On the Control of the Underactuated Ship : A Trajectory Planning Approach [1]

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Abstract

A trajectory planning approach is proposed for controlled path tracking maneuvers in an underactuated ship model. The trajectories of the position coordinates are planned in an off-line manner exploiting the partial *differential flatness* of the non-linear model and its *Liouvillian* character. The nominal state and control inputs trajectories are then used, in the traditional manner, to obtain an incremental feedback control law, computed on the basis of the controllable, time-varying, Jacobian linearization model.

1 Introduction

The control of a ship having two independent thrusters, located at the aft, has received sustained attention in the last few years. The interest in devising feedback control strategies for the underactuated ship model stems from the fact that the system does not satisfy Brockett's necessary condition for stabilization to the origin by means of time-invariant state feedback (see Brockett, [1]). Reyhanoglu [13] proposes a discontinuous feedback control which locally achieves exponential decay towards a desired equilibrium. A feedback linearization approach was proposed by Godhavn [8] for the regulation of the position variables without orientation control. In an article by Pettersen and Egeland [9], a time-varying feedback control law is proposed which exponentially stabilizes the state towards a given equilibrium point. Time varying quasi-periodic feedback control, as in Pettersen and Egeland [10], has been proposed exploiting the homogeneity properties of a suitably transformed model achieving simultaneous exponential stabilization of the position and orientation variables. A remarkable experimental set-up has been built which is described in the work of Pettersen and Fossen [11]. In this work the time varying feedback control found in [9] is extended to include integral control actions with excellent experimental results. High frequency feedback control signals, in combination with averaging theory and backstepping, has also been proposed by Pettersen and Nijmeijer [12] to obtain practical stabilization of the ship towards a desired equilibrium and also for trajectory tracking tasks.

In this article, we propose a feedback control scheme based on trajectory planning and approximate linearization around the off-line computed trajectory. For the trajectory planning apects, use is made of the fact that the system model exhibits a *differentially fla* subsystem (see the work of Fliess and his colleages [5]-[6] for a full discussion of the concept and its many implications). The flat subsystem is characterized by the angular orientation variable and the sway velocity. The control inputs can then be readily computed as *differential funcitons* of the flat variables, i.e., functions of the flat variables and a finite number of their time derivatives. The remaining variables (i.e., the position variables) can be expressed as *quadratures* of differential funcitons of the flat variables. Nonflat systems satisfying this last property are known as *Liouvillian* systems. They were first introduced in the work of Chelouah [3]. Liouvillian systems constitute an extension of the class of differentially flat systems with interesting implications in the *exact discretization* of nonlinear systems, as well as in other areas of nonlinear control.

In Section 2 of this article we present some generalities about Liouvillian systems and propose a feedback control scheme for such systems which is based on "trajectory planning". In Section 3 we describe the underactuated ship model to be used in our developments (this model, taken from [12], was developed by Fossen [7]). We show that the model is Liouvillian and proceed to describe an off-line trajectory planning by

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inverting the integro-differential parametrization defining the nonflat variables. A comnination of high-gain and linearizing control is used to exponentially stabilize the linearized tracking error variables towards the origin. Section 4 presents the simulation results and Section 5 presents the conclusions and some suggestions for furhter reserarch.

2 Liouvillian systems

Liouvillian systems constitute a natural extension of *differentially flat systems* into the area of systems which are non equivalent to controllable linear systems by means of endogenous feedback. The class of Liouvillian systems contains a subset of the class of non-flat systems with an identifiable flat subsystem of maximal dimension. A nonflat system is said to be Liouvillian, or *integrable by quadratures*, if the variables not belonging to the flat subsystem are expressible as elementary integrations of the flat outputs and a finite number of their time derivatives. An exposition about Liouvillian systems, from the perspective of Differential Galois theory in the context of Piccard-Vessiot extensions of differentially flat fields, may be found in [3].

