SLIDING MODE CONTROL OF A PM STEPPER MOTOR FROM THE PERSPECTIVE OF DIFFERENTIALLY FLAT SYSTEMS

Mohamed Zribi* , Hebertt Sira-Ramirez†, Andy Ngai Hin Sung‡
*ECE Depart. College of Engineering & Petroleum, Kuwait University
P. O. Box 5969, Safat 13060, Kuwait
Fax: (965) 481-7451, E-mail: mzribi@eng.kuniv.edu.kw
†Departamento Sistemas de Control, Escuela de Ingenieria de Sistemas,
Universidad de Los Andes, Merida 5101, Venezuela.
‡School of Electrical and Electronic Engineering, Nanyang Tech. University
Singapore, 639798, Singapore

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Abstract

In this paper, the sliding mode control of a permanent magnet (PM) stepper motor is addressed from the perspective of differentially flat systems. A static and a dynamic discontinuous feedback control schemes are proposed. Implementation results of these control schemes on an experimental set-up are given to illustrate the developments. Both, the results of the single stepping and multi-stepping cases are shown. In addition, comparisons of the performances of the two sliding mode control schemes with the performance of a feedback linearization control scheme are given.

1 Introduction

Due to their inherent robustness properties and conceptual simplicity, discontinuous feedback control schemes of the sliding mode type, have enjoyed deserved attention from researchers and practitioners. The fundamental results of sliding mode control are found in the many books already available on the subject. The interested reader is referred to the books by Utkin [1] and Zinober [2]. Sliding mode control of nonlinear multivariable systems has been addressed from different viewpoints. The differential geometric approach to the design problem received the attention of Sira-Ramirez [3]. A different approach to the multivariable sliding mode control problem for linear systems was given by Fliess and Sira-Ramirez [4]. In this approach, module theory is used in a special manner to formulate and uncover the fundamental differential algebraic nature of the problem. Sira-Ramirez [5] addressed the problem of sliding mode control of multivariable nonlinear systems, from the perspective of linear differential algebra, for a special class of linearizable systems.

In this article we propose the design of sliding mode controllers for a permanent magnet stepper motor. The design approach is based on the developed theory of differentially flat systems. Section 2 of this article discusses the generalities of multivariable sliding mode controller design for differentially flat systems. Section 3 contains a brief overview of stepper motors as well as the nonlinear model of the PM stepper motor. Section 4 deals with the design of a static sliding mode regulation of a multivariable permanent magnet stepper motor. Section 5 deals with the design of a dynamic sliding mode controller for the stepper motor. Section 6 discusses the implementation results of the proposed control schemes. Both, the single step case and the multi-step case are treated. A brief comparison of the performance of the proposed control schemes with the performance of a feedback linearization control scheme is also provided. Finally section 7 contains the conclusion.

2 Sliding Mode Control of Differentially Flat Systems

Flat systems were first introduced by Fliess and his co-workers in [6] and further developed and characterized by Fliess et al. [7]. Practical examples of some mechanical systems, such as the truck and the trailer, the jumping robot, and the crane were presented in [7]. Levine et al. [8] used the flatness property of the magnetic levitation model of a beam to design a nonlinear control scheme for its positioning. Martin and Rouchon [9] proposed a sampling control strategy for flat systems; this strategy was applied to induction motors. Rothfub et al. [10] exploited the flatness of a chemical reactor model to design a linearizing quasi-static feedback controller for it.

Differentially flat systems constitute a widespread class of dynamic systems which represent the simplest possible extension of controllable linear systems to the nonlinear systems domain [5]. Flat systems enjoy the property of possessing a finite set of differentially independent outputs called linearizing outputs. Flat systems are thus dynamic systems which are linearizable to a controllable linear system by means of endogenous feedback, i.e., one that does not require external variables to the system to be completely defined.

Since the control inputs to the system u are differential functions of the linearizing outputs y, then one may impose on the highest derivatives of such linearizing output components, which appear in the control input components expressions, a particular linear relation involving only smaller order derivatives of the same output component. One immediately obtains the required linearizing controller expression in terms of the involved linearizing outputs.

In the following, we consider the design of a multivariable sliding mode controller for a permanent magnet stepper motor. The system is first shown to be differentially flat with linearizing outputs given by physically meaningful variables constituted by a combination of the currents in phases A and B of the motor, and the rotor angular position of the shaft of the motor. A static sliding mode controller is then designed which stabilizes the system to the required equilibrium point. A dynamic sliding mode controller design is also carried out and comparisons with the static controller are made, on the basis of implementation results.

