A Passivity Based-Sliding Mode Control Approach for the Regulation of Power Factor Precompensators

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Abstract: A controller design method which combines passivity based control and sliding mode control is presented for the feedback regulation of a class of switched power converters, addressed as "power factor precompensators" (PFP). Aside from load voltage regulation to a prespecified constant level, a vital additional control objective is to avoid reactive losses by keeping the input power factor close to unity. A passivity plus sliding mode control approach is proposed which forces the input current to follow a suitable reference signal which is in phase with the rectified supplied voltage. This results in approximately satisfying both control objectives for the converter. Simulation results are furnished for assessing the performance of the proposed feedback control laws.

Keywords: Power circuits, converters, rectifiers, passive compensation, sliding mode control.

1. INTRODUCTION

Despite their widespread use in DC-to-DC power regulation, the traditional DC-to-DC Power Converter topologies (such as the boost, the buck-boost, the Cúk) have shown several disadvantages when used in rectified AC-to-DC power conversion schemes. One of the major drawbacks is related to the low-input-power factor usually attained with PFP's. Control strategies are sought which, simultaneously, enhance the low power factor while efficiently regulating the output load voltage.

In [4] new topologies are proposed for minimizing the input current distortion of boost rectifiers. Explicit performance parameters, such as the power factor, the total harmonic distortion, and others, are also proposed. In [6] a combination of open loop plus PI compensation is developed which improves the power factor at the expence of poor transient behaviour.

In this article, we propose a feedback controller whose closed loop performance approximately achieves, in a simultaneous fashion, the above stated control objectives. The controller design strategy is based on using a passivity approach in combination with a sliding mode control implementation. The boost converter topology is chosen for detailed illustration, but the approach is extendable to other traditional converter topologies as well. By defining a reference signal tracking problem on the input current of the converter, the power factor can be made very close to unity as long as the tracked signal is in phase with the rectified input voltage. For this we propose two alternative strategies, one based on following a rectified reference current signal, perfectly in phase with the input voltage and a second one using a biased sinusoidal input current reference signal, with its minima in phase with the rectified input voltage. In either case, the load voltage DC current component is explicitly computed in order to approximately determine, via a partial system inversion, the required reference signal amplitude for the input current. The passivity based-sliding mode controller performance is evaluated by explicitly computing the steady state power factor and the AC-line current distortion for each case.

The rest of the paper is organized as follows: in section 2 we introduce the passivity based plus sliding mode controller design methodology in rather general terms. In section 3 we apply the proposed methodology to a PFP of the Boost type. Illustrative digital computer simulations are provided in section 4 to assess the performance of the proposed control scheme.

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2. THE PASSIVITY-BASED PLUS SLIDING MODE CONTROL SCHEME

In this section we shall present a sliding mode control implementation of a common passivity based control scheme known as the "energy shapping plus damping injection" (ESDI) controller design scheme. We begin by briefly revisiting the basic ESDI controller design methodology, extensively used in passivity based control a concept first introduced in [3].

2.1. The Energy Shapping plus Damping Injection Method

Consider an n-dimensional nonlinear system of the form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \ ; \ x \in \mathcal{X} \subset \mathbb{R}^n \ , \ u \in \mathcal{U} \subset \mathbb{R} \\ y &= h(x) \ ; \ y \in \mathbb{R} \end{aligned} \tag{1}$$

where $\mathcal{X}$ denotes a sufficiently large open subset of $\mathbb{R}^n$, containing the system trajectories. $\mathcal{X}$ is addressed here as the *operating region* of the system. The set $\mathcal{U}$ denotes the set of available input values which, for the moment, is also considered to be a sufficiently large portion of the real line $\mathbb{R}$. The vector field $f(x)$ is such that $f(0) = 0$, i.e., the origin is a zero input equilibrium point for the system. The output map $h : \mathbb{R}^n \to \mathbb{R}$, satisfies $h(0) = 0$.

It is also assumed that the output function $h(x)$ of system (1) is *relative degree* equals to one i.e., the directional derivative, $L_g h(x)$, is bounded away from zero in the operating region of the system. Also, given a scalar locally C^1 function of time, $y^*(t)$, the system is assumed to be *stable* when the regulated output $y(t)$ is forced to locally track the signal $y^*(t)$ on, at least, a finite open time interval. In other words, the associated *zero dynamics* of the system, consistent with the time varying constraint $y = y^*(t)$, exhibits no internal finite escape time to infinity while the trajectory $y^*(t)$ is being perfectly followed. If $y^*(t)$ happens to be a constant reference signal, which can be followed in an indefinite manner, then the corresponding zero dynamics is assumed to be asymptotically stable towards an equilibrium point located in $\mathcal{X}$, i.e, the system is *minimum phase*.