Consider the following composite system, addressed, in general, as a "feedforward integrator" system,

$$\begin{aligned} \dot{z} &= g(x,u), \quad z \in R^{n_z}, u \in R^m \\ \dot{x} &= f(x,u), \quad x \in R^{n_x} \end{aligned} \tag{1}$$

Suppose that the subsystem described by the x variables is differentially flat. Hence, a set of m output variables, denoted by y, may be found such that, at least locally, a *differential parametrization* of the state vector x, and the control inputs u, can be obtained as follows,

$$x = \psi(y, \dot{y}, \dots, y^{(\alpha)}) \quad ; \quad u = \vartheta(y, \dot{y}, \dots, y^{(\alpha+1)}) \tag{2}$$

The above description of x and u implies that the nonflat variables z may be expressed as

$$\dot{z} = \gamma(y, \dot{y}, \dots, y^{(\alpha+1)}) \tag{3}$$

In some cases, a specification of the flat variables y as functions of time, denoted by $y^*(t)$, already fully describes the control objectives for the designer. The suitably initialized integration of (3), and the differential parametrization (2), allow then the computation, in an off-line fashion, of all the corresponding trajectories for the state variables of the system, $x^*(t)$, $z^*(t)$, as well as the associated nominal control inputs, $u^*(t)$. Such off-line computed state and input reference trajectories can be readily used in an approximate linearization based feedback control scheme for the design of the incremental feedback control inputs, u_δ. The feedback incremental inputs appropriately complement the nominal input signals, $u^*(t)$, for solving the trajectory tracking task. The underlying control design problem entitles the regulation, towards the origin, of a time-varying linearized system described by the incremental state variables $x_\delta = x - x^*(t)$ and $z_\delta = z - z^*(t)$. The regulation task is to be accomplished by means of an appropriately designed feedback control law for the incremental control inputs, $u_\delta = u_\delta(x^*(t), z^*(t), x_\delta, z_\delta)$.

The more interesting case, however, is constituted by that in which the nonflat variables z are to track a given prespecified trajectory $z^*(t)$, proposed as the control objective by the designer. Generally speaking, the Jacobian matriz $\partial\gamma/\partial y^{(\alpha+1)}$ is non-invertible and the relation (3) does not always properly define a set of differential, or algebraic, equations for the flat variables y's defined now in terms of the given $z^*(t)$, viewed as given *differential parameters*. In some particular cases only a subset of the flat variables y may be directly obtained in terms of a subset of the z's and their corresponding time derivatives. In such cases, the computation of such flat variables can be performed without integrating differential equations. In general, however, provided that suitable initial conditions are known for the flat variables, y and a finite number of their time derivatives, one is led to obtain, from (3), a differential-algebraic relation of the following form,

$$\mu(z, \dot{z}, \dots, z^{(\beta)}, y, \dot{y}, \dots, y^{(\delta)}) = 0 \tag{4}$$

From a relation of this sort, the flat variables trajectories, $y^*(t)$, can be locally computed in correspondence with the specified non-flat variables desired reference trajectories, $z^*(t)$. This, of course, entitles solving the following set of *implicit* differential algebraic equations for the flat variables y, with appropriate initial conditions.

$$\mu(z^*(t), \dot{z}^*(t), \dots, \frac{d^\beta}{dt^\beta} z^*(t), y, \dot{y}, \dots, y^{(\delta)}) = 0 \tag{5}$$

3 The underactuated ship model

In a recent paper by Pettersen and Nijmeijer, [12], the following mathematical model is proposed for the kinematics of an underactuated ship, which includes a sway acceleration constraint,

$$\begin{aligned} \dot{x} &= \cos(\psi)u_1 - \sin(\psi)z \\ \dot{y} &= \sin(\psi)u_1 + \cos(\psi)z \\ \dot{\psi} &= u_2 \\ \dot{z} &= -\gamma u_1 u_2 - \beta z \end{aligned} \tag{6}$$

In the above model, x, y and ψ determine, respectively, the position and orientation of the ship in reference to a fixed earth frame. The control input variables u_1 and u_2 represent, in fact, the surge (i.e., forward) and yaw velocities. The state variable z represents the sway velocity. The constants γ and β are strictly positive constants with $\gamma < 1$.

