Proceedings of the 37th IEEE Conference on Decision & Control • Tampa, Florida USA • December 1998 TA15-3 10:40

# A Hybrid Passivity Based Controller Design for a Three Phase Voltage Sourced Reversible Boost Type Rectifier [1]

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**Abstract**

In this article, we present a criteria to design the switching control sequence for a nonlinear system in a normal form when the control input lives in a finite set. The criteria is based on the minimization of a Lyapunov function and the concept of *output regulated subspaces ORS* recently presented in [2]. Moreover, we propose to modify the controller by adding a passivity based algorithm in order to improve the transient behavior and reduce the control effort. The technique is applied to the well known *three phase Boost type rectifier* for which simulations results are provided in order to show the improvement in the transient response. Results are compared with the controller based in sliding mode approach reported in [7].

## 1 Introduction

In many processes the controllers are restricted to live in finite sets. This is the case in most of the circuits in the field of power electronics, where the control input is normally a binary signal to be introduced in the gate of a thyristor acting as a switch. Classical examples of these systems are the DC–DC converters, AC–DC converters better known as rectifiers, DC–AC converters more frequently referred as inverters, among others. The control solution for such systems is normally based in the concept of pulse with modulation PWM, even though also some controllers have been proposed that use the sliding mode approach.

In this paper we present a methodology to design the switching control sequence for systems in the *normal form* when the control input is restricted to live in a finite set. Vital in our approach is the concept of *equivalent control* $u_{eq}$ which is a continuous signal such that, applied to the system in steady state, restricts its trajectories to a desired manifold. since the control input is restricted to live in a finite set, we use the concept of *output regulated subspaces ORS* introduced in [2] and a Lyapunov based criteria in order to implement the switching control sequence. Moreover, *energy shaping plus damping injection ESDI* stages are added to the original controller which results in an improved transient response of the system with a decrease in the control effort. The closed loop system belongs to the class of *hybrid systems* since it contains signals of different nature, continuous and discontinuous, where the last is due to the fact that the controller lives in a finite set. The technique is applied to the well known *three phase boost based rectifier* and simulations results are provided that shown the advantages of our approach. By defining a reference signal tracking problem on the input currents of the converter, the power factor can be made very close to unity as long as the tracked signal is in phase with the rectified input voltages. The load voltage DC component is explicitly computed in steady state in order to approximately determine, via a partial system inversion, the required reference signal amplitude for the input currents.

This work was motivated by the seminal paper [6] where the problem of output voltage regulation in the three phase rectifier is formulated and some guidelines are given to implement a controller based on the definition of subspaces in the control input space. Later in [7] they presented an approach based on the concept of sliding modes and the computation of the equivalent control. They propose to divide the input space into four quadrants according to the different signs of the sliding surfaces. Then, they select the control vector that is contained in the *good* quadrant, i.e., the one that fulfills a sliding criteria. Special attention is given to the case where no vectors or more than one are contained in such quadrant, in such case, they propose to select the nearest control vector to the equivalent control.

## 2 Problem formulation

Consider an $n$-dimensional nonlinear system of the form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{2.1}$$

where $x \in \mathcal{X}$ denotes sufficiently large open subset of $\mathbb{R}^n$, $y \in \mathbb{R}^m$ and $u \in \mathcal{U} = \{U_1, U_2, ..., U_N\} \subset \mathbb{R}^m$, $N \in \mathbb{Z}$; $\mathcal{X}$ is addressed here as the *operating region* of the system that contains the system trajectories. The vector field $f(x)$ is such that $f(0) = 0$, i.e., the origin is a zero input equilibrium point for the system. The output map $h : \mathbb{R}^n \to \mathbb{R}^m$, satisfies $h(0) = 0$. We need the following assumptions.

**Assumption A.1** The output function $h(x)$ of system (2.1) has *vector relative degree* equals $\{1, 1, ...., 1\}$, i.e., the directional derivative, $L_g h(x)$, is bounded away from zero in the operating region of the system.