If the system is relative degree one in $\mathcal{X}$ then, there exists an invertible state coordinate transformation in $\mathcal{X}$

$$(z, y) = (T(x), h(x)) \tag{2}$$

which locally takes the system into the following *normal form* given by

$$\begin{aligned} \dot{z} &= q(z, y) \\ \dot{y} &= a(z, y) + b(z, y)u \end{aligned} \tag{3}$$

The scalar function $L_g h(z, y) = b(z, y)$, is nonzero in $\mathcal{X}$. Note that from the assumptions on $f(x)$ and the diffeomorphic nature of the transformation, it follows that $q(0,0) = 0$ and $a(0,0) = 0$. Also, the minimum phase assumption implies that the system $\dot{z} = q(z, y^*(t))$ is a stable system.

It is also a straightforward consequence of the relative degree one and the minimum phase assumptions, that by specifying an appropriate *regular static state feedback* of the form $u = \alpha(z, y) + \beta(z, y)v$, where $\beta(x)$ is a nonzero scalar function (see Byrnes, Isidori and Willems, [1]), the system (3) can be made passive with respect to some C^1 positive definite storage function $V(z, y)$. The function $V(z, y)$ can always be chosen to be zero at the origin of coordinates $(z, y) = (0, 0)$.

We further assume that $V(z, y)$ is such that the gradient components $\partial V/\partial z$ and $\partial V/\partial y$ of the storage function only vanish identically at the origin $(z, y) = (0, 0)$. The following nonlinear generalization of the classical Kalman-Yacubovitch-Popov properties, proposed in (see [2]), are satisfied by the transformed system ,

$$\frac{\partial V(z, y)}{\partial z} q(z, y) + \frac{\partial V(z, y)}{\partial y} a(z, y) \leq 0 \tag{4}$$

and,

$$\frac{\partial V(z, y)}{\partial y} b(z, y) = y \tag{5}$$

Let (z_d, y_d), with $z_d \in \mathbb{R}^{n-1}$ and $y_d \in \mathbb{R}$, represent the state components of an auxiliary dynamical system yet to be specified. Furthermore, let the vector (ξ, η) be defined as the error vector, $(\xi, \eta) = (z - z_d, y - y_d)$. Let also R_z and R_y stand for a constant, symmetric $(n-1) \times (n-1)$ matrix and a strictly positive constant scalar, respectively, such that the composite matrix,

$$R_{zy} = \begin{bmatrix} R_z & 0 \\ 0 & R_y \end{bmatrix}$$

is positive definite in $\mathbb{R}^n$.

In general terms, the ESDI controller design methodology is based on the consideration of the following *auxiliary dynamical system*, characterized by an *exogenous* composite state variable (z_d, y_d), defined by,

$$\begin{aligned} \dot{z}_d &= q(z, y) - q(\xi, \eta) + R_z \frac{\partial V(\xi, \eta)}{\partial \xi}\bigg|_{\substack{\xi = z - z_d \\ \eta = y - y_d}} \\ \dot{y}_d &= a(z, y) - a(\xi, \eta) + b(z, y)u + R_y \frac{\partial V(\xi, \eta)}{\partial \eta}\bigg|_{\substack{\xi = z - z_d \\ \eta = y - y_d}} \end{aligned} \tag{6}$$

We have the following lemma:

Lemma 2.1 *If there exists an input function u and a vector (z_d, y_d) such that the differential equation (6) is satisfied, then the error vector $(\xi, \eta) = (z - z_d, y - y_d)$ asymptotically converges to zero.*

Proof: The proof is based on obtaining a differential equation describing the error vector dynamics,

$$\begin{aligned} \dot{\xi} &= q(\xi, \eta) - R_z \frac{\partial V(\xi, \eta)}{\partial \xi} \\ \dot{\eta} &= a(\xi, \eta) - R_y \frac{\partial V(\xi, \eta)}{\partial \eta} \end{aligned} \tag{7}$$

Computing the time derivative of the positive definite storage function $V(\xi, \eta)$ along the trajectories of the autonomous error vector equation (7), we obtain

$$\begin{aligned} \dot{V}(\xi, \eta) &= \frac{\partial V(\xi, \eta)}{\partial \xi^T} q(\xi, \eta) + \frac{\partial V(\xi, \eta)}{\partial \eta} a(\xi, \eta) \\ &\quad - \frac{\partial V(\xi, \eta)}{\partial \xi^T} R_z \frac{\partial V(\xi, \eta)}{\partial \xi} - R_y \left[\frac{\partial V(\xi, \eta)}{\partial \eta}\right]^2 \\ &\leq 0 \end{aligned} \tag{8}$$

The negativity of $\dot{V}(\xi, \eta)$ follows directly from the first KYP property in (4) and the positivity of the composite constant matrix R_{zy} in $\mathbb{R}^n$