System (6) is not *differentially flat*, but it is, nevertheless, *Liouvillian*. This means that it contains a flat subsystem, here represented by the states ψ and z, while the remaining variables, x and y, can be expressed as quadratures of *differential functions* of the flat variables (i.e., functions of the flat variables and a finite number of their time derivatives).

3.1 The Liouvillian character of the underactuated ship model

Consider the system variables $\mathcal{F} = \psi$ and $\mathcal{L} = z$. The subsystem characterized by the states ψ and z is differentially flat, as the following differential parametrization demonstrates,

$$\begin{array}{rclrcl} \psi & = & \mathcal{F}, & z & = & \mathcal{L} \\ u_1 & = & -\frac{\dot{\mathcal{L}}+\beta\mathcal{L}}{\gamma\dot{\mathcal{F}}}, & u_2 & = & \dot{\mathcal{F}} \end{array} \qquad (7)$$

An integro-differential parametrization of the nonflat position variables $\mathcal{W} = x$ and $\mathcal{S} = y$, in terms of the flat variables $\mathcal{F}$ and $\mathcal{L}$, is given by

$$\begin{aligned} \dot{\mathcal{W}} &= \left(-\frac{\dot{\mathcal{L}}+\beta\mathcal{L}}{\gamma\dot{\mathcal{F}}}\right)\cos(\mathcal{F}) - \mathcal{L}\sin(\mathcal{F}) \\ \dot{\mathcal{S}} &= \left(-\frac{\dot{\mathcal{L}}+\beta\mathcal{L}}{\gamma\dot{\mathcal{F}}}\right)\sin(\mathcal{F}) + \mathcal{L}\cos(\mathcal{F}) \qquad (8) \end{aligned}$$

Hence, displacement variables $\mathcal{W}$ and $\mathcal{S}$ are expressible as *quadratures* of *differential functions* of the flat outputs, i.e.,

$$\begin{aligned} \mathcal{W} &= \int\left\{-\left(\frac{\dot{\mathcal{L}}+\beta\mathcal{L}}{\gamma\dot{\mathcal{F}}}\right)\cos\mathcal{F} - \mathcal{L}\sin\mathcal{F}\right\}dt \\ \mathcal{S} &= \int\left\{-\left(\frac{\dot{\mathcal{L}}+\beta\mathcal{L}}{\gamma\dot{\mathcal{F}}}\right)\sin\mathcal{F} + \mathcal{L}\cos\mathcal{F}\right\}dt \qquad (9) \end{aligned}$$

Notice that the Jacobian matrix

$$\frac{\partial\left(\dot{\mathcal{W}},\dot{\mathcal{S}}\right)}{\partial\left(\dot{\mathcal{F}},\dot{\mathcal{L}}\right)} = \begin{bmatrix} \frac{\dot{\mathcal{L}}+\beta\mathcal{L}}{\gamma\dot{\mathcal{F}}^2}\cos\mathcal{F} & -\frac{\cos\mathcal{F}}{\gamma\dot{\mathcal{F}}} \\ \frac{\dot{\mathcal{L}}+\beta\mathcal{L}}{\gamma\dot{\mathcal{F}}^2}\sin\mathcal{F} & -\frac{\sin\mathcal{F}}{\gamma\dot{\mathcal{F}}} \end{bmatrix} \qquad (10)$$

is singular and, therefore, a set of simultaneous differential equations for $\mathcal{F}$ and $\mathcal{L}$, parametrized in terms of differential functions of $\mathcal{W}$ and $\mathcal{S}$ cannot be obtained. However, one may still *invert* by other means the integrodifferential parametrization (8) and proceed to obtain, after some algebraic manipulations, the following *explicit* set of differential-algebraic equations for $\mathcal{F}$ and $\mathcal{L}$

$$\begin{aligned} \dot{\mathcal{F}} &= \frac{1}{(\gamma-1)\left(\dot{\mathcal{W}}(t)\cos\mathcal{F}+\dot{\mathcal{S}}(t)\sin\mathcal{F}\right)} \times \\ &\quad \left[\left(\ddot{\mathcal{W}}(t)+\beta\dot{\mathcal{W}}(t)\right)\sin\mathcal{F} - \left(\ddot{\mathcal{S}}(t)+\beta\dot{\mathcal{S}}(t)\right)\cos\mathcal{F}\right] \\ \mathcal{L} &= -\dot{\mathcal{W}}(t)\sin\mathcal{F} + \dot{\mathcal{S}}(t)\cos\mathcal{F} \qquad (11) \end{aligned}$$

