Flatness in the Passivity Based Control of DC–to–DC Power Converters [1]

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Abstract

In this article an advantageous combination of differential flatness and passivity based control is proposed for the feedback regulation of dc-to-dc power converters of the "boost" type. Controllers are designed for a single stage "boost" converter and also for a two-stage multivariable version of the "boost" converter. The results are illustrated by means of digital computer simulations.

Keywords : DC–to–DC power converters, passivity, differential flatness, trajectory planning.

1 Introduction

Dc-to-dc power supplies constitute an interesting and ubiqutous class of nonlinear switch regulated systems which are frequently controlled by means of pulse-width-modulation (PWM) or sliding mode control techniques (see the textbooks by Severns and Bloom [1], and by Kassakian *et al* [2]).

In [5], passivity based control of average models of dc-to-dc power converters have been shown to yield sufficiently simple, yet quite robust, dynamical feedback controllers. Adaptive versions of the proposed passivity based controllers have also been developed in [6] for unknown but constant resistive loads. Regulation of this class of converters is achieved in an *indirect* fashion due to the non-minimum phase character of the output voltage.

In [3] it has been shown that average models of dc-to-dc power converters are exactly linearizable by static state feedback. This means that the system enjoys a structural property addressed as *differential flatness* (see the articles by Fliess and his coworkers [7],[8]). Roughly speaking, a system is *flat* if there exists certain special outputs, called the *flat outputs*, equal in number to the inputs, which are functions of the state and a finite number of its time derivatives. Additionally, the flat outputs are such that every variable in the system (states, outputs, control inputs) can, in turn, be expressed as functions of the flat outputs and a finite number of their time derivatives. Differential flatness is a remarkable *structural property* of the system which allows to establish all the important properties of the system in connection with a particular feedback control design methodology (see the article by Fliess and Sira-Ramírez [9]). Differential flatness has been succesfully exploited in the systematic design of exactly linearizing feedback laws arising from trajectory planning problems in rather complex multivariable nonlinear systems.

In this article, we naturally combine the passivity based control of average models of dc-to-dc power converters with the differential flatness property of the considered PWM average model of the regulated converter system. Using differential flatness one establishes a desired trajectory for the minimum phase outputs in terms of corresponding open loop trajectories for the flat outputs. Differential flatness allows for the needed static relationship between the required output equilibrium transfer and the corresponding equilibrium points for the minimum phase outputs and the flat outputs. This relationship is basic in the trajectory planning aspects of the nonminimum phase outputs regulation problem. The resulting minimum phase output trajectories, placed in terms of the flat outputs trajectories, are then used in the derived passivity based dynamical feedback controller.

Section 2 presents a controller derivation based on passivity and flatness for the single stage "boost" converter while section 3 presents the corresponding developments for the regulation of the cascade, multivariable, connection of two "boost" converters. Section 4 contains the conclusions of the article.

[1] This research was supported by the Consejo Nacional de Investigaciones Científicas de Venezuela (CONICIT) under Research Grant S1-95-000886.

2 A Passivity–based controller for the "boost" converter

2.1 The average model of the "boost" converter

Consider the average PWM model of a "boost" converter circuit (see [10]), as shown in figure 1

$$\begin{aligned} \dot{z}_1 &= -\frac{1}{L}\mu z_2 + \frac{E}{L} \\ \dot{z}_2 &= \frac{1}{C}\mu z_1 - \frac{1}{RC}z_2 \end{aligned} \tag{2.1}$$

where z_1 represents the inductor current, z_2 is the capacitor voltage. The positive quantity E represents the constant value of the external source voltage. The variable μ denotes the ***duty ratio function*** taking values in the closed interval $[0,1]$ of the real line and acting as a control input variable. The control objective is to design a feedback control law for the duty ratio funtion μ such that the output voltage $y = z_2$ attains a desired constant reference level, denoted by V_d.

The system (2.1) is known to be non-minimum phase with respect to the regulated output y, while it is minimum phase for the input current z_1 taken as an output (see Sira–Ramírez and Lischinsky–Arenas [4]). For this reason an *indirect* feedback control strategy must be adopted in order to regulate a transfer of $y = z_2$ between two equilibrium values.