It is easy to see that the set of error states (ξ, η) where $\dot{V}(\xi, \eta) = 0$ is represented only by the origin of coordinates $(\xi, \eta) = (0, 0)$. Indeed, $\dot{V}(\xi, \eta) = 0$ for $(\xi, \eta) \neq (0, 0)$ implies, that

$$\begin{aligned} &\frac{\partial V(\xi, \eta)}{\partial \xi^T} q(\xi, \eta) + \frac{\partial V(\xi, \eta)}{\partial \eta} a(\xi, \eta) \\ &= \frac{\partial V(\xi, \eta)}{\partial \xi^T} R_z \frac{\partial V(\xi, \eta)}{\partial \xi} + R_y \left[\frac{\partial V(\xi, \eta)}{\partial \eta}\right]^2 \\ &= \begin{bmatrix} \frac{\partial V(\xi, \eta)}{\partial \xi^T} & \frac{\partial V(\xi, \eta)}{\partial \eta} \end{bmatrix} \begin{bmatrix} R_z & 0 \\ 0 & R_y \end{bmatrix} \begin{bmatrix} \frac{\partial V(\xi, \eta)}{\partial \xi} \\ \frac{\partial V(\xi, \eta)}{\partial \eta} \end{bmatrix} \\ &\geq 0 \end{aligned} \tag{9}$$

which is a contradiction of property (4), unless the following equality is satisfied

$$\begin{bmatrix} \frac{\partial V(\xi, \eta)}{\partial \xi^T} & \frac{\partial V(\xi, \eta)}{\partial \eta} \end{bmatrix} \begin{bmatrix} R_z & 0 \\ 0 & R_y \end{bmatrix} \begin{bmatrix} \frac{\partial V(\xi, \eta)}{\partial \xi} \\ \frac{\partial V(\xi, \eta)}{\partial \eta} \end{bmatrix} = 0 \tag{10}$$

From the assumptions about the positivity of the composite matrix R_{zy}, and the nonvanishing assumption about the gradients of V, it follows that the relation (11) is valid if and only if $(\xi, \eta) = (0, 0)$. In other words, the set of points where $\dot{V}(\xi, \eta) = 0$ is precisely constituted by the origin of coordinates $(\xi, \eta) = (0, 0)$. Invoking LaSalle's invariance principle, the origin $(\xi, \eta) = (0, 0)$ is an asymptotically stable equilibrium point.

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Remark: If the system equations are of the particular but important form

$$\mathcal{D}\dot{x} = \mathcal{J}(x, u)x - \mathcal{R}(x)x + g(x)u$$

where $\mathcal{D}$ is a positive definite constant matrix, $\mathcal{J}(x, u)$ is an skew-symmetric matrix, for all (x, u) and $\mathcal{R}(x)$ is a positive semidefinite symmetric matrix for all x, which is also absolutely continuous, then one can exploit the semilinearity in the state variable x by proposing an auxiliary system of the form,

$$\mathcal{D}\dot{x}_d = \mathcal{J}(x, u)x_d - \mathcal{R}(x)x_d + g(x)u + \mathcal{R}_I(x - x_d)$$

where $\mathcal{R}_I$ is a constant positive definite matrix representing the damping injection term.

The above choice of the auxiliary dynamics yields a time-varying linear differential equation for the error $e = x - x_d$ given by,

$$\mathcal{D}\dot{e} = [\mathcal{J}(x,u) + \mathcal{R}(x) + \mathcal{R}_I]\, e$$

which under all the above assumptions is asymptotically stable to zero as it can be verified from the Lyapunov function candidate $V(e) = \frac{1}{2}e^T\mathcal{D}e$.

The energy shapping aspect of the above method usually refers to the time varying storage function modification, implicit in the expression $V(\xi,\eta) = V(z - z_d, y - y_d)$. The damping injection, on the other hand, refers to the stabilizing effect, on the error dynamics (7), of the terms $-R_z\partial V(\xi,\eta)/\partial\xi$ and $-R_y\partial V(\xi,\eta)/\partial\xi$.

Since the error variables (ξ,η) converge to zero, then the original system state (z,y) converges to the auxiliary system state (z_d, y_d). As stated, the control objective is to solve a reference signal tracking problem for the regulated output y. The energy shapping plus damping injection method proceeds to set the value of the auxiliary variable y_d, in the last differential equation of (6), to a desired time varying scalar function $y_d^*(t)$. This signal, of course, represents the reference signal to be tracked by the output of the system, i.e., $y_d^*(t) = y^*(t)$. Once the particularization, $y_d(t) = y_d^*(t)$, has been carried out in both equations (6), the method proceeds to solve for the feedback control input u from the last equation in (6) and to regard the *zero dynamics*, corresponding to the restriction $y_d(t) = y_d^*(t)$, as a *dynamical feedback controller* state equation. Such a dynamical feedback controller is characterized then by an $n-1$ dimensional state vector, denoted here by ζ, which formally replaces the auxiliary dynamics vector z_d.