**Assumption A.2** The system is *input-to-state-stable (ISS)*. That is, given a locally $C^1$ function of time, $y^*(t)$,

[1] This research was supported by the Consejo Nacional de Ciencia y Tecnología de México (CONACYT)

the system is assumed to be *stable* when the regulated output $y(t)$ is locally tracking the signal $y^*(t)$. In other words, the associated *zero dynamics* of the system, consistent with the time varying constraint $y = y^*(t)$, is stable.

**Normal form.** If the system is relative degree one in $\mathcal{X}$ and the distribution span$\{g_1, g_2, ..., g_m\}$ is involutive around the equilibrium point then, there exists an invertible state coordinate transformation in $\mathcal{X}$ (see [3])

$$(z, y) = (T(x), h(x)) \tag{2.2}$$

which locally takes the system into the following *normal form* given by

$$\begin{aligned} \dot{z} &= q(z, y) \\ \dot{y} &= a(z, y) + b(z, y)u \end{aligned} \tag{2.3}$$

where $z \in \mathbb{R}^{n-m}$, $a(z, y) = L_f h(z, y)$ and $b(z, y) = L_g h(z, y)$ is a full rank matrix in $\mathcal{X}$ called the *decoupling matrix* [3]. Note that from the assumptions on $f(x)$ and the diffeomorphic nature of the transformation, it follows that $q(0, 0) = 0$ and $a(0, 0) = 0$. Also, the ISS assumption implies that the system $\dot{z} = q(z, y^*(t))$ is a stable system.

**Assumption A.3** The system (2.3) can be passified with respect to some $C^1$ positive definite storage function $V(z, y)$ by specifying an appropriate *regular static state feedback* $u = \alpha(z, y) + \beta(z, y)v$. Thus, without loss of generality, we assume that system systgral and therefore systnormal are two different versions of an already passive system.

**Assumption A.4** The function $V(z, y)$ can always be chosen to be zero at the origin of coordinates $(z, y) = (0, 0)$ moreover its gradient components $\partial V/\partial z$ and $\partial V/\partial y$ of the storage function only vanish identically at the origin $(z, y) = (0, 0)$.

**Control objective.** Consider a $m$-dimensional time varying reference signal $y^*(t)$, and define the tracking error signal as $e = y(t) - y^*(t)$. Then the control objective consists of designing the switching sequence of control vectors $u \in \mathcal{U}$ such that the error $e \to 0$ as $t \to \infty$, i.e., tracking is ensured (as close as possible) maintaining all the internal signals bounded.

## 3 Output regulation subspaces

In this section we recall the *output regulation subspaces ORS* as they were defined in [2], i.e., the subspaces of the input space $\mathbb{R}^m$ where each $\dot{y}_i = 0$. They are defined by

$$\text{ORS}(\dot{y}_i) = \left\{ u \triangleq -\frac{a_i(z, y)}{|b_i(z, y)|^2} b_i(z, y) + c_i \mathcal{J} b_i(z, y) \right\}$$

where $\mathcal{J} + \mathcal{J}^T = 0$, $c_i \in \mathbb{R}$ and $i = 1, 2, ..., m$.

The *ORS* define hyperplanes with the characteristic that points "above" the $i$-th hyperplane yield $\dot{y}_i > 0$ while for those "below" we have $\dot{y}_i < 0$. See Fig. 1. Moreover, the slope of those hyperplanes is given by the rows of the matrix $b_i(z, y)$ and the intersection point is given by $\frac{a_i(z,y)}{|b_i(z,y)|^2} b_i(z, y)$.

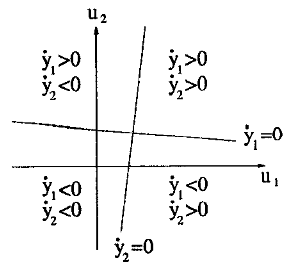


**Figure 1:** Output regulation subspaces in $\mathbb{R}^2$

From relative degree assumption **A.1** we have that the *OSR* will not be parallel, thus inducing a partition of the input space in function of the signs of $\dot{y}_i$. This gives us the capacity to increase or decrease a particular output by selecting a control vector above (below) the corresponding *ORS*.