2.2 A transfer problem for the output voltage

It is desired to transfer the non–minimum phase output $y = z_2$ from a given equilibrium value at time $t_1 > 0$, given by $z_2(t_1) = V_{d1}$, towards a second equilibrium value $z_2(t_2) = V_{d2}$ in a finite amount of time $t_2 - t_1 = \Delta t > 0$. Such a control problem is usually solved by considering an *indirect* regulation problem on the basis of a minimum phase output represented by the inductor current z_1 (see [4]). Such a transfer requires to move the new output z_1 from the value $z_1(t_1) = V_{d1}^2/RE$ towards the value $z_1(t_2) = V_{d2}^2/RE$ in Δt units of time. Instead, we shall use a flat output, which is always minimum-phase, in order to carry out the indirect regulation.

2.3 Flatness of the "boost" converter

It has been shown in [3] that the average boost converter system (2.1) is feedback linearizable by means of static state feedback. The average circuit model is therefore differentially flat with flat output F given by the average stored energy of the circuit, expressed as:

$$F = \frac{1}{2}Lz_1^2 + \frac{1}{2}Cz_2^2 \tag{2.2}$$

The time derivate of the output F is obtained as

$$\dot{F} = Ez_1 - \frac{z_2^2}{R} \tag{2.3}$$

The flatness property of the system allows us, by use of (2.2) and (2.3), to express the state variables z_1, z_2 and the control input μ, as ***differential functions*** of the flat output F, as follows,

$$\begin{aligned} z_1 &= -\frac{ERC}{2L} + \left[\left(\frac{ERC}{2L}\right)^2 + \left(\frac{2F + RC\dot{F}}{L}\right)\right]^{\frac{1}{2}} \\ z_2 &= \left\{-R\dot{F} - \frac{R^2CE^2}{2L} + ER\left[\left(\frac{ERC}{2L}\right)^2 \right.\right. \\ &\quad \left.\left. + \left(\frac{2F + RC\dot{F}}{L}\right)\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \\ \mu &= \frac{LRC}{[2Lz_1(F,\dot{F}) + ERC]z_2(F,\dot{F})}\left[\frac{E^2}{L} \right. \\ &\quad \left. + \frac{2}{R^2C}z_2^2(F,\dot{F}) - \ddot{F}\right] \end{aligned} \tag{2.4}$$

where use has been made of the fact in the "amplifier" mode of the converter, the average inductor current z_1 and the output voltage z_2 are both positive quantities.

Note that all important properties of the system (2.1) are contained in such a differential parametrization (2.4).

2.4 An indirect output equilibrium transfer in terms of a flat output transfer

The control objetive of regulating the output voltage z_2 from the equilibrium value $z_2(t_1) = \bar{z}_2(t_1) = V_{d1}$ towards a given second equilibrium value $z_2(t_2) = \bar{z}_2(t_2) = V_{d2}$, in a prespecified amount of time $\Delta t = t_2 - t_1 > 0$, can be translated into a corresponding transfer of the minimum phase flat output F from the equilibrium value $F(t_1) = \bar{F}_1$ towards the equilibrium value $F(t_2) = \bar{F}_2$, where

$$\begin{aligned} F(t_1) &= \bar{F}_1 = \frac{1}{2}L\left(\frac{V_{d1}^2}{RE}\right)^2 + \frac{1}{2}CV_{d1}^2 \\ F(t_2) &= \bar{F}_2 = \frac{1}{2}L\left(\frac{V_{d2}^2}{RE}\right)^2 + \frac{1}{2}CV_{d2}^2 \end{aligned}$$

2.5 Trajectory planning

An open loop trajectory $F^*(t)$ for the flat output F, achieving a transfer between two equilibrium points $\bar{F}_1$ and $\bar{F}_2$, may be specified in terms of a suitable polynomial, as follows,

$$\begin{aligned} F^*(t) &= \bar{F}_1 + \left[21\left(\frac{t-t_1}{\Delta t}\right)^5 - 35\left(\frac{t-t_1}{\Delta t}\right)^6 \right. \\ &\quad \left. + 15\left(\frac{t-t_1}{\Delta t}\right)^7\right](\bar{F}_2 - \bar{F}_1) \end{aligned} \tag{2.5}$$