The obtained control is then given by

$$\begin{aligned} u(z,y,\zeta,y_d^*) &= b(z,y)^{-1}\Big[\dot{y}_d^* - a(z,y) + a(z-\zeta, y-y_d^*) \\ &\quad -R_y \frac{\partial V(z-\zeta,\eta)}{\partial\eta}\bigg|_{\substack{\xi = z-\zeta \\ \eta = y - y_d^*}}\Big] \end{aligned} \tag{11}$$

and the dynamical controller state equation is obtained as,

$$\dot{\zeta} = q(z,y) - q(z-\zeta, y-y_d^*) + R_z\frac{\partial V(\xi,\eta)}{\partial\xi}\bigg|_{\substack{\xi = z-\zeta \\ \eta = y-y_d^*}} \tag{12}$$

where, from the *minimum phase* assumption, also $\zeta \in \mathcal{L}_\infty$.

2.2. A Sliding Mode Control Implementation

In this section we shall be making reference to well known results in sliding mode control. The reader unfamiliar with this theory is referred to the book by Utkin [7].

In many applications of the ESDI synthesis method, the control input u of the system is only allowed to take values on a discrete set $\mathcal{U}$. Specifically, in the class of switched systems to be considered in this paper, the set of available control values is represented by the binary set $\mathcal{U} = \{0,1\}$.

In order to obtain a discontinuous feedback control solution of the tracking problem proposed on the original system, the ESDI solution method, previously revisited, has to be suitably reinterpreted in terms of the traditional elements of the sliding mode control theory.

It turns out that forcing the restrictions $y_d(t) = y_d^*(t)$ and $\dot{y}_d(t) = \dot{y}_d^*(t)$ upon the auxiliary dynamics (6) effectively corresponds to a simultaneous *ideal sliding dynamics* and an *equivalent control* determination for a corresponding *sliding surface* defined by a time varying relation of the form,

$$\sigma(t) = y_d - y_d^*(t) \tag{13}$$

from the associated sliding mode *invariance conditions*

$$\sigma = 0 \;\; ; \;\; \dot{\sigma} = 0 \tag{14}$$

It should be clear that, for switched regulated systems, the limited availability of control input values renders unfeasible the synthesis of the equivalent control as an actual feedback controller. The invariance conditions $\sigma = 0$ and $\dot{\sigma} = 0$ can be made valid only in an *average sense* after an appropriate switching policy is deviced, usually characterized by a very large switching frequency, which renders valid the invariance conditions $\sigma = 0$ and $\dot{\sigma} = 0$.

Formally, however, the equivalent control is defined only on the basis of the condition $\dot{\sigma} = 0$ (see [7]). As such, the equivalent control represents a *virtual feedback control action* that, in the absence of perturbations and modelling errors, ideally keeps the system responses evolving on a manifold represented by the condition $\sigma =$ *constant*. In particular, the equivalent control ideally keeps the trajectories on the sliding surface $\sigma = 0$, when the initial conditions of the system are set precisely on such a surface. The equivalent control expression, computed on the sliding surface $\sigma = 0$, however, is rather useful in determining the region of existence of a sliding regime on such a surface.

The following theorem may be proven in quite general terms,[7],

Theorem 2.2 *A sliding regime exists on an open region of the time-varying $n-1$ dimensional manifold $\sigma = 0$ if and only if the equivalent control u_{eq}, computed from the condition $\dot{\sigma} = 0$, satisfies:*

$$0 \le u_{eq}\Big|_{\sigma=0} \le 1 \tag{15}$$

We proceed to rewrite the auxiliary system equations in new coordinates (z_d, σ), as follows

$$\begin{aligned} \dot{z}_d &= q(z,y) - q(z - z_d, y - \sigma - y_d^*) \\ &\quad + R_z\frac{\partial V(\xi,\eta)}{\partial\xi}\bigg|_{\substack{\xi = z - z_d \\ \eta = y - \sigma - y_d^*}} \\ \dot{\sigma} &= -\dot{y}_d^* + a(z,y) - a(z - z_d, y - \sigma - y_d^*) \\ &\quad + b(z,y)u + R_y\frac{\partial V(\xi,\eta)}{\partial\eta}\bigg|_{\substack{\xi = z - z_d \\ \eta = y - \sigma - y_d^*}} \end{aligned} \tag{16}$$

In the context of the ESDI controller design methodology, the equivalent control corresponding to the condition $\sigma = y_d - y_d^*(t) = 0$ can be computed from the last equation in (16), as

$$\begin{aligned} u_{eq}(z,y,z_d,\sigma,y_d^*) &= b(z,y)^{-1}\Big[\dot{y}_d^* - a(z,y) \\ &\quad + a(z - z_d, y - \sigma - y_d^*) - R_y\frac{\partial V(\xi,\eta)}{\partial\eta}\bigg|_{\substack{\xi = z - z_d \\ \eta = y - \sigma - y_d^*}}\Big] \end{aligned} \tag{17}$$