## 4 Basic controller design

In this section the basic controller hybrid strategy is presented. Consider the error model as given by equations

$$\begin{aligned} \dot{z} &= q(z, y) \\ \dot{e} &= a(z, y) - \dot{y}^* + b(z, y)u \end{aligned} \tag{4.4}$$

The method proceeds to solve for the feedback control $u$ from the last equation. This feedback controller is referred as the *equivalent control*

$$u_{eq} = -b^{-1}(z, y)(a(z, y) - \dot{y}^*) \tag{4.5}$$

the controller so obtained represents a *virtual feedback control action* that, in the absence of perturbations and modeling errors, ideally keeps the system responses evolving on a manifold represented by the condition $\dot{e} = 0$.

Nevertheless for the class of systems studied here, the controller is only allowed to take values in a discrete set $\mathcal{U} = \{U_1, U_2, ..., U_N\}$. So efforts will be done in order to obtain a discontinuous feedback control solution of the tracking problem proposed on the original system.

Let us propose the (partial) Lyapunov function $V = \frac{1}{2}e^2$. Its time derivative along the trajectories of (4.4) can be expressed in terms of $u_{eq}$ as

$$\dot{V} = e^T \dot{e} = e^T b(z, y)(u - u_{eq}) \tag{4.6}$$

So now the problem is translated into the selection of a vector $u \in \mathcal{U}$ in such a way that $\dot{V}$ is render negative definite.

In the case of the tracking problem the *ORS* can be reinterpreted in the following way. The *ORS* is now defined for $\dot{e}_i$, i.e., $OSR(\dot{e}_i)$. Moreover the *ORS* can be rewritten in terms of the *equivalent control* as follows

$$\text{ORS}(\dot{e}_i) = \{u = u_{eq} + c_i \mathcal{J} b_i(z, y)\} \tag{4.7}$$

Fig. 2 shows graphically this reinterpretation of *ORS* for the tracking problem, considering the *equivalent control* concept.

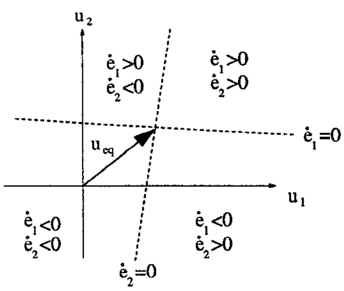


**Figure 2:** Output regulation subspaces for $\dot{e}$ in $\mathbb{R}^2$

**Proposed strategy.** We propose to select the control vector according to the following three steps strategy.

**S.1** We select the control vector $u$ that makes negatives simultaneously all the products $e_i\dot{e}_i$, $(i = 1, ..., m)$, that is,

$$u = U_i, i = arg\{(e_1\dot{e}_1 < 0) \cap (e_2\dot{e}_2 < 0) \cap ... \cap (e_m\dot{e}_m < 0)\}$$

**S.2** If $l$ $(1 < l \leq N)$ control vectors fulfill this condition,, then we select among them, the nearest $U_i$ to $u_{eq}$, that is,

$$u = U_i \left\{ i = \arg \min_{k\in\{1,\ldots,l\}} |(U_k - u_{eq})| \right\}$$

**S.3** Finally, if no vector fulfills the condition, then we select the one that minimizes $\dot{V}$, that is,

$$u = U_i \left\{ i = \arg \min_{k\in\{1,\ldots,N\}} \dot{V}(x, t, U_k) \right\}$$

with this at least one of the errors $e_i$ is being minimized if the equivalent control doesn't leave the convex hull formed by the control vectors in the finite set $\mathcal{U} = \{U_1, U_2, ..., U_N\}$. Moreover, if the equivalent controller doesn't escape from the convex hull then there is always a switching sequence that makes $\dot{V} < 0$. Moreover the manifold $e = 0$ is reached in a finite time, and under assumption of fast switching, the trajectories stay all the time in that manifold.

## 5 The energy shaping plus damping injection method

In order to improve the transient response of the system, and to try to reduce the control effort (at least in an average way), we propose to add the *ESDI* technique to the basic controller proposed above.

$$\begin{aligned} \dot{z} &= q(z,y) \\ \dot{y} &= a(z,y) + b(z,y)u \end{aligned} \tag{5.8}$$

with $q(z,y)$, $a(z,y)$, $b(z,y)$ given as before, $z \in \mathbb{R}^{n-m}$ and $y \in \mathbb{R}^m$.

