On the Passivity Based Regulation of a Class of Delay Differential Systems [1]

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Abstract

In this article a general canonical form is proposed for a class of delay differential systems which explicitly identifies the conservative, the dissipative and the locally destabilizing forces in the uncontrolled delayed drift vector field. The proposed canonical form is helpful in the design of passivity based controllers. A design example is presented along with computer simulations.

Keywords : Passivity, Delayed systems, Nonlinear systems

1 Introduction

In this article we propose an extension of the canonical form derived in [2], for a class of nonlinear delay differential systems, which clearly exhibits the dissipative, the conservative and the destabilizing forces acting on the system dynamics. A simple input coordinate transformation which respects the beneficial nonlinearities is shown to make the system output passive and ready for a direct application of the "energy shapping plus damping injection" method.

2 Basic result

Consider the following delay differential system, with fixed delay τ, given by

$$\begin{aligned} \dot{x} &= f(x, x(t-\tau)) + g(x)u \\ y &= h(x) \end{aligned} \qquad (2.1)$$

where $x \in \mathcal{X} \subset R^n$, $u \in \mathcal{U} \subset R$, $\tau > 0$ and $y \in \mathcal{Y} \subset R^n$.

It is assumed that for a constant control input, $u = U$, the system exhibits a nonzero constant state equilibrium vector $\overline{x}$. The system is supposed to operate on an open region of the state space, $\mathcal{X}$, where the equilibrium points are found.

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Let $V(x)$ be a positive definite, C^1 storage function. $\partial V/\partial x$ is the gradient of V as a column vector. The *transversality condition*$L_g V(x) = \frac{\partial V}{\partial x^T} g(x) \neq 0 \quad \forall x \in \mathcal{X}$ is assumed to hold valid in all of $\mathcal{X}$, Notice that if the transversality condition is not satisfied, the following shapping of the storage function does satisfy the transversality condition, $W(x) = V(x) + \beta \left(L_f^{r-1} V(x) \right)^2 \quad ; \; \beta > 0$ where r is the *relative degree* of $V(x)$ with respect to the input u in all of $\mathcal{X}$.

Proposition 2.1 *Let $\tilde{x}$ denote the composite vector $(x^T(t), x^T(t-\tau))^T$. The system (2.1) can always be rewritten, after affine feedback of the form $u = \phi(\tilde{x}) + \theta(x)v$, as*

$$\begin{aligned} \dot{x} &= \mathcal{J}(\tilde{x}) \frac{\partial V}{\partial x} + \mathcal{S}_n(\tilde{x}) \frac{\partial V}{\partial x} + \gamma(x)v \\ y &= \gamma^T(x) \frac{\partial V}{\partial x} \end{aligned} \qquad (2.2)$$

where $\mathcal{J}(\tilde{x})$ is a skew symmetric matrix, and $\mathcal{S}_n(\tilde{x})$ is either a negative semidefinite or a negative definite matrix.

Proof The proof is based on a decomposition of the delayed vector field $f(\tilde{x})$ into two components. The first component of $f(\tilde{x})$ is represented by its *projection onto the tangent space, at x, of the constant level sets of $V(x)$, along the span of the control input vector field $g(x)$*. The second component is just the difference between the delayed vector field $f(\tilde{x})$ and the previously defined projection. The second component evidently belongs to the span of $g(x)$. (For further details see Sira-Ramírez and Angulo-Núñez [2]). Once the delayed drift vector field is decomposed, the locally unstable components of the resulting symmetric matrix $\mathcal{S}(\tilde{x})$ are *neutralized* by means of feedback.

3 A Feedback Controller Design Example

Consider the following second-order system

$$\begin{aligned} \dot{x}_1 &= x_2 - \frac{0.7x_1(t-\tau)}{\sqrt{1+x_1^2(t-\tau)}} \\ \dot{x}_2 &= 0.7x_2(t-\tau) - x_1 + u \\ y &= x_2 \end{aligned} \tag{3.1}$$

The regulated output $y = x_2$ is evidently a relative degree one variable which is also minimum phase, as it can be easily demonstrated.