3 The Stepper Motor System

3.1 A brief overview of stepper motors

In recent years, the rapid growth of digital electronics has indirectly influenced the development of the stepper motor technology [11]. Stepper motors are now widely used in numerous motion-control applications such as: robots, printers, process control systems, index table for automatic assembly machines, etc.

When operated in open loop mode, stepper motors are generally oscillatory in their positional response. Driving a high inertia load at low speeds aggravates the oscillations of the motor shaft about the equilibrium position.

Over the years, many control algorithms that can be used to improve the performance of stepper motors have been examined. Zribi and Chiasson [12] developed an exact feedback linearization control method for controlling a permanent magnet stepper motor. Bodson and Chiasson [13] presented a control scheme based on feedback linearization with adaptive rules that can be used to estimate the parameters of the system. Speagle and Dawson [14] developed an adaptive tracking controller for stepper motors.

3.2 Model of the PM stepper motor

Consider the following model of a permanent magnet stepper motor [12]

$$\begin{aligned}
\frac{di_a}{dt} &= \frac{1}{L}(v_a - Ri_a + K_m\omega\sin(N_r\theta)) \\
\frac{di_b}{dt} &= \frac{1}{L}(v_b - Ri_b - K_m\omega\cos(N_r\theta)) \qquad (1)\\
\frac{d\omega}{dt} &= \frac{1}{J}(-K_m i_a\sin(N_r\theta) + K_m i_b\cos(N_r\theta) - B\omega) \\
\frac{d\theta}{dt} &= \omega
\end{aligned}$$

where, i_a is the current in winding A, i_b is the current in winding B, θ is the angular displacement of the shaft of the motor, ω is the angular velocity of the shaft of the motor, v_a is the voltage across the windings of phase A, and v_b is the voltage across the windings of phase B. Also, N_r is the number of rotar teeth, J is the rotar and load inertia, B is the viscous friction coefficient, L and R are the inductance and the resistance of each phase winding, and K_m is the motor torque (back-emf) constant.

Equations (1) which are used to described the stepper motor model, are highly nonlinear. A nonlinear transformation, known as the Direct-Quadrature (DQ) transformation [12] can be used to transform these equations into a form which is more suitable for designing nonlinear controllers. The DQ transformation is defined as:

$$\begin{bmatrix} i_d \\ i_q \\ \omega \\ \theta \end{bmatrix} = \begin{bmatrix} \cos(N_r\theta) & \sin(N_r\theta) & 0 & 0 \\ -\sin(N_r\theta) & \cos(N_r\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ \omega \\ \theta \end{bmatrix}$$

In addition, define the new inputs as:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \cos(N_r\theta) & \sin(N_r\theta) \\ -\sin(N_r\theta) & \cos(N_r\theta) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix}$$

where v_d is the direct voltage, v_q is the quadrature voltage, i_d is the direct current, i_q is the quadrature current.

In terms of these new inputs and state variables, the transformed model of the stepper motor can be written as,

$$\begin{aligned}
\frac{di_d}{dt} &= \frac{1}{L}(v_d - Ri_d + N_r\omega L i_q) \\
\frac{di_q}{dt} &= \frac{1}{L}(v_q - Ri_q - N_r\omega i_d - K_m\omega) \qquad (2)\\
\frac{d\omega}{dt} &= \frac{1}{J}(K_m i_q - B\omega) \\
\frac{d\theta}{dt} &= \omega
\end{aligned}$$

Let,

$x_1 = i_d$, $x_2 = i_q$, $x_3 = \omega$, $x_4 = \theta$, $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$,

$k_1 = \frac{R}{L}$, $k_2 = \frac{K_m}{L}$, $k_3 = \frac{K_m}{J}$, $k_4 = \frac{B}{J}$, $k_5 = N_r$, $u_1 = \frac{v_d}{L}$, $u_2 = \frac{v_q}{L}$

Then, equations (2) can be written as,

$$\begin{aligned}
\dot{x}_1 &= -k_1x_1 + k_5x_2x_3 + u_1 = f_1 + u_1 \\
\dot{x}_2 &= -k_1x_2 - k_5x_1x_3 - k_2x_3 + u_2 = f_2 + u_2 \qquad (3)\\
\dot{x}_3 &= k_3x_2 - k_4x_3 \\
\dot{x}_4 &= x_3
\end{aligned}$$

where,

$$f_1 = -k_1x_1 + k_5x_2x_3$$

$$f_2 = -k_1x_2 - k_5x_1x_3 - k_2x_3$$

It is desired that the rotor angular position tracks a given constant reference position θ_d. It is also desired that the direct current tracks a given constant reference direct current, I_{dd}. Therefore, the linearizing as well as the controlled outputs of the system are such,

$$z = y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_4 \end{bmatrix}$$

The stepper motor system is differentially flat with linearizing output coordinates given by $y_1 = x_1$ and $y_2 = x_4$. This is the case because all variables in the system can be written as differential functions of the linearizing outputs such that,

$$x_1 = y_1$$
$$x_2 = (\dot{x}_3 + k_4 x_3)/k_3 = (\ddot{y}_2 + k_4 \dot{y}_2)/k_3$$
$$x_3 = \dot{y}_2$$
$$x_4 = y_2$$