Remark 3.1 *Notice, that by straightforward manipulations on the system equations (6) one may also obtain the following* static *relation for $\mathcal{F}$,*

$$\mathcal{F}(t) = \arctan\left(\frac{\dot{\mathcal{S}}(t)}{\dot{\mathcal{W}}(t)}\right) - \arctan\left(\frac{\mathcal{L}(t)}{u_1(t)}\right) \qquad (12)$$

which clearly depicts the specific influence of the sway velocity $\mathcal{L}$ on the instantaneous value of the orientation angle $\mathcal{F}$.

3.2 Off-line trajectory planning

Suppose that a desired reference trajectory is specified for the nonflat outputs, $\mathcal{W}$ and $\mathcal{S}$, as the time-varying signals, $\mathcal{W}^*(t)$, $\mathcal{S}^*(t)$, respectively. Then, the algebraic-differential system (11), particularized for the specified $\mathcal{W}^*(t)$, $\mathcal{S}^*(t)$, is to be viewed as an *off-line trajectory planning solver* which produces the required corresponding trajectories for $\mathcal{F}^*(t)$ and $\mathcal{L}^*(t)$. The initial condition for the differential equation defining $\mathcal{F}^*$ is obtained from the nominal initial value $\psi(t_0)$ of the orientation angle ψ corresponding to the reference trajectory initial conditions $x(t_0) = \mathcal{W}^*(t_0)$, $y(t_0) = \mathcal{S}^*(t_0)$, $\dot{x}(t_0) = \dot{\mathcal{W}}^*(t_0)$ and $\dot{y}(t_0) = \dot{\mathcal{S}}^*(t_0)$. In fact, if $z(t_0) = \mathcal{L}(t_0) = 0$, then,

$$\mathcal{F}(t_0) = \psi(t_0) = \arctan\frac{\dot{\mathcal{S}}(t_0)}{\dot{\mathcal{W}}(t_0)} \qquad (13)$$

The nominal control inputs, $u_1^*(t)$, $u_2^*(t)$, corresponding to the off-line computed trajectories may also be readily obtained in terms of the planned signals $\mathcal{W}^*(t)$, $\mathcal{S}^*(t)$ and $\mathcal{F}^*(t)$ as follows,

$$\begin{aligned} u_1^*(t) &= \dot{\mathcal{W}}^*(t)\cos(\mathcal{F}^*(t)) + \dot{\mathcal{S}}^*(t)\sin(\mathcal{F}^*(t)) \\ u_2^*(t) &= \dot{\mathcal{F}}^*(t) \qquad (14) \end{aligned}$$

The Jacobian linearization of system (6) around the planned trajectories $\mathcal{W}^*(t)$, $\mathcal{S}^*(t)$, $\mathcal{F}^*(t)$, $\mathcal{L}^*(t)$, yields the following time-varying linear system

$$\begin{aligned} \dot{x}_\delta &= -\dot{\mathcal{S}}^*(t)\psi_\delta - \sin(\mathcal{F}^*(t))z_\delta + \cos(\mathcal{F}^*(t))u_{1\delta} \\ \dot{y}_\delta &= \dot{\mathcal{W}}^*(t)\psi_\delta + \cos(\mathcal{F}^*(t))z_\delta + \sin(\mathcal{F}^*(t))u_{1\delta} \\ \dot{\psi}_\delta &= u_{2\delta} \\ \dot{z}_\delta &= -\beta z_\delta - \gamma u_2^*(t)\, u_{1\delta} - \gamma u_1^*(t)\, u_{2\delta} \qquad (15) \end{aligned}$$

where,

$$x_\delta = x - \mathcal{W}^*(t) \; ; \; y_\delta = y - \mathcal{S}^*(t)$$
$$\psi_\delta = \psi - \mathcal{F}^*(t) \; ; \; z_\delta = z - \mathcal{L}^*(t) \quad (16)$$