Rewritting the sliding surface dynamics by adding and substracting in the second equation of (16) the quantity $b(z,y)u_{eq}$ and using the expression for the equivalent control (17) one readily obtains,

$$\begin{aligned}\dot{\sigma} &= -\dot{y}_d^* + a(z,y) - a(z-z_d, y-\sigma-y_d^*) + b(z,y)u_{eq} \\ &\quad + b(z,y)(u-u_{eq}) + R_y \left.\frac{\partial V(\xi,\eta)}{\partial \eta}\right|_{\substack{\xi = z - z_d \\ \eta = y - \sigma - y_d^*}} \\ &= b(z,y)\left[u - u_{eq}(z,y,z_d,\sigma,y_d^*)\right] \end{aligned} \quad (18)$$

We then have the following result

Proposition 2.3 *Let (z, y) be a given n-dimensional vector of time-varying components then, the feedback switching policy:*

$$u = \begin{cases} 1 & for \quad b(z,y)\sigma < 0 \\ 0 & for \quad b(z,y)\sigma > 0 \end{cases} \quad (19)$$

locally drives the trajectories of (18) towards $\sigma = 0$ and, moreover, it locally sustains a sliding regime on the sliding surface (i.e., $\sigma = 0$ and $\dot{\sigma} = 0$ are rendered valid in an average sense) provided the equivalent control, given by (17), satisfies the following relation,

$$0 < u_{eq}(z, y, z_d, \sigma, y_d^*) < 1 \quad (20)$$

Proof: Taking the singular Lyapunov function candidate $V(\sigma) = \frac{1}{2}\sigma^2$ we obtain upon evaluating $\dot{V}(\sigma)$ along the trajectories of the regulated surface coordinate function σ,

$$\dot{V}(\sigma) = \sigma\dot{\sigma} = b(z,y)\sigma\left[u - u_{eq}(z,y,z_d,\sigma,y_d^*)\right] \quad (21)$$

It is evident that if the switching policy (19) is used and provided (20) is valid for $\sigma \neq 0$, then $\dot{V}(\sigma) < 0$ for all $\sigma \neq 0$. The invariance set $\dot{V}(\sigma) = 0$ is hit in finite time and a sliding regime is subsequently created on the manifold $\sigma = 0$.

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3. SWITCH-REGULATED BOOST CONVERTER AS A PFP

In this section we will apply the previously described methodology to a PFP of the boost type, whose circuit is shown in figure 1. Moreover, the power factor will be explicitly computed in order to evaluate the performance of the proposed controller.

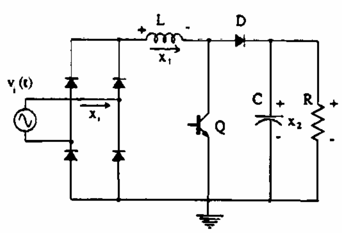

Fig. 1: Switch-regulated PFP Boost circuit

The differential equations describing the circuit are,

$$\begin{aligned} L\dot{x}_1 &= -ux_2 + V|\sin(wt)| \\ C\dot{x}_2 &= ux_1 - \frac{1}{R}x_2 \end{aligned} \quad (22)$$

where x_1 and x_2 are the input inductor current and the output capacitor voltage variables, respectively; $V|\sin(wt)| > 0$ is the rectified voltage of the ac-line source; R is the nominal constant value of the output resistance; u, which takes values in the discrete set $\{0, 1\}$, denotes the switch position function, and acts as a control input. The control objective is twofold. First, the output x_2 should be driven to some constant desired value $V_d > V$. Second, in order to guarantee a power factor near unity, the inductor current x_1 should follow a rectified sinusoidal signal of the same frequency and also in phase with the ac-line voltage source.

3.1. Controller design

Consider the boost model (22), rewritten in matrix form,

$$\mathcal{D}\dot{x} = \mathcal{J}(u)x - \mathcal{R}x + \tau \quad (23)$$

where $x = [x_1,\ x_2]^T$, $\mathcal{D} = \text{diag}\{L,\ C\}$, $\mathcal{R} = \text{diag}\{0,\ \frac{1}{R}\}$, $\tau = [V|\sin(wt)|,\ 0]^T$, and $\mathcal{J}(u)$ is a skew symmetric matrix given by,

$$\mathcal{J}(u) = \begin{bmatrix} 0 & -u \\ u & 0 \end{bmatrix}$$

Consider also the auxiliary system,

$$\mathcal{D}\dot{x}_d = \mathcal{J}(u)x_d - \mathcal{R}x_d + \tau + \mathcal{R}_I(x - x_d) \quad (24)$$

where $x_d = [x_{1d},\ x_{2d}]^T$, $\mathcal{R}_I(x - x_d)$ is the damping injection terms, with matrix $\mathcal{R}_I = \text{diag}\{R_1, R_2^{-1}\}$, $R_1, R_2 \in \mathbb{R}^+$.

We know from [5] that in the boost converter, the output x_1 yields a minimum phase system, while the capacitor voltage x_2, yields a non minimum phase system. For this reason control actions are geared to indirectly regulate x_2 through x_1.