The proposed trajectory guarantees the following initial and terminal conditions for the flat output transfer,

$$
\begin{array}{ll}
F^*(t_1) = \bar{F}_1 & F^*(t_2) = \bar{F}_2 \\
\dot{F}^*(t_1) = 0 & \dot{F}^*(t_2) = 0 \\
\ddot{F}^*(t_1) = 0 & \ddot{F}^*(t_2) = 0 \\
\frac{d^3}{dt^3}F^*(t_1) = 0 & \\
\frac{d^4}{dt^4}F^*(t_1) = 0 &
\end{array}
$$

Thus, from the first equation in (2.4), we can specify a trajectory for the indirect output $z_1^*(t)$,

$$
z_1^*(t) = \begin{cases} V_{d1}^2/RE & \text{for} \quad t \le t_1 \\ \Phi & \text{for} \quad t_1 < t < t_2 \\ V_{d2}^2/RE & \text{for} \quad t \ge t_2 \end{cases} \tag{2.6}
$$

where

$$
\Phi = -\frac{ERC}{2L} + \left[\left(\frac{ERC}{2L} \right)^2 + \left(\frac{2F^*(t) + RC\dot{F}^*(t)}{L} \right) \right]^{\frac{1}{2}}
$$

2.6 Controller design for "boost" converter

A passivity–based controller can be proposed for system (2.1), as already done in [5]. We first propose an auxiliary damped system which represents a reference system to the original plant.

$$
\begin{aligned}
L\dot{z}_{1d} &= -\mu z_{2d} + E + R_1(z_1 - z_{1d}) \\
C\dot{z}_{2d} &= \mu z_{1d} - \frac{1}{R} z_{2d}
\end{aligned} \tag{2.7}
$$

where $R_1 > 0$ is a design parameter providing a suitable "damping injection" to the desired state dynamics (2.7). This system exhibits the same properties as the original plant regarding minimum phase outputs, relative degree and flatness.

Letting $z_{1d}(t) = z_1^*(t)$ defined in (2.6), one obtains the following dynamical controller expression, where $z_{2d}(t)$ has been substituted by the controller state variable ξ.

$$
\begin{aligned}
\dot{\xi} &= \frac{z_1^*(t)}{C\xi}\left[E - L\dot{z}_1^*(t) + R_1(z_1 - z_1^*(t))\right] - \frac{1}{RC}\xi \\
\mu &= \frac{1}{\xi}\left[E - L\dot{z}_1^*(t) + R_1(z_1 - z_1^*(t))\right]
\end{aligned} \tag{2.8}
$$

2.7 Simulation results

Using the state feedback control (2.8), two voltage transitions were performed. First, from arbitrary initial conditions, it was set as a control objetive the stabilization of the voltage z_2 around a constant equilibrium value $z_2 = V_{d1} = 30$ Volts. From the reached equilibrium, a second transfer, starting at time $t_1 = 0.05$ sec., was enforced to reach, at time $t_2 = 0.15$ sec., a new equilibrium value given by, $z_2(t_2) = V_{d2} = 60$ Volts. The simulations results shown in Figure 2 correspond to the following set of converter parameter values: $L = 20$ mH, $C = 20$ μF, $R = 30$ Ω, $E = 15$ Volts.

Figure 2 shows the closed loop response of the system (2.1), the controller state and the synthesized control input.

3 A Passivity–based controller for the "boost–boost" converter

3.1 The "boost-boost" converter circuit

Consider now a cascade connection of two "boost" DC–to–DC power converters. The average PWM model of the cascaded set of "boost" converters, shown in Figure 3, is given by,

$$
\begin{aligned}
\dot{z}_1 &= -\frac{1}{L_1}\mu_1 z_2 + \frac{E}{L_1} \\
\dot{z}_2 &= \frac{1}{C_1}\mu_1 z_1 - \frac{z_3}{C_1} \\
\dot{z}_3 &= -\frac{1}{L_2}\mu_2 z_4 + \frac{z_2}{L_2} \\
\dot{z}_4 &= \frac{1}{C_2}\mu_2 z_3 - \frac{z_4}{RC_2}
\end{aligned} \tag{3.1}
$$

where z_1 and z_3 represent the average inductor currents. The variables z_2 and z_4 are the average capacitor voltages, while $\mu_1 \in [0,1]$ and $\mu_2 \in [0,1]$ are the duty ratio functions associated with the PWM operation of the regulating switches.