Let $(z_d, y_d)$, with $z_d \in \mathbb{R}^{n-m}$ and $y_d \in \mathbb{R}^m$, represent the state components of an auxiliary dynamical system yet to be specified. Furthermore, let the vector $(\xi, \eta)$ be defined as the error vector, $(\xi, \eta) = (z - z_d, y - y_d)$. Let also $R_z$ and $R_y$ stand for a constant, symmetric $(n-m) \times (n-m)$ and $(m) \times (m)$ matrices, respectively, such that the composite matrix

$$R_{zy} = \begin{bmatrix} R_z & 0 \\ 0 & R_y \end{bmatrix}$$

is positive definite in $\mathbb{R}^n$.

In general terms, the *ESDI* controller design technique is based on the consideration of the following *auxiliary dynamical system*

$$\begin{aligned} \dot{z}_d &= q(z,y) - q(\xi,\eta) + R_z \left.\frac{\partial V(\xi,\eta)}{\partial \xi}\right|_{\substack{\xi = z - z_d \\ \eta = y - y_d}} \\ \dot{y}_d &= a(z,y) - a(\xi,\eta) + b(z,y)u + R_y \left.\frac{\partial V(\xi,\eta)}{\partial \eta}\right|_{\substack{\xi = z - z_d \\ \eta = y - y_d}} \end{aligned} \tag{5.9}$$

The *ESDI* method proceeds to set the value of the auxiliary variable $y_d$, in the last differential equation of (5.9), to a desired time varying scalar function $y_d^*(t)$. Once the particularization, $y_d(t) = y_d^*(t)$, has been carried out in both equations (5.9), the method proceeds to solve for the feedback control input $u$, referred as the *equivalent control*, from the last equation in (5.9) and to regard the *zero dynamics*, corresponding to the restriction $y_d(t) = y_d^*(t)$, as a *dynamical feedback controller* state equation.

For implementation of this controller as a discontinuous strategy, the *ESDI* solution method, previously revisited, has to be reinterpreted. First of all, the state $y_d$ is not directly set to $y_d^*$ but it will be forced to follow the desired signal $y_d^*$ by means of a discontinuous controller strategy quite similar to the previously studied. This strategy should ensure that $y_d$ reaches $y_d^*$ in a finite time and once the manifold $\dot{e} = e = 0$ is reached it will stay there for all the time, with $e = y_d - y_d^*$.

We proceed to rewrite the auxiliary system equations in new coordinates $(z_d, e)$ as follows

$$\begin{aligned} \dot{z}_d &= q(z,y) - q(z - z_d, y - e - y_d^*) \\ &\quad + R_z \left.\frac{\partial V(\xi,\eta)}{\partial \xi}\right|_{\substack{\xi = z - z_d \\ \eta = y - e - y_d^*}} \\ \dot{e} &= -\dot{y}_d^* + a(z,y) - a(z - z_d, y - e - y_d^*) \\ &\quad + b(z,y)u + R_y \left.\frac{\partial V(\xi,\eta)}{\partial \eta}\right|_{\substack{\xi = z - z_d \\ \eta = y - e - y_d^*}} \end{aligned} \tag{5.10}$$

In the context of the *ESDI* controller design methodology, the *equivalent control* corresponding to the condition $\dot{e} = \dot{y}_d - \dot{y}^*(t) = 0$ can be computed from the last equation in (5.10) as

$$u_{eq}(z, y, z_d, e, y_d^*) = b(z,y)^{-1} \Big[ \dot{y}_d^* - a(z,y) + $$
$$+ a(z - z_d, y - e - y_d^*) - R_y \left.\frac{\partial V(\xi,\eta)}{\partial \eta}\right|_{\substack{\xi = z - z_d \\ \eta = y - e - y_d^*}} \Big] \tag{5.11}$$