In order to fullfill the two control objectives, namely, power factor close to unity and a constant output voltage level, we propose the following sliding surface on the auxiliary system,

$$\sigma = x_{1d} - x_{1d}^* = x_{1d} - K|\sin(wt)| \quad (25)$$

where K is a constant factor to be determined through steady state considerations.

The time derivative of the sliding surface is,

$$\dot{\sigma} = \frac{1}{L}[-ux_{2d} + V|\sin(wt)| + R_1(x_1 - x_{1d}) - L\dot{x}_{1d}^*] \quad (26)$$

Is clear that $\dot{x}_{1d}^*$ does not exist at the instants $wt = k\pi,\ k = 0, ..., n$. As a consequence we can only track the proposed signal during open intervals of time.

The equivalent control, on $\sigma = 0$ is given by,

$$u_{eq} = \frac{1}{x_{2d}}[V|\sin(wt)| + R_1(x_1 - x_{1d}) - L\dot{x}_{1d}^*], \quad x_{2d} \neq 0 \quad (27)$$

which exists as long as $wt \neq k\pi,\ k = 0, ..., n$. For the positive half cycle we have,

$$u_{eq} = \frac{1}{x_{2d}}[V\sin(wt) - KLw\cos(wt) + R_1(x_1 - x_{1d})], x_{2d} \neq 0 \quad (28)$$

The existence conditions (15) for a sliding regime to exist are translated into the following expressions,

$$wt \geq \arctan(\gamma), \quad \gamma = \frac{2V_d^2 Lw}{RV^2} \quad (29)$$

$$V_d \geq \sqrt{V^2 + \left(\frac{2V_d^2 Lw}{RV}\right)^2} \quad (30)$$

i.e., the equivalent control is defined in the period $[\arctan(\gamma),\ \pi)$, and outside this time interval it takes negative values. The second condition implies that V_d should be strictly larger than V.

The following switching policy guarantees a finite time reachability of the sliding surface and the creation of a local sliding regime,

$$u = \frac{1}{2}(1 + \text{sign}(\sigma)) \quad (31)$$

3.2. Desired inductor current.

To determine the required amplitude K of the desired inductor current we consider that x_1 is described with sufficient accuracy when its first harmonic is retained while x_2 is assumed to be nearly a dc–quantity. This assumptions are reasonable if the output capacitor is large enough.

The total energy in the system is given by,

$$H = \frac{L}{2}x_1^2 + \frac{C}{2}x_2^2$$

Its time derivative is given by,

$$\dot{H} = x_1 V|\sin(wt)| - \frac{1}{R}x_2^2$$

In steady state this energy is mantained constant in an averaged way, since the system is stable. Thus, by taking only the dc–components, and assuming in steady state $x_{2SS} = V_d$ and $x_{1SS} = x_{1d}^* = K|\sin(wt)|$, we have,

$$0 = \langle KV\sin^2(wt)\rangle_{DC} - \frac{V_d}{R}$$

where $\langle\cdot\rangle_{DC}$ indicates the dc–component.

Solving the last equation for K, the desired current results in,

$$x_{1d}^* = \frac{2V_d^2}{RV}|\sin(wt)| \tag{32}$$

3.3. Power factor analysis

In this subsection we will obtain an explicit expression for the power factor (See [4] for further details). We start by obtaining an explicit expression for the trayectory of the input current. For this, we assume that the system is already in the steady state and we consider only the positive half cycle.

As discussed before, u_{eq} will try to adopt a negative value at the begining of the cycle, so the best the control can do is to set $u = 0$. With this choice of u the differential equation for $\dot{x}_1$ from (22), reduces to,

$$L\dot{x}_1 = V\sin(wt)$$

which can be solved considering the initial condition $x_1(0) = 0$, this yields,

$$x_1(t) = \frac{V}{Lw}(1 - \cos(wt))$$

Moreover, the controller will be maintained at the value $u = 0$ until the state trajectory reaches the tracking reference signal at $wt = \beta$, where β may be explicitely computed as follows,

$$\beta = 2\arctan(\frac{2V_d^2 wL}{RV^2}) = 2\arctan(\gamma) \tag{33}$$

The trajectory of the inductor current, in steady state, is thus described as,

$$x_1(t) = \begin{cases} \frac{V}{wL}(1 - \cos(wt)), & 0 \le wt \le \beta \\ \frac{2V_d^2}{RV}\sin(wt), & \beta < wt \le \pi \end{cases} \tag{34}$$

Let $x_I(t)$ denote the *ac–line current signal* $x_I(t)$, at the input of the diode rectifier. This signal has the alternate symetrical form shown in fig. 2 and it is expressed as,

$$x_I(t) = x_1(t)\mathrm{sign}(\sin(wt)) \tag{35}$$

The *fundamental components* of $x_I(t)$, i.e., the real and imaginary parts of the first harmonic of $x_I(t)$ are given by,

$$X_1 = \frac{V}{\pi wL}(\sin(\beta) - \beta) \tag{36}$$

$$X_2 = \frac{2V_d^2 wL}{\pi RV}(\sin(\beta) + \pi - \beta) \tag{37}$$

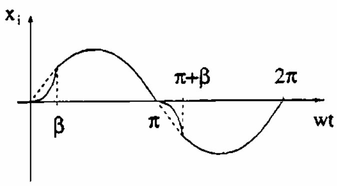

Fig. 2: Input current time response.