3.2 A transfer problem for the output variables

It is desired to transfer, in a finite amount of time $\Delta t = t_1 - t_2 > 0$, the non–minimum phase outputs $y_1 = z_2$ and $y_2 = z_4$ from given equilibrium values at time $t_1 > 0$, given by $z_2(t_1) = V_{d1}$ and $z_4(t_1) = V_{d2}$, towards a second set of equilibrium values $z_2(t_2) = V_{d3}$ and $z_4(t_2) = V_{d4}$. Exactly as in the "boost" converter, the control problem is usually solved by considering an *indirect* regulation problem on the basis of minimum phase outputs represented by the inductor currents z_1 and z_3. Such a transfer requires to transfer the new outputs z_1 and z_3 from the values $z_1(t_1) = V_{d2}^2/RE$ and $z_3(t_1) = V_{d2}^2/RV_{d1}$, towards the values $z_1(t_2) = V_{d4}^2/RE$ and $z_3(t_2) = V_{d4}^2/RV_{d3}$, respectively.

3.3 Flatness of the "boost–boost" converter

The average state model (3.1) is differentially flat with flat outputs F_1 and F_2, associated with the stored energy in each converter:

$$
\begin{aligned}
F_1 &= \frac{1}{2}L_1 z_1^2 + \frac{1}{2}C_1 z_2^2 \\
F_2 &= \frac{1}{2}L_2 z_3^2 + \frac{1}{2}C_2 z_4^2
\end{aligned} \tag{3.2}
$$

The time derivate of the outputs F_1 and F_2 are given by

$$
\dot{F}_1 = Ez_1 - z_2 z_3
$$

$$\dot{F}_2 = z_2 z_3 - \frac{1}{R} z_4^2 \quad (3.3)$$

Using (3.2) and (3.3), the system can be expressed in terms of the state variables z_1, z_2, z_3, z_4 and the control inputs μ_1 and μ_2, as ***differential functions*** of the flat outputs F_1 and F_2, i.e.

$$z_i = \phi_i \left(F_1, \dot{F}_1, F_2, \dot{F}_2 \right) \quad ; \quad i = 1,2,3,4$$

These are difficult to compute ***explicitly*** but they can, nevertheless, be numerically evaluated from (3.2) and (3.3) for the purposes of computer simulations.

3.4 An indirect output equilibrium transfer in terms of a flat outputs transfer

The control objetive of regulating the outputs z_2 and z_4 from given constant equilibrium values $z_2(t_1) = \bar{z}_2(t_1) = V_{d1}$ and $z_4(t_1) = \bar{z}_4(t_1) = V_{d2}$, towards given second equilibrium values $z_2(t_2) = \bar{z}_2(t_2) = V_{d3}$ and $z_4(t_2) = \bar{z}_4(t_2) = V_{d4}$, in a prespecified amount of time $\Delta t = t_1 - t_2 > 0$, can be now translated into a corresponding transfer of the minimum phase flat outputs F_1 and F_2 from the initial values $F_1(t_1) = \bar{F}_1^1$ and $F_2(t_1) = \bar{F}_2^1$ towards the final equilibrium values $F_1(t_2) = \bar{F}_1^2$ and $F_2(t_2) = \bar{F}_2^2$.

$$\begin{aligned}
F_1(t_1) &= \bar{F}_1^1 = \frac{1}{2} L_1 \left(\frac{V_{d2}{}^2}{RE} \right)^2 + \frac{1}{2} C_1 V_{d2}^2 \\
F_2(t_1) &= \bar{F}_2^1 = \frac{1}{2} L_2 \left(\frac{V_{d2}^2}{RV_{d1}} \right)^2 + \frac{1}{2} C_2 V_{d2}^2 \\
F_1(t_2) &= \bar{F}_1^2 = \frac{1}{2} L_1 \left(\frac{V_{d4}^2}{RE} \right)^2 + \frac{1}{2} C_1 V_{d4}^2 \\
F_2(t_2) &= \bar{F}_2^2 = \frac{1}{2} L_2 \left(\frac{V_{d4}^2}{RV_{d3}} \right)^2 + \frac{1}{2} C_2 V_{d4}^2
\end{aligned}$$

