigma}} \end{array} \tag{9}$$

Letting $\ddot{\overline{L}} = 0$ yields $\overline{\varsigma} = g$ and then $\dot{\overline{\varsigma}} = \ddot{\overline{\varsigma}} = 0$. In other words, $\overline{L}^{(3)} = \overline{L}^{(4)} = 0$. As a consequence, $\overline{v}_2 = 0$.

Letting, on the other hand, $F^{(4)} = 0$ one obtains $\overline{v}_1 = 0$ and $\overline{u}_2 = 0$. From the above expressions one also has, $\overline{u}_1 = \overline{\varsigma} = g$ and the equilibrium values of all system variables is, therefore, completely determined.

2.4 An indirect non-minimum phase output equilibria transfer in terms of a corresponding transfer of the flat outputs

The control objective of regulating the outputs x and z from given constant equilibrium values $x(T_1) = \overline{x}_{in}$ and $z(T_1) = \overline{z}_{in}$, towards a given second equilibrium value $x(T_2) = \overline{x}_f$ and $z(T_2) = \overline{z}_f$, in a prespecified amount of time $\Delta T = T_2 - T_1 > 0$, can be now translated into a corresponding transfer of the minimum phase flat outputs F and L from the initial equilibrium values $F(T_1) = \overline{F}_{in} = \overline{x}_{in}$ and $L(T_1) = \overline{L}_{in} = \overline{z}_{in} + \epsilon$ towards the final equilibrium value $F(T_2) = \overline{F}_f = \overline{x}_f$ and $L(T_2) = \overline{L}_f = \overline{z}_f + \epsilon$.

2.5 Trajectory planning

A set of open loop trajectories $F^*(t)$ and $L^*(t)$ for the flat outputs F and L, achieving a transfer between two equilibrium points $(\overline{F}_{in}, \overline{L}_{in})$ and $(\overline{F}_f, \overline{L}_f)$, may be specified in terms of suitable polynomial splines, as follows,

$$\begin{aligned} F^*(t) &= \overline{F}_{in} + \psi(t, T_1, T_2)(\overline{F}_f - \overline{F}_{in}) \\ L^*(t) &= \overline{L}_{in} + \psi(t, T_1, T_2)(\overline{L}_f - \overline{L}_{in}) \end{aligned} \tag{10}$$

where $\psi(t, t_0, T)$ is a polynomial function satisfying, $\psi(T_1, T_1, T_2) = 0$ and $\psi(T_2, T_1, T_2) = 1$ with a sufficient number of time derivatives being zero at time T_1 and at time T_2, thus guranteeing sufficiently smooth departures and arrivals. As an example $\psi(t, T_1, T_2)$ may be specified as,

$$\begin{aligned} \psi(t, T_1, T_2) = & \left[252\left(\frac{t-T_1}{\Delta T}\right)^5 - 1050\left(\frac{t-T_1}{\Delta T}\right)^6 \right. \\ & +1800\left(\frac{t-T_1}{\Delta T}\right)^7 - 1575\left(\frac{t-T_1}{\Delta T}\right)^8 \\ & \left. +700\left(\frac{t-T_1}{\Delta T}\right)^9 - 126\left(\frac{t-T_1}{\Delta T}\right)^{10}\right] \end{aligned} \tag{11}$$

This particular choice of trajectories for the flat outputs F and L, guarantees that at time T_1, the first four time derivatives of $F^*(t)$ and $L^*(t)$ are all zero, while at time T_2 the first five time derivatives of the planned flat outputs are also zero, thu s avoiding noticeable discontinuities in the dynamically generated control inputs u_1 and u_2 as well as on the auxiliary inputs v_1 and v_2.

2.6 A state feedback controller for the PVTOL aircraft system

One proceeds to impose on the flat output tracking errors $e_F(t) = F - F^*(t)$ and $e_L(t) = L - L^*(t)$ the following asymptotically stable behaviours,

$$\begin{aligned} e_F^{(4)}(t) + a_3 e_F^{(3)}(t) + a_2 \ddot{e}_F(t) + a_1 \dot{e}_F(t) + a_0 e_F(t) &= 0 \\ e_L^{(4)}(t) + b_3 e_L^{(3)}(t) + b_2 \ddot{e}_L(t) + b_1 \dot{e}_L(t) + b_0 e_L(t) &= 0 \end{aligned} \tag{12}$$

where the sets of coefficients $\{a_3, a_2, a_1, a_0\}$ and $\{b_3, b_2, b_1, b_0\}$ are chosen so that the corresponding polynomials in the complex variable s,

$$\begin{aligned} p_F(s) &= s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 \\ p_L(s) &= s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0 \end{aligned} \tag{13}$$

are both *Hurwitz* polynomials, i.e., with all their roots having strictly negative real parts.