Rewritting the error dynamics in terms of $u_{eq}$ by using (5.11) one readily obtains

$$\dot{e} = b(z,y)\,[u - u_{eq}(z,y,z_d,e,y_d^*)] \qquad (5.12)$$

Taking the (partial) Lyapunov function candidate $V(e) = \frac{1}{2}e^2$ we obtain upon evaluating $\dot{V}(e)$ along the trajectories of the regulated surface coordinate function $e$

$$\dot{V}(e) = e^T\dot{e} = e^T b(z,y)\,[u - u_{eq}(z,y,z_d,e,y_d^*)] \qquad (5.13)$$

It is evident that the same discontinuous strategy discused before can be applied here with the only considerations that the definition of the tracking error $e$ and the computation of the *equivalent control* have changed.

## 6 Application to the three phase Boost type rectifier

**Mathematical model.** Consider the three phase Boost type rectifier shown in Fig. 3.

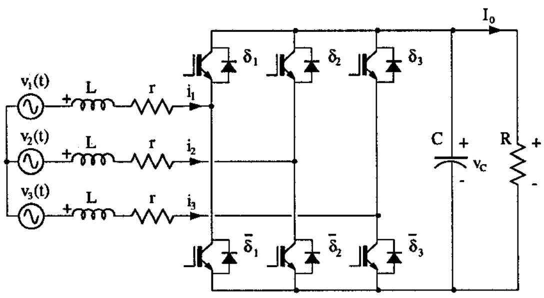


**Figure 3:** Circuit schematic of a three phase Boost type rectifier

Assume that the system is balanced, i.e., the voltage and current source signals have the same amplitudes but are displaced $\frac{2}{3}\pi$ rad one with respect to the other and the passive elements on each line have the same values.

Also assume that the vector of source line voltages is composed by purely sinusoidal signals, i.e.,

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} V\cos(wt) \\ V\cos(wt - \frac{2}{3}\pi) \\ V\cos(wt - \frac{4}{3}\pi) \end{bmatrix}$$

where $V$ is the amplitude of the voltage signal in [Volts] and $w$ its frequency in [rad/sec].

By neglecting the parasitic inductance resistance $r_L$ and considering the output load curren $I_0$ is due to an output load resistance, and after a *Blondel-Parks*-transformation, the model of this circuit, i.e., the $\alpha\beta$-Model is given by

$$\begin{aligned} \begin{bmatrix} L\frac{d}{dt}i_\alpha \\ L\frac{d}{dt}i_\beta \end{bmatrix} &= \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} - \frac{v_C}{2}\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \\ C\frac{d}{dt}v_C &= \frac{1}{2}\begin{bmatrix} u_\alpha & u_\beta \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} - \frac{v_C}{R} \end{aligned} \qquad (6.14)$$

where $[i_\alpha,\ i_\beta]^T$ is the vector of line or inductance currents; $v_C$ is the output capacitor voltage; $[v_\alpha,\ v_\beta]^T$ is the vector of the source line voltages; $[u_\alpha,\ u_\beta]^T$ is the vector of control inputs taking values in the discrete set described in table 1; $L$ is the inductance filter at the line source inputs and $C$ is the output capacitor.

Line source voltages $[v_\alpha,\ v_\beta]^T$ take now the values

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{3}{2}}V\begin{bmatrix} \cos(wt) \\ \sin(wt) \end{bmatrix}$$

Table 1 resumes the transformation of the control input vectors. The vectors $u_{\alpha\beta} = [u_\alpha,\ u_\beta]^T$, form the commonly called *input space* which in this specific problem can be drawn on a plane as shown in figure 4.