3.5 Trajectory planning

A set of open loop trajectories $F_1^*(t)$ and $F_2^*(t)$ for the flat outputs F_1 and F_2, achieving a transfer between two equilibrium points $(\bar{F}_1^1, \bar{F}_2^1)$ and $(\bar{F}_1^2, \bar{F}_2^2)$, may be specified in terms of suitable polynomials, as follows,

$$\begin{aligned}
F_i^*(t) &= \bar{F}_i^1 + \left[21 \left(\frac{t - t_1}{\Delta t} \right)^5 - 35 \left(\frac{t - t_1}{\Delta t} \right)^6 \right. \\
&\quad \left. + 15 \left(\frac{t - t_1}{\Delta t} \right)^7 \right] (\bar{F}_i^2 - \bar{F}_i^1) \quad (3.4)
\end{aligned}$$

for $i = 1, 2$.

The proposed trajectory guarantees the following initial and terminal conditions for the flat output transfer,

$$\begin{array}{ll}
F_i^*(t_1) = \bar{F}_i^1 & F_i^*(t_2) = \bar{F}_i^2 \\
\dot{F}_i^*(t_1) = 0 & \dot{F}_i^*(t_2) = 0 \\
\ddot{F}_i^*(t_1) = 0 & \ddot{F}_i^*(t_2) = 0 \\
\frac{d^3}{dt^3} F_i^*(t_1) = 0 & \\
\frac{d^4}{dt^4} F_i^*(t_1) = 0 &
\end{array}$$

with $i = 1, 2$.

The trajectories of the outputs $z_1^*(t)$ and $z_3^*(t)$ are numerically obtained from (3.2) and (3.3) due to the difficulty in obtaining an explicit expression for these state variables in terms of the flat outputs. These are given by,

$$z_1^*(t) = \begin{cases} V_{d2}^2/RE & \text{for} \quad t \le t_1 \\ \Phi_1 & \text{for} \quad t_1 < t < t_2 \\ V_{d4}^2/RE & \text{for} \quad t \ge t_2 \end{cases} \quad (3.5)$$

$$z_3^*(t) = \begin{cases} V_{d2}^2/RV_{d1} & \text{for} \quad t \le t_1 \\ \Phi_3 & \text{for} \quad t_1 < t < t_2 \\ V_{d4}^2/RV_{d3} & \text{for} \quad t \ge t_2 \end{cases} \quad (3.6)$$

where $\Phi_i = \phi_i(F_1^*(t), \dot{F}_1^*(t), F_2^*(t), \dot{F}_2^*(t))$, $i = 1, 3$.

3.6 Controller design for the "boost–boost" converter

We rewrite the average system (3.1) in matrix form as follows,

$$\mathcal{D}_{BB}\dot{z} + \mathcal{J}_{BB}z + \mathcal{R}_{BB}z = \mathcal{E}_{BB} \quad (3.7)$$

where

$$\begin{aligned}
\mathcal{D}_{BB} &= diag\{L_1, C_1, L_2, C_2\} \\
\mathcal{J}_{BB} &= \begin{bmatrix} 0 & \mu_1 & 0 & 0 \\ -\mu_1 & 0 & 1 & 0 \\ 0 & -1 & 0 & \mu_2 \\ 0 & 0 & -\mu_2 & 0 \end{bmatrix} \\
\mathcal{R}_{BB} &= diag\{0, 0, 0, 1/R\} \\
\mathcal{E}_{BB}^T &= [E \;\; 0 \;\; 0 \;\; 0] \quad (3.8)
\end{aligned}$$

Note the matrices $\mathcal{J}_{BB}$ and $\mathcal{R}_{BB}$ satisfy: $\mathcal{J}_{BB} + \mathcal{J}_{BB}^T = 0$ and $\mathcal{R}_{BB} > 0$.