The incremental control inputs $u_{1\delta}$, $u_{2\delta}$, complement the nominal precomputed control signals $u_1^*(t)$, $u_2^*(t)$, in the usual manner,

$$u_1 = u_1^*(t) + u_{1\delta}$$
$$u_2 = u_2^*(t) + u_{2\delta} \quad (17)$$

with $u_1^*(t)$ and $u_2^*(t)$ being the (nominal) open loop reference control inputs that would, ideally, steer the ship along the nominal reference trajectory $(W^*(t), S^*(t), \mathcal{F}^*(t), \mathcal{L}^*(t))$, provided the actual initial conditions were precisely set at the nominal values and no perturbations were ever present along the prescribed path.

Is is easy to show that the linear system (15) is controllable. This may be checked using the well known controllability rank criterion for time-varying linear systems of the form, $\dot{\xi} = A(t)\xi + B(t)u$, $\xi \in R^n$, given by (sec, for instance, Fliess, [4]), rank $[B(t), (A(t) - \frac{d}{dt})B(t), \ldots, (A(t) - \frac{d}{dt})^{n-1}B(t)] = n$ where, in this case, we must take

$$A(t) = \begin{bmatrix} 0 & 0 & -\dot{\mathcal{S}}^*(t) & -\sin\mathcal{F}^*(t) \\ 0 & 0 & \dot{\mathcal{W}}^*(t) & \cos\mathcal{F}^*(t) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta \end{bmatrix}$$
$$B(t) = \begin{bmatrix} \cos\mathcal{F}^*(t) & 0 \\ \sin\mathcal{F}^*(t) & 0 \\ 0 & 1 \\ -\gamma u_2^*(t) & -\gamma u_1^*(t) \end{bmatrix} \quad (18)$$

3.3 A feedback control law for trajectory tracking error stabilization

Consider next a time-varying state coordinate transformation for z_δ, written in terms of a new state variable, denoted by σ_δ, which has a linearizing effect on the system equations when the incremental state trajectories are constrained to the manifold $\sigma_\delta = 0$. Within this context, consider also the following incremental state feedback control law for $u_{1\delta}$ and $u_{2\delta}$, given by

$$z_\delta = -u_1^*(t)\psi_\delta + k_x x_\delta \sin(\mathcal{F}^*(t)) - k_y y_\delta \cos(\mathcal{F}^*(t)) + \sigma_\delta$$
$$u_{1\delta} = \mathcal{L}^*(t)\psi_\delta - k_x x_\delta \cos(\mathcal{F}^*(t)) - k_y y_\delta \sin(\mathcal{F}^*(t))$$
$$u_{2\delta} = \frac{1}{(1-\gamma)u_1^*(t)} \Big\{ \beta z_\delta + \gamma u_2^*(t) \times \Big[\mathcal{L}^*(t)\psi_\delta - k_x x_\delta \cos(\mathcal{F}^*(t)) - k_y y_\delta \sin(\mathcal{F}^*(t))\Big]$$
$$-\dot{u}_1^*(t)\psi_\delta + k_x \dot{x}_\delta \sin(\mathcal{F}^*(t)) + k_x \dot{\mathcal{F}}^*(t) x_\delta \cos(\mathcal{F}^*(t))$$
$$-k_y \dot{y}_\delta \cos(\mathcal{F}^*(t)) + k_y \dot{\mathcal{F}}^*(t) y_\delta \sin(\mathcal{F}^*(t)) - k_\sigma \theta(\sigma_\delta) \Big\} \quad (19)$$

where k_x, k_y and k_σ are taken to be strictly positive design constants while $\theta(\sigma_\delta)$ is a nonlinear function of the new state coordinate σ_δ, to be specified as a "high-gain" function, or as a suitable "smooth" approximation to a "switch" function of the *signum* type, defined on the basis of the values of σ_δ around the manifold $\sigma_\delta = 0$. The variable σ_δ thus replaces the incremental sway velocity z_δ.