The *fundamental power factor* (FPF) is defined using the fundamental components given above as,

$$FPF = \frac{X_2}{\sqrt{X_1^2 + X_2^2}} = \frac{1}{\sqrt{1 + \left(\frac{2}{\frac{V_d^2 wL}{RV^2}\left(1 + \frac{\pi}{\sin(\beta) - \beta}\right)}\right)^2}} \tag{38}$$

The *root mean square value* of $x_I(t)$, denoted by x_{IRMS} is, on the other hand, given by

$$x_{IRMS} = \left(\frac{V^2}{\pi L^2 w^2}\left[\left(\frac{3}{2}\beta - 2\sin(\beta) + \frac{1}{4}\sin(2\beta)\right) + \frac{2V_d^4}{\pi R^2 V^2}\left(\pi - \beta + \frac{1}{2}\sin(2\beta)\right)\right]\right)^{\frac{1}{2}} \tag{39}$$

The *power factor* (PF) is then computed in the following manner,

$$PF = \frac{X_2}{\sqrt{2}x_{IRMS}} = \frac{\left(\frac{\sin(\beta)}{\pi} + 1 - \frac{\beta}{\pi}\right)}{\left[\frac{V^4 R^2}{2V_d^4 \pi w^2 L^2}\left(\frac{3\beta}{2} - 2\sin(\beta) + \frac{\sin(2\beta)}{4}\right) + \left(1 - \frac{\beta}{\pi} + \frac{\sin(2\beta)}{2\pi}\right)\right]} \tag{40}$$

Using the above results we can state the following proposition.

Proposition 3.1 *Consider the PFP power converter of the "boost" type described by (22), and consider also the auxiliary dynamics given by eqs.(24).*

If the switching policy is defined as,

$$u = \frac{1}{2}(1 - sign(\sigma))$$

where $\sigma = x_{1d} - x_{1d}^*$ is the sliding surface, with x_{1d}^* given by (32), then the closed loop system exhibits regulation of the output capacitor voltage towards the desired constant value V_d in an averaged sense. Moreover, the switching policy locally creates a stable sliding regime on open sets of the form $wt \in [\beta + k\pi, (k+1)\pi), (k = 0, 1, .., n)$ provided,

$$wt \ge \arctan(\frac{2V_d^2 Lw}{RV^2}), \quad V_d \ge \sqrt{V^2 + \left(\frac{2V_d^2 Lw}{RV}\right)^2}$$

And, the steady state form of the current inductor trajectory has a power factor given by the expression (40), whith $PF \to 1$ as $\beta \to 0$.

Proof: From the previous developements, is clear that the sliding regime will be stablished on open sets of the form $wt \in [\beta + k\pi, (k+1)\pi), (k = 0, 1, .., n)$, wich are strictly included in the open sets where the equivalent control is defined, i.e., $wt \in [\gamma + k\pi, (k+1)\pi), (k = 0, 1, .., n)$. Actually, β is twice the value of γ. Thus, the form of the current trajectory will be as given in fig. 2. From eq. (40) is easy to see that as $\beta \to 0$, $PF \to 1$. Its corresponding rectified signal will have a slightly different dc-component than the required to assure $x_2 \to V_d$. So we can only assure that x_2 will converge towards a small neighborhood of V_d, and $x_2 \to V_d$ as $\beta \to 0$. The latter can be established by computing the error between the dc-components of the inductor current in steady state and its desired value,

$$\langle \overline{x}_1 \rangle_{DC} - \langle x_{1d}^* \rangle_{DC} = \frac{2V}{\pi w L}(\beta - \sin(\beta)) + \frac{4V_d^2}{\pi R V}(1 - \cos(\beta))$$

□□□

3.4. Following a softer desired current signal

An alternative strategy that avoids the discontinuities presented in the previous scheme, consists in approximating the rectified sinusoidal reference signal by a signal of the form,

$$x_{1d}^* = \frac{4V_d^2}{\pi R V}\left(1 - \frac{2}{3}\cos(2wt)\right) \quad (41)$$

The time derivative of this signal is,

$$\dot{x}_{1d}^* = \frac{16V_d^2 w}{3\pi R V}\sin(2wt) \quad (42)$$

For such a reference signal, the existence condition for a sliding motion is given by,

$$0 \leq V\sin(wt) - \frac{16V_d^2 wL}{3\pi R V}\sin(2wt) + R_1(x_1 - x_{1d}^*) \leq x_{2d}$$

which in steady state, is further reduced to

$$4V_d\sqrt{\frac{2wL}{3\pi R}} \leq V \leq V_d \quad (43)$$

As a consequence, by appropriately choosing the involved parameters we can guarantee perfect tracking during the entire period.