We use the ***modified storage function***

$$V_d(z, z_d) = \frac{1}{2}(z - z_d)^T \mathcal{D}_B(z - zd)$$

where z_d is an auxiliary state vector. We let the auxiliary vector z_d, satisfy the following system of the differential equations

$$\mathcal{D}_{BB}\dot{z}_d = -\mathcal{R}_{BB}z_d - \mathcal{J}_{BB}z_d + \mathcal{R}_{di}(z - z_d) + \mathcal{E}_B \quad (3.9)$$

where $-\mathcal{R}_{di}$ is a damping injection term. The time derivate of $V_d(z, z_d)$ satisfies

$$\dot{V}_d(z, z_d) = -(z - z_d)^T \mathcal{R}_m (z - z_d) \le -\frac{a}{b} V_d(z, z_d)$$

where $a = \max\{R_1, 1/R_2, R_3, 1/R\}$ and $b = \min\{L_1, C_1, L_2, C_2\}$.

It follows that the vector $z(t)$ exponentially asymptotically converges towards the prescribed auxiliary vector trajectory $z_d(t)$.

Thus, the auxiliary system (3.9) is explicitly written as

$$\begin{aligned} L_1\dot{z}_{1d} &= -\mu_1 z_{2d} + E + R_1(z_1 - z_{1d}) \\ C_1\dot{z}_{2d} &= \mu_1 z_{1d} - z_{3d} + \frac{1}{R_2}(z_2 - z_{2d}) \\ L_2\dot{z}_{3d} &= -\mu_2 z_{4d} + z_{2d} + R_3(z_3 - z_{3d}) \\ C_2\dot{z}_{4d} &= \mu_2 z_{3d} - \frac{1}{R} z_{4d} \end{aligned} \quad (3.10)$$

where $R_i > 0$, $i = 1, 2, 3$, are design parameters.

Letting $z_{1d}(t) = z_1^*(t)$ and $z_{3d}(t) = z_3^*(t)$, one obtains the following dynamical controller expression, where $z_{2d}(t)$ and $z_{4d}(t)$ have been substituted by the controller state variable ξ_1 and ξ_2, respectively.

$$\begin{aligned} \dot{\xi}_1 &= \frac{z_1^*(t)}{C_1\xi_1}[E - L_1\dot{z}_1^*(t) + R_1(z_1 - z_1^*(t))] - \frac{1}{C}z_3^*(t) \\ &\quad + \frac{1}{RC_2}(z_2 - \xi_1) \\ \dot{\xi}_2 &= \frac{z_3^*(t)}{C_2\xi_2}[\xi_1 - L_2\dot{z}_3^*(t) + R_3(z_3 - z_3^*(t))] - \frac{1}{RC_2}\xi_2 \\ \mu_1 &= \frac{1}{\xi_1}[E - L_1\dot{z}_1^*(t) + R_1(z_1 - z_1^*(t))] \\ \mu_2 &= \frac{1}{\xi_2}[\xi_1 - L_2\dot{z}_3^*(t) + R_3(z_3 - z_3^*(t))] \end{aligned} \quad (3.11)$$

3.7 Simulation results

Simulations were performed for the closed loop behaviour of a two stage cascaded average "boost" converter and the passivity based indirect feedback controller (3.11). Two voltage transitions were proposed. First, from arbitrary initial conditions, it was set as a control objetive the stabilization of the voltages z_2 and z_4 around constant equilibrium values $z_2 = V_{d1} =$ 30 Volts and $z_4 = V_{d2} = 60$ Volts, respectively. From the reached equilibrium, a second transfer, starting at time $t_1 = 0.05$ sec., the system was forced to reach, at time $t_2 = 0.15$ sec., a new set of equilibrium values given by, $z_2(t_2) = V_{d3} = 60$ Volts and $z_4(t_2) = V_{d4} = 120$ Volts, respectively. The simulation results, shown in Figure 4, correspond to the following set of converter parameter values: $L_1 = L_2 = 20$ mH, $C_1 = C_2 = 20$ μF, $R = 30$ Ω, $E = 15$ Volts.

Figure 4 shows the closed loop response of the system (3.1), the controller states and the synthesized control inputs.