| $No.$ | $\delta_1$ | $\delta_2$ | $\delta_3$ | $u_\alpha$ | $u_\beta$ |
|---|---|---|---|---|---|
| $U_0$ | $-1$ | $-1$ | $-1$ | $0$ | $0$ |
| $U_1$ | $1$ | $-1$ | $-1$ | $2\sqrt{\frac{2}{3}}$ | $0$ |
| $U_2$ | $1$ | $1$ | $-1$ | $\sqrt{\frac{2}{3}}$ | $\sqrt{2}$ |
| $U_3$ | $-1$ | $1$ | $-1$ | $-\sqrt{\frac{2}{3}}$ | $\sqrt{2}$ |
| $U_4$ | $-1$ | $1$ | $1$ | $-2\sqrt{\frac{2}{3}}$ | $0$ |
| $U_5$ | $-1$ | $-1$ | $1$ | $-\sqrt{\frac{2}{3}}$ | $-\sqrt{2}$ |
| $U_6$ | $-1$ | $-1$ | $1$ | $\sqrt{\frac{2}{3}}$ | $-\sqrt{2}$ |
| $U_7$ | $1$ | $1$ | $1$ | $0$ | $0$ |

**Table 1:** Permitted switch positions in the $\alpha\beta$-model

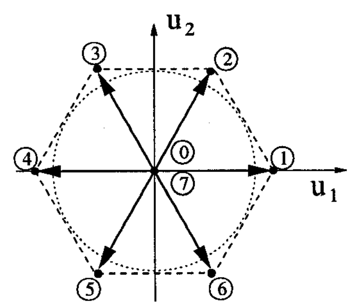


**Figure 4:** Input space

**Control Objective.** The control objective consists into design the switching sequence of control vectors $u_{\alpha\beta}$ to drive the output capacitor voltage $v_C$, of the system modeled by equations (6.14), to a desired constant value given by $V_d$, maintaining all the internal signals bounded. An important constraint in the design is that the input control vector $u_{\alpha\beta}$ can only take the elements $U_i$ ($i \in \{1, ..., 7\}$) from the table 1. Moreover the current signals $i_\alpha$ and $i_\beta$ should follow sinusoidal functions in phase with the source line voltages $v_\alpha$ and $v_\beta$, respectively, in order to ensure a near the unity power factor functioning.

The sinusoidal signals are computed from the steady state analysis, and its amplitude should take a value that ensures simultaneously the regulation of the output voltage $v_C$ towards the desired value $V_d$. In this case they take the following values

$$i_\alpha^* = \sqrt{\frac{2}{3}\frac{V_d^2}{RV}}\cos(wt), \quad i_\beta^* = \sqrt{\frac{2}{3}\frac{V_d^2}{RV}}\sin(wt) \qquad (6.15)$$

Considering the change of variables $y_1 = i_\alpha$, $y_2 = i_\beta$, $z = \frac{L}{2}i_\alpha^2 + \frac{L}{2}i_\beta^2 + \frac{C}{2}v_C^2$, the model (6.14) is written in the *normal form*

$$\begin{aligned} \dot{z} &= q(z,y) \\ \dot{y} &= a(z,y) + b(z,y)u \end{aligned} \qquad (6.16)$$

where $y = [y_1, y_2]$, $u = [u_\alpha \ u_\beta]^T$,

$$q(z,y) = y^T v_{\alpha\beta} - \frac{L}{RC}(y^T y) - \frac{2}{RC}z, \quad a(z,y) = [v_\alpha \ v_\beta]^T,$$

$$b(z,y) = -\frac{1}{2\sqrt{C}}\sqrt{2z - Ly^T y}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**Proposition 6.1** *The system is vector relative degree* $\{1,1\}$ *if the current signals* $i_\alpha$, $i_\beta$ *are considered as outputs.*

**Proof:** This can be easily proved since the *decoupling matrix* $b(z,y)$ in the expression (6.16) is a full rank matrix.

□□□

**Proposition 6.2** *The system is minimum phase with respect to output* $y$ *(the inductor currents* $i_{\alpha\beta}$*), i.e., given a function of time* $y^*(t) \in \mathcal{C}^1$*, the associated zero dynamics of the system, consistent with the time varying constraint* $y(t) = y^*(t)$ *is stable. In other words, the system is stable when the regulated output* $y(t)$ *is forced to track the signal* $y^*(t)$.

As was done in previous section, to solve the tracking problem, we propose to define the tracking error $e = [e_1 \ e_2]^T = [(y_1 - y_1^*) \ (y_2 - y_2^*)]$.