The control actions can be expressed using the linearizing output coordinates such that,

$$u_1 = \dot{y}_1 + k_1 y_1 - \frac{k_5}{k_3}(\ddot{y}_2 + k_4 \dot{y}_2)\dot{y}_2$$

$$u_2 = \frac{1}{k_3}\left(y_2^{(3)} + k_4 \ddot{y}_2\right) + \frac{k_1}{k_3}(\ddot{y}_2 + k_4 \dot{y}_2) + k_5 y_1 \dot{y}_2 + k_2 \dot{y}_2$$

From the expressions of u_1 and u_2 it follows that the linearized equations for the system are simply given by

$$\dot{y}_1 = v_1$$
$$y_2^{(3)} = v_2$$

In the following, a multivariable sliding mode controller which asymptotically regulates the output variables towards their desired equilibrium positions is proposed.

4 Static Sliding Mode Controller Design for a Stepper Motor

The static sliding mode control design entitles specifying the auxiliary, endogenous, control input variables v_1 and v_2 as sliding mode feedback control laws, such that the forced evolution of the linearized variables asymptotically converge towards their desired positions. Due to the physical nature of the actual control input variables u_1 and u_2, representing voltage signals for which a switching strategy is entirely feasible, we may proceed to specify a static sliding mode controller.

A sliding surface for the tracking of the linearizing output coordinate y_1 towards its equilibrium point I_{dd}, is constituted by the direct current stabilization error given by,

$s_1 = y_1 - I_{dd} = x_1 - I_{dd}$

For the regulation of the second linearizing coordinate, $y_2 = x_4$, a sliding surface expression is proposed which depicts a desired second order dynamic response for the controlled angular position $y_2 = x_4$, towards its desired equilibrium point $x_4 = \theta_d$. We propose that,

$$s_2 = \ddot{y}_2 + \alpha_1 \dot{y}_2 + \alpha_2 (y_2 - \theta_d)$$
$$= k_3 x_2 - k_4 x_3 + \alpha_1 x_3 + \alpha_2 (x_4 - \theta_d)$$

where α_1 and α_2 are positive design parameters.

Denote "sgn" for the signum function. To ensure that the linearizing output coordinates y_1 and y_2 will converge to I_{dd} and θ_d respectively, in a finite time, we impose the following sliding mode controlled dynamics on the evolution of the sliding surface coordinate functions s_1 and s_2,

$$\dot{s}_1 = -W_1 \operatorname{sgn}(s_1) \quad (4)$$
$$\dot{s}_2 = -W_2 \operatorname{sgn}(s_2) \quad (5)$$

where W_1 and W_2 are positive design parameters. Thus, the sliding mode dynamics in (4)-(5) yield the following required dynamics of the linearizing output coordinates y_1 and y_2.

$$\dot{y}_1 = -W_1 \operatorname{sgn}(s_1) \quad (6)$$
$$y_2^{(3)} + \alpha_1 \ddot{y}_2 + \alpha_2 \dot{y}_2 = -W_2 \operatorname{sgn}(s_2)$$

The first closed loop dynamics in (6) guarantees finite time reachability of the desired motor current equilibrium value I_{dd}

The second imposed dynamics in (6) implies a second order controlled response for the angular position x_4. Note that since s_2 is driven to zero in finite time, the linearizing output $y_2 = x_4$ is also governed, after such a finite amount of time, by the second order dynamics $\ddot{y}_2 + \alpha_1 \dot{y}_2 + \alpha_2 (y_2 - \theta_d) = 0$.