The specification of the tracking errors dynamics (12) results in the following feedback controller explicitly based on the open loop specification of the flat outputs,

$$\begin{aligned} v_1 &= F^{(4)} = F^{*(4)}(t) + a_3\left(F^{(3)}(t) - F^{*(3)}(t)\right) \\ &\quad + a_2\left(\ddot{F}(t) - \ddot{F}^*(t)\right) + a_1\left(\dot{F}(t) - \dot{F}^*(t)\right) \\ &\quad + a_0\left(F(t) - F^*(t)\right) \\ &= F^{*(4)}(t) + a_3\left(-\dot{\varsigma}\sin\theta - \varsigma\dot{\theta}\cos\theta - F^{*(3)}(t)\right) \\ &\quad + a_2\left(-\varsigma\sin(\theta) - \ddot{F}^*(t)\right) \\ &\quad + a_1\left(\dot{x} - \epsilon\dot{\theta}\cos\theta - \dot{F}^*(t)\right) \\ &\quad + a_0\left(x - \epsilon\sin\theta - F^*(t)\right) \\ v_2 &= L^{(4)} = L^{*(4)}(t) + b_3\left(L^{(3)}(t) - L^{*(3)}(t)\right) \\ &\quad + b_2\left(\ddot{L}(t) - \ddot{L}^*(t)\right) + b_1\left(\dot{L}(t) - \dot{L}^*(t)\right) + \\ &\quad b_0\left(L(t) - L^*(t)\right) \\ &= L^{*(4)}(t) + b_3\left(\dot{\varsigma}\cos(\theta) - \varsigma\dot{\theta}\sin(\theta) - L^{*(3)}(t)\right) \\ &\quad + b_2\left(\varsigma\cos(\theta) - g - \ddot{L}^*(t)\right) \\ &\quad + b_1\left(\dot{z} + \epsilon\dot{\theta}\sin\theta - \dot{L}^*(t)\right) \\ &\quad + b_0\left(z + \epsilon\cos\theta - L^*(t)\right) \end{aligned} \tag{14}$$

3 Simulation results

Using the multivariable state feedback control scheme (14), a maneuver transfering the PVTOL aircraft center of mass outputs (x, z) from a given initial equilibrium position towards a prescribed second equilibrium position was performed. The initial equilibrium point was set at $(\overline{x}_{in}, \overline{z}_{in}) = (0, 0)$ while the second equilibrium position for the center of mass was set to be located at $(\overline{x}_f, \overline{z}_f) = (1, 1)$.

The maneuver was set to smoothly begin at $T_1 = 6$ time units, and it was prescribed to be completed at $T_2 = 14$

time units. The simulation results shown in Figure 3 correspond to the following set of system (normalized) parameter values,

$$\epsilon = 0.5 \;\; ; \;\; g = 1$$

The controller design parameters were chosen so that the polynomials $p_F(s)$ and $p_L(s)$ each had four roots located at the point $-2 + 0j$ in the real axis of the complex plane, i.e,

$$a_3 = 8 \;\; ; \;\; a_2 = 24 \;\; ; \;\; a_1 = 32 \;\; ; \;\; a_0 = 16$$

$$b_3 = 8 \;\; ; \;\; b_2 = 24 \;\; ; \;\; b_1 = 32 \;\; ; \;\; b_0 = 16$$

4 Conclusions

The regulation of nonminimum phase system outputs between prescribed constant equilibria is usually tackled by resorting to an indirect control scheme whereby a minimum phase output is commanded to stably converge towards an equilibrium point uniquely related (i.e., parameterized) to the corresponding required equilibrium of the nonminimum phase output.

The control scheme presented in this article relies on the differential flatness of the given system. The flat outputs are indeed nonminimum phase outputs in the sense that they exhibit no zero dynamics whatsoever. The differential parametrization of all system state, output and control variables in terms of the flat outputs usually provides the key element to be exploited regarding all relevant static aspects of the required equilibrium parameterization in terms of the corresponding equilibrium transfer problem.

Several other problems can also be explored using similar ideas to the ones presented here. Particularly important is the trajectory tracking problem for nonminimum phase systems. The problem of predictive control is closely related to such a program and, also, the gain scheduling problem can benefit from the fact that for differentially flat systems the differential parametrization contains all relevant information about the system.

References

[1] L. Benvenuti, M. D. di Benedetto, and J. W. Grizzle, "Approximate output tracking for nonlinear non-minimum phase systems with applications to flight control" Report CGR-92-20, Michigan Control Group Reports, University of Michigan, Ann Arbor Michigan, USA, 1992.