The *equivalent control* is computed from equations (4.5), (6.16), this gives

$$u_{eq} = -\frac{2}{v_C}\left[-v_{\alpha\beta} + L\frac{d}{dt}i_{\alpha\beta}^*\right] \qquad (6.17)$$

**Output regulation subspaces.** The *ORS* in case of rectifier with $c_1, c_2 \in \mathbb{R}^+$, are given as follows,

$$\begin{aligned} ORS(\dot{e}_1) &= \{u = u_{\alpha eq} - \frac{c_1}{2}v_C\mathcal{J}[1\ 0]^T \\ ORS(\dot{e}_2) &= \{u = u_{\beta eq} - \frac{c_2}{2}v_C\mathcal{J}[0\ 1]^T \end{aligned} \qquad (6.18)$$

The *ORS* define in this case two hyperplanes which divide the input space into four quadrants. In this case the *ORS* is easily obtained and composed by a vertical and horizontal lines perpendicular to each other. To each subspace it is assigned a combination of signs for $\dot{e}_1$ and $\dot{e}_2$ as shown in figure 5.

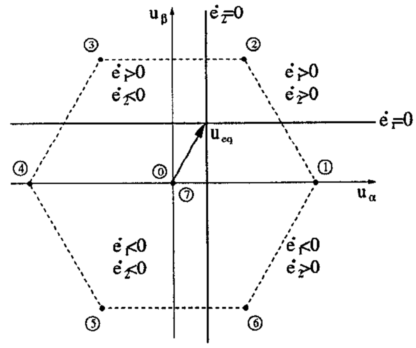


**Figure 5:** Input space, output regulation subspaces **OSR**

**Proposition 6.3** *Existence of a switching sequence under fast switching assumption is guaranteed whenever the equivalent control vector is contained in the hexagon formed by the points* $U_i$, $i \in \{1,...,6\}$ *in figure 4. A more conservative condition is that the equivalent control vector should be contained in the inscrit circle to the hexagon, this yields the following condition*

$$\sqrt{3V^2 + \frac{4V_d^4 w^2 L^2}{3R^2V^2}} \leq v_C \qquad (6.19)$$

Condition (6.19) reveals the amplification characteristic of the *Boost rectifier*, which for the steady state gives approximately, $\sqrt{3}V < V_d$.

**Energy shaping plus damping injection approach.**

The auxiliary system for this system is given by

$$\begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & C \end{bmatrix}\begin{bmatrix} \frac{d}{dt}\xi_1 \\ \frac{d}{dt}\xi_2 \\ \frac{d}{dt}\xi_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 0 & -u_\alpha \\ 0 & 0 & -u_\beta \\ u_\alpha & u_\beta & -\frac{2}{R} \end{bmatrix}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix}$$

$$+\begin{bmatrix} v_\alpha \\ v_\beta \\ 0 \end{bmatrix} + \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3^{-1} \end{bmatrix}\begin{bmatrix} (i_\alpha - \xi_1) \\ (i_\beta - \xi_2) \\ (v_C - \xi_3) \end{bmatrix} \qquad (6.20)$$

where $\xi_1$, $\xi_2$, $\xi_3$ are the auxiliary system states, $R_1$, $R_2$, $R_3 \in \mathbb{R}^+$ are design parameters used to introduce damping.

Let us define the tracking error $e$ as

$$e = \xi_{12} - y^* = [e_1 \ e_2]^T = [(\xi_1 - y_1^*) \ (\xi_2 - y_2^*)]^T \qquad (6.21)$$

The equivalent control is computed as follows

$$u_{eq} = g^{-1}(\xi_3)\left[-v_{\alpha\beta} - R_{12}\left(i_{\alpha\beta} - i_{\alpha\beta}^*\right) + L\frac{d}{dt}i_{\alpha\beta}^*\right] \qquad (6.22)$$

In the steady state, the existence condition for a switching sequence is reduced to the same condition proposed in (6.19).

The *ORS* are exactly the same as those given in 6.18, with the only consideration that $u_{eq}$ and the error vector $e$ are computed now as in (6.22) and (6.21). So the implementation of the discontinuous strategy is performed in a similar way.