Consider as the energy storage function $V(x)$ the quadratic form $V(x) = \frac{1}{2}\left(x_1^2 + x_2^2\right)$. The transversality condition, $L_gV(x) \neq 0$, results, in this case, in $L_gV(x) = x_2$. The region of singularity for the transversality condition is then represented by the x_1 axis in the x_1-x_2 plane. The required equilibrium transfers or the tracking of signals must then be such that the x_1 axis is never crossed. We assume that the operating region is constitued by the first quadrant in the x_1-x_2 plane.

The system may be rewritten in the form (2.2):

$$\dot{x} = \mathcal{J}(\bar{x})x + \mathcal{S}(\bar{x})x + g(x)u \tag{3.2}$$

where

$$\begin{aligned} \mathcal{J}(\bar{x}) &= \begin{bmatrix} 0 & \frac{1}{2} - \frac{0.35x_1(t-\tau)}{x_2\sqrt{1+x_1^2(t-\tau)}} \\ \frac{0.35x_1(t-\tau)}{x_2\sqrt{1+x_1^2(t-\tau)}} - \frac{1}{2} & 0 \end{bmatrix} \\ \mathcal{S}(\bar{x}) &= \begin{bmatrix} 0 & \frac{1}{2} - \frac{0.35x_1(t-\tau)}{x_2\sqrt{1+x_1^2(t-\tau)}} \\ \frac{1}{2} - \frac{0.35x_1(t-\tau)}{x_2\sqrt{1+x_1^2(t-\tau)}} & -\frac{x_1}{x_2} + \frac{0.7x_2(t-\tau)}{x_2} \end{bmatrix} \\ g(x) &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned}$$

Define the neutralizing state-dependent input coordinate transformation

$$u = \frac{0.7x_1(t-\tau)x_1}{x_2\sqrt{1+x_1^2(t-\tau)}} - 0.7x_2(t-\tau) + v \tag{3.3}$$

Transformation (3.3) results in a passive system operator relating the new input v and the output variable x_2.

3.1 Passivity-based feedback controller design

A dynamical passivity-based feedback controller may be obtained for the system by using the "energy shapping plus damping injection" design methodology (see [1].

$$\begin{aligned} \dot{\xi} &= \bar{x}_2 - \frac{0.7x_1(t-\tau)\bar{x}_2}{x_2\sqrt{1+x_1^2(t-\tau)}} + R_1\left(x_1 - \xi\right) \\ v &= \xi - \frac{0.7x_1(t-\tau)\xi}{x_2\sqrt{1+x_1^2(t-\tau)}} - R_2\left(x_2 - \bar{x}_2\right) \end{aligned}$$

The required equilibrium point for x_2 was set to be $\bar{x}_2 = 0.5$, this corresponds with the steady state values: $\bar{x}_1 = 1.02$ and $\bar{u} = 0.67$. The delay τ was supposed to be $\tau = 5$. The starting function for the delayed variables initial condition was set to be zero in the interval $[-5.0]$ (See Figure 1)

4 Conclusions

A canonical form has been proposed for a class of delay differential systems which clearly exhibits the conservative, the dissipative and the locally destabilizing forces of the drift vector field. The canonical form is useful in identifying the beneficial nonlinearities that should be respected in an input coordinate transformation geared to make the system feedback passive. The partially closed loop system is readily suitable for a direct application of the popular "energy shapping + damping injection" controller design methodology. A controller design example was presented for a two dimensional nonlinear delayed system, along with computer simulations.

References

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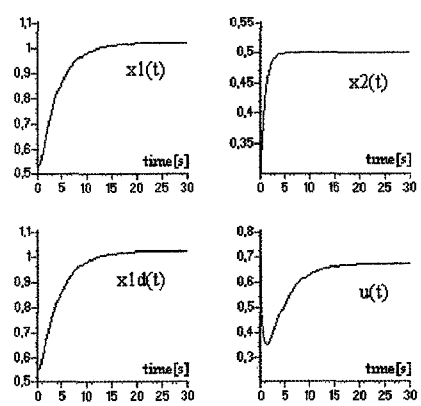

Figure 1: Simulation results of the passivity–based regulated nonlinear system with delay.