[2] J. S. Hauser, S. Sastry, and G. Meyer, "Nonlinear control design for slightly nonminimum phase systems: Application to v/stol aircraft" *Automatica*, Vol. 28, 1992, pp. 665-679.

[3] H. Sira-Ramírez, S. Ahmad, and M. Zribi, "Dynamical Feedback Control of Robotic Manipulators with Joint Flexibility," *IEEE Transactions on Systems Man and Cybernetics.* Vol. 22, No. 4, July/August 1992, pp. 736-747.

[4] H. Sira-Ramírez, and P. Lischinsky-Arenas, " The Differential Algebraic Approach in Nonlinear Dynamical Compensator Design for DC-to-DC Power Converters, " *International J. of Control*, Vol. 54, No. 1, July 1991, pp. 111-134.

[5] O. Llanes-Santiago, and H. Sira-Ramírez, "A Controller Resetting Strategy for the Stabilization of DC-to-DC Power Converters towards Nonminimum Phase Equilibria", *Proceedings of the 33d IEEE Conference on Decision and Control*, Lake Buena-Vista, Florida USA, December 1994. pp. 2920-2925.

[6] M. Fliess, J. Lévine, P. Martin and P. Rouchon, "Flatness and defect of nonlinear systems: introductory theory and examples," *Internat. J. Control*, Vol. 61, 1995, pp. 1327-1361.

[7] M. Fliess, J. Lévine, P. Martin and P. Rouchon, "Design of trajectory stabilizing feedback for driftless flat systems," *Proceedings of the 3rd European Control Conference*, pp. 1882-1887, Rome, 1995.

[8] McDonnell Aircraft Company, "YAV-8V Simulation and Modelling". McDonnell Douglas Corporation Report. 1982.

[9] Ph. Martin, "Contribution á l'étude des Systémes Différentiellement Plats" PhD thesis, Ecole des Mines de Paris, France.

[10] Ph. Martin, S. Devasia, and B. Paden, "A different look at output tracking: Control of a VTOL aircraft" *Automatica*, Vol. 32, No. 1, 1996, pp. 101-107.

[11] Ph. Martin, R. Murray, and P. Rouchon, "Flat systems" Plenary Lectures and Minicourses of the 1997 European Control Conference, Chapter 8, pp. 211-264, G. Bastin and M. Gevers (Eds.) 1997

[12] M. Fliess, H. Sira-Ramírez and R. Márquez, "Regulation of nonminimum-phase outputs: A flatness based approach" in *Perspectives in Control* D. Normand-Cyrot (Ed.), Springer-Verlag, 1998.

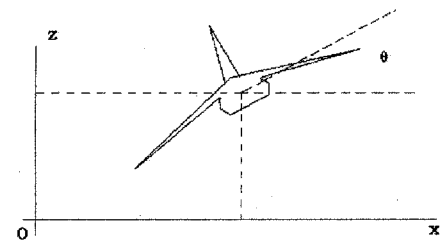

Figure 1: Planar Vertical Take-Off and Landing Aircraft System.

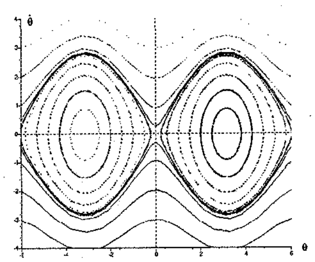

Figure 2: Unstable nature of the zero dynamics ($\epsilon = 0.5$, $g = 1$).

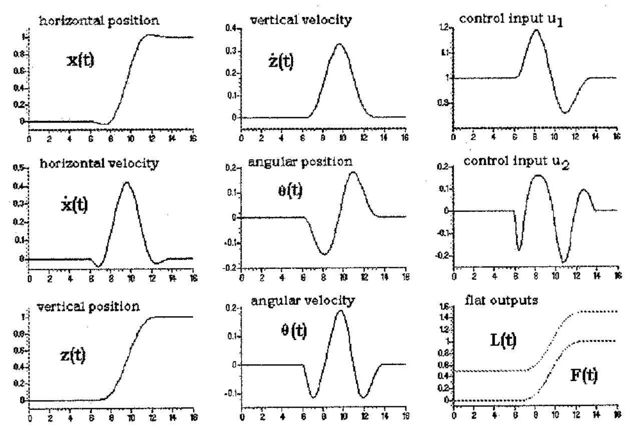

Figure 3: State variables, control inputs and flat outputs in a position transfer maneuver for a PVTOL example.