After some simplifications, the closed loop transformed system is given by the following set of time-varying differential equations.

$$\dot{x}_\delta = -k_x\, x_\delta - \sin(\mathcal{F}^*(t))\sigma_\delta$$
$$\dot{y}_\delta = -k_y\, y_\delta + \cos(\mathcal{F}^*(t))\sigma_\delta$$
$$\dot{\psi}_\delta = \frac{1}{(1-\gamma)u_1^*(t)} \Big\{ -k_\sigma\theta(\sigma_\delta) - \Big[\dot{u}_1^*(t) + \beta u_1^*(t) - \gamma u_2^*(t)\mathcal{L}^*(t)\Big]\psi_\delta + \Big[\beta - k_x \sin^2(\mathcal{F}^*(t)) - k_y \cos^2(\mathcal{F}^*(t))\Big]\sigma_\delta$$
$$+\Big[(\beta - k_x)\sin(\mathcal{F}^*(t)) + (1-\gamma)\, u_2^*(t)\cos(\mathcal{F}^*(t))\Big] k_x x_\delta$$
$$-\Big[(\beta - k_y)\cos(\mathcal{F}^*(t)) - (1-\gamma)\, u_2^*(t)\sin(\mathcal{F}^*(t))\Big] k_y y_\delta \Big\}$$
$$\dot{\sigma}_\delta = -k_\sigma\theta(\sigma_\delta) \quad (20)$$

A rather natural choice of the design parameters and involved functions is given by

$$k_x = k_y = \beta \; ; \; \theta(\sigma_\delta) = \frac{\sigma_\delta}{|\sigma_\delta| + \epsilon} \quad (21)$$

where ϵ is a small, striclty positive, constant.

A Lyapunov stability theory argument shows that the trajectories $\sigma_\delta(t)$ of the variable σ_δ globally exponentially converge to zero. The closed loop linear time-varying system is thus excited by an external $\mathcal{L}_2$ signal. The time-varying eigenvalues of the linear part of the system are given by

$$\lambda_1 = \lambda_2 = -\beta$$
$$\lambda_3(t) = -\frac{\Big[\dot{u}_1^*(t) + \beta u_1^*(t) - \gamma u_2^*(t)\mathcal{L}^*(t)\Big]}{(1-\gamma)u_1^*(t)} \quad (22)$$

It is not difficult to show, in view of the relations (11) and (14), that the linear subsystem eigenvalue, $\lambda_3(t)$, is bounded above by a strictly negative constant. Furthermore, the closed loop linear system matrix has a bounded norm and this norm has a continuous first order time derivative. As a result, it follows from linear systems theory (see Callier and Desoer [2]), that the trajectories of the incremental states, describing the closed loop behavior of the tracking errors, are globally exponentially stable to zero.

4 Simulation Results

Simulations were carried out for assessing the performance of the designed feedback controller on the ship model. Following [12], we chose a typical trajectory on the $(\mathcal{W}, \mathcal{S})$-plane, parametrized by time, and given by

$$\mathcal{W}^*(t) = 10\sin(0.01\, t)\ ;\ \mathcal{S}^*(t) = 10\,(1 - \cos(0.01\, t))$$

The above set of parametric equations correspond to a circumpherence of radius 10, centered at the point $(x, y) = (0, 5)$. Figure 2 shows the results of the off-line trajectory planning task. The graph for the orientation angle $\mathcal{F}^*$ was obtained by solving the differential equation for $\mathcal{F}$ in (11), with initial condition $\mathcal{F}^*(t_0) = 0$ and taking $\mathcal{W}^*(t), \dot{\mathcal{W}}^*(t), \ddot{\mathcal{W}}^*(t)$ and $\mathcal{S}^*(t), \dot{\mathcal{S}}^*(t), \ddot{\mathcal{S}}^*(t)$, as given data. The corresponding nominal sway velocity $\mathcal{L}^*(t)$ was directly computed using (11). The nominal, open loop, control signals $u_1^*(t)$ and $u_2^*(t)$ are also shown in this figure.