4 Conclusions

In this article we have proposed a combination of passivity based control and differential flatness for the "stabilization by tracking" of a class of dc-to-dc power converters which are exactly linearizable by means of endogenous feedback. The procedure is based on exploiting differential flatness in order to plan a suitable trajectory which is in correspondance with the trajectory demanded by the designed passivity based feedback controller. Differential flatness guarantees that the explicit *differential* relation linking the flat output and the minimum phase controlled outputs is not only a relation between minimum phase outputs (thus largely avoiding instability problems associated to the direct regulation of a possibly nonminimum phase regulated output) but also, it is a relationship which links the controlled output to a privileged output naturally related to every other variable in the system.

This article should be considered as a first step in the development of a systematic procedure aimed at exploiting differential flatness in the design of passivity based feedback controllers. In fact, all fundamental features of the system variables which are essential in the design of passivity based controllers can be systematically determined from the differential parameterization characterizing flatness. For instance, the minimum or nonminimum phase character of particular variables, zero state detectability and all important static parametrized relations between the regulated outputs and minimum phase outputs can be directly determined from the differential parametrization allowed by flatness (See [9] for some of these issues in the context of regulation of non–minimum phase multivariable nonlinear systems).

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Acknowledgments The ideas presented in this paper were obtained through discussions with Prof. Michel Fliess of the Centre de Mathematiques et leurs Applications, Ecole Normale Superieure de Cachan (France). Prof. Fliess' insight was crucial in the development of the connections between passivity and flatness, and could not have been made possible without his inspiring remarks and enthusiasm. Discussions with Professors Jean Levine and Romeo Ortega of the Lobratoire des Signaux et Systèmes (CNRS), France, are also greatfully acknowledged.

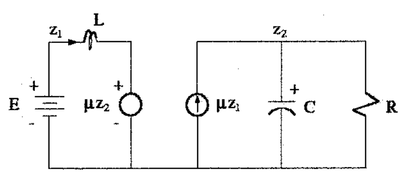

Figure 1: Equivalent circuit of the average PWM model of the "boost" converter circuit.

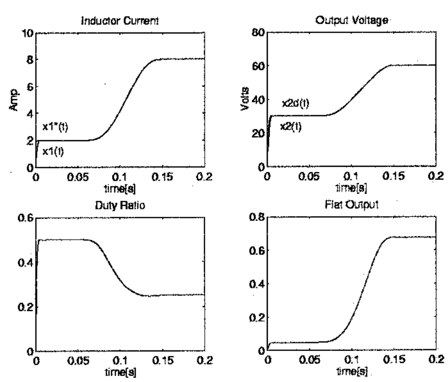

Figure 2: Simulation of state feedback controlled "boost" converter.

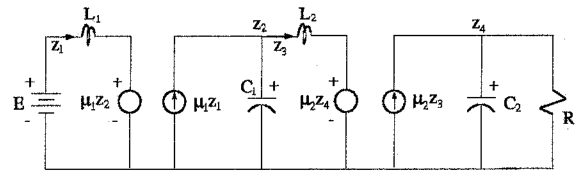

Figure 3: Equivalent circuit of the average PWM model of the "boost–boost" converter circuit.

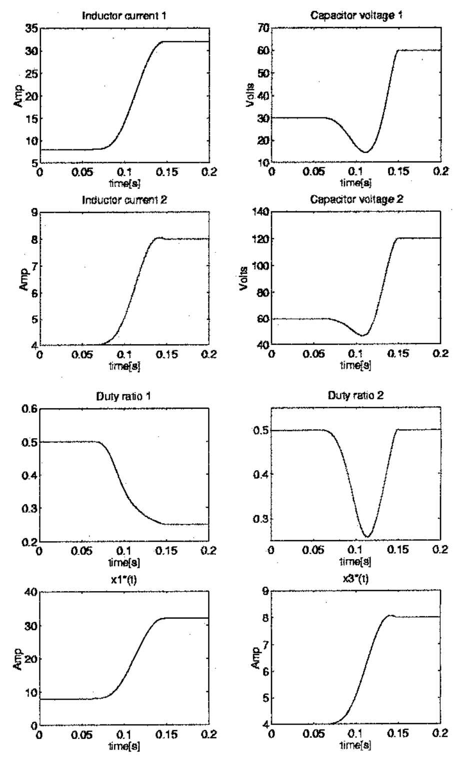

Figure 4: Simulation of state feedback controlled "boost–boost" converter.