The *ac-line current signal* $x_I(t)$, at the input of the diode rectifier takes now the alternate symetrical form shown in fig. 3.

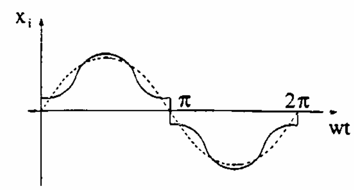

Fig. 3: Input current time response. Approximate version

The *Power factor analysis* is carried out in a similar manner as before. The FPF is now given by,

$$FPF = cos\phi_1 = \frac{X_2}{|\langle x_I \rangle_1|} = 1 \quad (44)$$

The *root mean square value* of $x_I(t)$ is computed as,

$$x_{IRMS} = \frac{4V_d^2}{\pi R V}\sqrt{1 + \frac{2}{9}} \quad (45)$$

The *ac-line current distortion* denoted by CDF is defined as,

$$CDF = \frac{|\langle x_I \rangle_1|}{\sqrt{2}x_{IRMS}} \quad (46)$$

which in our case turns out to be,

$$CDF = \frac{2\sqrt{22}}{3\pi} \approx 0.99 \quad (47)$$

The *power factor* PF is now obtained as

$$PF = \frac{2\sqrt{22}}{3\pi} \approx 0.99 \quad (48)$$

4. SIMULATION RESULTS

Digital computer simulations were performed for evaluating of the proposed feedback controller. The following parameters were used, $R = 100\Omega$, $L = 10mH$, $C = 2200\mu F$, $V = \sqrt{2} \times 115$ Volts, $w = 2\pi \times (60)$ rad/seg. $V_d = 215$ Volts, $R_1 = 1$, $R_2 = 1$. These parameters were taken from the implementation of a PFC control of ac-mains current supplying the battery charging power in a 5kW high performance off-line uninterruptible power supply system reported in [4]. For this example the power factor takes the value PF =0.999.

In fig. 4 the responses of the Boost PFP under sliding mode plus passivity based controller are shown. In this case, the desired current signal to be followed is a rectified sinusoidal signal.

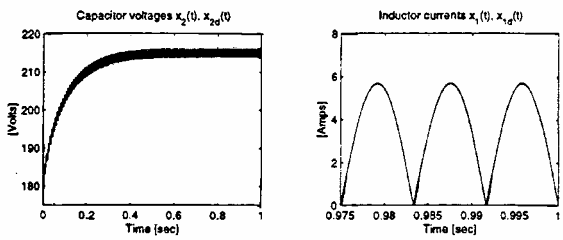

Fig. 4: Time response for the Boost PFP

In fig. 5 the closed loop responses are shown for the PFP converter using the sinusoidal biased approximation of the rectified sinusoidal signal.

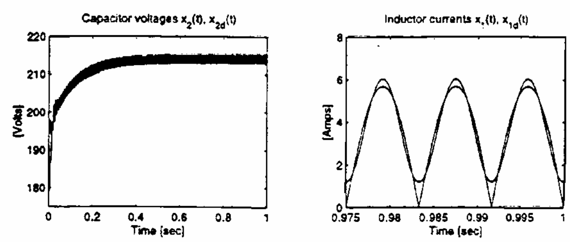

Fig. 5: Time response for an approximative desired current

REFERENCES

[1] CH. BYRNES, A. ISIDORI AND J. C. WILLEMS. "Passivity, Feedback Equivalence and the Global Stabilization of Minimum Phase Nonlinear Systems" *IEEE TAC*, Vol. 36, No. 11, 1228-1240, 1991.

[2] D. HILL AND P. MOYLAN. "The Stability of Nonlinear Dissipative Systems", *IEEE TAC*, pp. 708-711. 1976.

[3] R. ORTEGA AND M. SPONG. "Adaptive Motion Control of Rigid Robots: A Tutorial". *Automatica*, **25**, 877-888. 1989.

[4] SALMON J.C. "Techniques for Minimizing the Input Current Distortion of Current-Controlled Single-Phase Boost Rectifiers", *IEEE Trans. on Power Electronics*, Vol. 8, No. 4, pp. 509-520, 1993.

[5] SIRA-RAMÍREZ, H., M. PÉREZ-MORENO. R. ORTEGA AND M. GARCÍA-ESTEBAN, "Passivity-Based Controllers for the Stabilization of DC-to-DC Power Converters", *Automatica*, Vol. 39, No. 4, pp. 499-513, 1997.

[6] SIVAKUMAR S., K. NATARAJAN AND R. GUDELEWICZ "Control of Power Factor Correcting Boost Converter without Instantaneous Measurement of Input Current". *IEEE Trans. on Power Electronics*, Vol. 10, No. 4, pp. 435-445, 1995.

[7] V.I, UTKIN, *Sliding Modes in the Theory of Variable Structure Systems*, MIR Editors, Moscow 1978.