Hence, one immediately obtains the auxiliary control inputs v_1 and v_2 as,

$$v_1 = -W_1 \operatorname{sgn}(s_1)$$
$$v_2 = -\alpha_1 \ddot{y}_2 - \alpha_2 \dot{y}_2 - W_2 \operatorname{sgn}(s_2)$$

Therefore, the required actual controls inputs u_1 and u_2,

$$u_1 = -W_1 \operatorname{sgn}(s_1) + k_1 y_1 - \frac{k_5}{k_3} \dot{y}_2 (\ddot{y}_2 + k_4 \dot{y}_2)$$

$$u_2 = \frac{1}{k_3}\left[(k_1 + k_4 - \alpha_1)\ddot{y}_2 + (k_1 k_4 - \alpha_2)\dot{y}_2 - W_2 \operatorname{sgn}(s_2)\right] + k_5 y_1 \dot{y}_2 + k_2 \dot{y}_2$$

The obtained static sliding mode controllers can be expressed in terms of the original state variables of the system by simply substituting the linearizing coordinates y_1 and y_2, and their time derivatives, in terms of the original state variables. After some straightforward algebraic manipulations we obtain the following expressions for the static discontinuous feedback controllers,

$$u_1 = -W_1 \operatorname{sgn}(s_1) + k_1 x_1 - k_5 x_2 x_3 \quad (7)$$
$$u_2 = k_1 x_2 + k_5 x_1 x_3 + k_2 x_3 + k_4 x_2 - \alpha_1 x_2$$
$$- \frac{1}{k_3}\left[k_4^2 x_3 - \alpha_1 k_4 x_3 + \alpha_2 x_3 + W_2 \operatorname{sgn}(s_2)\right]$$

The previous analysis allows us to state the following theorem.

Theorem 1:

The discontinuous static feedback controller (7) when applied to the stepper motor system (3), asymptotically stabilizes the outputs of the system to their desired values.

In order to alleviate the chattering problem, the next section illustrates the design of a dynamic sliding mode controller for the PM stepper motor.

5 Dynamic Sliding Mode Controller Design for a Stepper Motor

From simulation results, chattering is observed when the sliding mode controller discussed in the previous section is used. This chattering is due to the basic assumption in variable structure control that control can be switched from one value to another at any moment and with almost zero time delay. However, in practical systems, it is not easy to achieve such switching control. This is the case because of two reasons. Firstly, there are time delays due to computations of the control actions. Secondly, there are physical limitations on the actuators used in the plants; most of the actuators contain coils that have rather high inductance values, hence input currents to these actuators cannot be switched at infinitely fast rates. It should also be mentioned that at steady state, chattering might cause oscillations about the desired equilibrium point of the system. In order to attenuate the chattering induced on the linearizing coordinates y_1 and y_2 by the bang-bang nature of the proposed static sliding mode control signals u_1 and u_2, we propose a dynamic sliding mode control scheme.

A sliding surface for the tracking of the linearizing output coordinate y_1 towards its equilibrium point I_{dd}. is is chosen as,

$$\sigma_1 = \dot{y}_1 + \lambda'(y_1 - I_{dd}) = -k_1 x_1 + k_5 x_2 x_3 + u_1 + \lambda'(x_1 - I_{dd})$$

where I_{dd} is the desired value of I_d and λ' is a positive design parameter.

For the regulation of the second linearizing coordinate, $y_2 = x_4$, a sliding surface expression is proposed which depicts a desired third order dynamic response for the controlled angular position $y_2 = x_4$, towards its desired equilibrium point $x_4 = \theta_d$. We propose,

$$\begin{aligned}\sigma_2 &= y_2^{(3)} + \alpha_1' \ddot{y}_2 + \alpha_2' \dot{y}_2 + \alpha_3'(y_2 - \theta_d) \\ &= -k_1 k_3 x_2 - k_3 k_5 x_1 x_3 - k_2 k_3 x_3 + k_3 u_2 \\ &\quad + (\alpha_1' - k_4)(k_3 x_2 - k_4 x_3) + \alpha_2' x_3 + \alpha_3'(x_4 - \theta_d)\end{aligned}$$

where θ_d is the desired value of $\theta = x_4$. The constants α'_1, α'_2 and α'_3 are chosen such that the polynomial $s^3 + \alpha'_1 s^2 + \alpha'_2 s + \alpha'_3$ is Hurwitz.

To ensure that the linearizing output coordinates y_1 and y_2 will converge to I_{dd} and θ_d respectively, in a finite time, we impose the following sliding mode controlled dynamics on the evolution of the sliding surface coordinate functions σ_1 and σ_2,

$$\dot{\sigma}_1 = -W_1' \operatorname{sgn}(\sigma_1)$$
$$\dot{\sigma}_2 = -W_2' \operatorname{sgn}(\sigma_2)$$

where W_1' and W_2' are positive design parameters. Because of these dynamics, σ_1 and σ_2 will converge to zero in finite time. Hence, after finite time $\dot{y}_1 = -\lambda'(y_1 - I_{dd})$ and $y_2^{(3)} = -\alpha_1' \ddot{y}_2 - \alpha_2' \dot{y}_2 - \alpha_3'(y_2 - \theta_d)$. It should be noted that because of the choice of λ', α'_1, α'_2, α'_3, we are guaranteed that $y_1 = x_1 = I_d$ converges to I_{dd} and $y_2 = x_4 = \theta$ converges to θ_d.