We remark that, as it follows from (11) and the above prescription for $\mathcal{W}^*(t)$ and $\mathcal{S}^*(t)$, that the periodic reference signal $\mathcal{L}^*$ has indeed a small amplitude, of the order of 0.1. Also, the reference solution for $\mathcal{F}^*(t)$, as obtained from (11), is given by a steadily growing, slightly oscillatory, signal representing the naturally growing orientation angle along the prescribed circular trajectory.

Figure 3 shows the closed loop controlled trajectory of the ship in the (x, y) plane, along with the actual orientation angle ψ, the controlled sway velocity z and the feedback control input signals. The following perturbed values were set for the required initial conditions in the presented simulations.

$$x(t_0) = -1.0\ ;\ y(t_0) = -1.0\ ;\ z(t_0) = 0\ ;\ \psi(t_0) = -1$$

In the simulations, the system parameters and the design parameters were taken to be,

$$\begin{aligned} \gamma &= 0.58\ ;\ \beta = 0.07\ ;\ k_x = 0.07\ ;\ k_y = 0.07 \\ k_\sigma &= 0.1\ ;\ \theta(\sigma_\delta) = \frac{\sigma_\delta}{|\,\sigma_\delta\,| + \epsilon}\ ;\ \epsilon = 1 \end{aligned}$$

5 Conclusions

In this article we have proposed a feedback control scheme for trajectory tracking in a class of non-differentially flat systems of the Liouvillian type. The approach consists in first performing the necessary off-line trajectory computations based on the partial flatness of the system and inversion of the integro-differential parametrization relating the flat variables and the nonflat variables. The nature of the desired trajectories, whether specified in terms of the flat or nonflat variables, determines the degree of difficulty in computing the required open loop trajectories for all of the state variables and the corresponding nominal (open loop) control inputs. Once the reference state and control input trajectories are determined, as time varying functions, a classical approximate linearization approach may be used to devise an incremental (on line) feedback controller which complements the nominal control inputs and suitably corrects the deviations, due to modeling errors and external perturbations, of the actual state trajectory with respect to the prespecified one.

Systematic controller design procedures are needed for the regulation and tracking tasks of non-differentially flat systems. Challenging nonflat mechanical and electrical systems await for analytical treatment and the proposal of reasonable and conceptually clear feedback control solution schemes.

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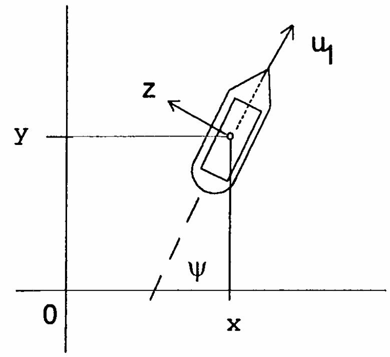

Figure 1: Position variables in earth cartesian coordinates and surge and sway velocities

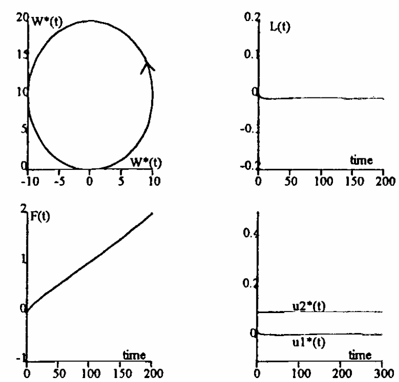

Figure 2: Open loop planned trajectories for the ship variables

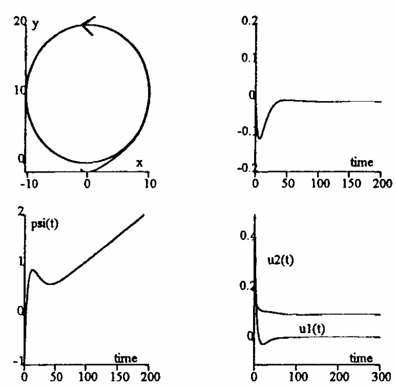

Figure 3: Closed loop performance of the feedback controller