## 7 Simulation results

The two implementation strategies for the switching sequence discused before were applied to the rectifier system. Moreover, in order to see the improvement in the transient behavior also the strategy proposed in [7] was implemented, we will refer to this as **strategy (1)**, we then propose to add to this simple strategy the *ESDI* procedure which is referred as **strategy (2)**. The first strategy proposed in this paper is referred as **strategy (3)**, while the strategy incorporating *ESDI* procedure is called **strategy (4)**. Notice that both strategies require as inputs the *equivalent control vector* $u_e q$ and the error vector definition $e = [e_1,\ e_2]^T$.

The simulations were carried out using the actual nonlinear system model (6.14), with the parameters $L = 10\mu$H, $\omega_0 = 120\pi$rad/sec, $C_0 = 1mF$, $V = 110V$, $R = 25\Omega$, $V_d = 300V$, $R_1 = 2$, $R_2 = 2$, $R_3 = 1$.

Fig. 6 shows the phase plots $i_\beta(i_\alpha)$, for the four control strategies. This figure exhibits the improvement in the transitory state of the current signals. Here, the strategy (4), i.e., the one incorporating simultaneously passivity and the proposed criteria to select the control vector, exhibits the best transient behavior.

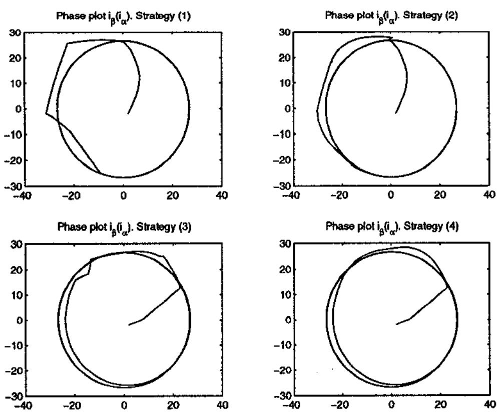


**Figure 6:** Phase plots $i_\beta(i_\alpha)$

Fig. 7 shows the phase plots of the equivalent control signals for the four strategies studied. This plot exhibits the lower control effort required in strategies using passivity based control.

In Fig. 8 the time response of the output capacitor voltage is presented. This signal reaches the desired final value $V_d = 300$ Volts in the steady state. From this figure, the nonminimum phase behavior of the system response can be observed.

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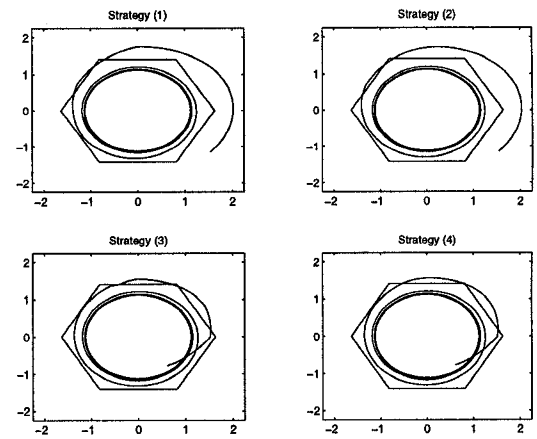


**Figure 7:** Phase plots of the equivalent control $u_\beta^{eq}(u_\alpha^{eq})$

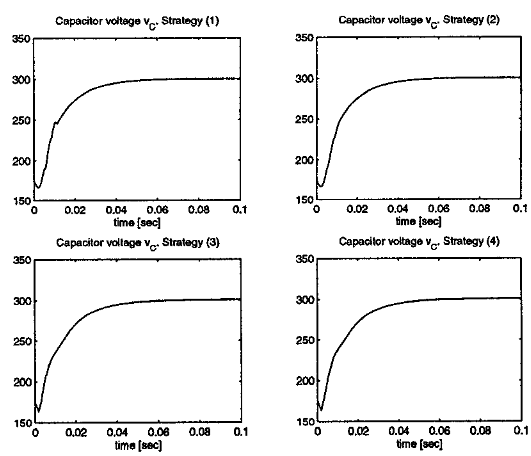


**Figure 8:** Time responses of the output capacitor voltage $v_C(t)$