After some manipulations, we obtain the following control scheme,

$$\begin{aligned}\dot{u}_1 &= -f_3 - \lambda'(f_1 + u_1) - W_1' \operatorname{sgn}(\sigma_1) \\ \dot{u}_2 &= \frac{1}{k_3}[-f_4 - \alpha_1'(k_3 f_2 - k_3 k_4 x_2 + k_4^2 x_3 + k_3 u_2) \\ &\quad - \alpha_2'(k_3 x_2 - k_4 x_3) - \alpha_3' x_3 - W_2' \operatorname{sgn}(\sigma_2)]\end{aligned} \quad (8)$$

Equations (8) constitute the equations for a dynamic sliding mode feedback controller. Such a controller is then characterized by the solutions of the underlying differential equations for the control inputs u_1 and u_2. Indeed, the previous equations may be immediately rewritten as time-varying nonlinear ordinary differential equations, with discontinuous right hand side, for the original control inputs u_1 and u_2.

The following theorem can now be stated.

Theorem 2:

The dynamic feedback controller (8) when applied to the stepper motor system (3), asymptotically stabilizes the outputs of the system to their desired values.

6 Implementation Results of The Sliding Mode Controllers

The designed controllers are implemented using the experimental set-up depicted in Fig. 1. The system consists of a PM stepper motor, different loads to be attached to the shaft of the motor, drive circuitry, two current sensors, an optical encoder, and a controller board. The controllers were implemented using a digital signal processor (DSP).

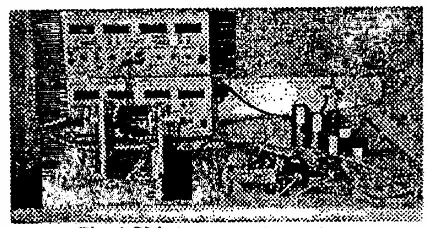

Fig. 1 PM stepper motor system

The block diagram representation of the overall system is shown in Fig. 2.

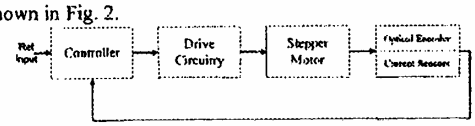

Fig. 2 Block diagram of the overall system

The motor used in the implementation is a unipolar hybrid permanent magnet stepper motor (Vexta, model PH268-22B). The motor has a shaft at both of its ends. Therefore, the attachment of different loads and an optical encoder to the shaft is simple. An optical encoder is used to measure the position of the PM stepper motor shaft. Two current sensors are also used to measure the currents supplied to the two coils of the PM stepper motor. The measured data is then fed back to the controller board for processing. An observer is used to estimate the angular velocity of the shaft of the motor [15]. The brain of the system is a digital signal processor (the DS1102 floating point processor board which is based on Texas Instrument TMS320C31 floating point DSP). The least square estimation technique was used to estimate the parameters of the system. It was found that these parameters are such: R = 19.1388Ω, L = 40 mH, K_m = 0.1349 Nm/A, J = 4.1295 × 10^{-4} kgm^2, B = 0.0013 Nm/rad/sec and N_r = 50.

The objective of the implementation is for the shaft of the motor to rotate by one step (one step = 1.8°=0.03142 radians) as fast as possible and with the least amount of overshoot (critically damped response). Later on, we will briefly discuss the multi-stepping case. Such a behavior is a typical representation of fast and accurate positioning for robotic applications.

To simulate the different loads experienced by the stepper motor, different weights are attached to the shaft of the motor by an extension arm of about 6 cm. For this implementation, a wide range of loads are used to simulate different loads on the motor. The results shown in this paper correspond to a load of 520 grams.

The design parameters for the static controller are chosen such that W_1=1000, $W_2=7\times10^5$, α_1=550 and $\alpha_2=7.5\times10^4$. The design parameters for the dynamic controller are chosen such that W_1'=2000, $W_2'=5.5\times10^7$, λ'=480 and α_1'=1250, $\alpha_2'=47\times10^4$, $\alpha_3=5.2\times10^7$.
The state variables of the system with the static sliding mode controller are shown in figure 3. The state variables of the system with the dynamic sliding mode controller are shown in figure 5. The state variables of the system with a feedback linearization controller (cancellation of the nonlinearities of the system and placement of the closed loop poles using the pole placement technique) are shown in Figure 7.

The states are seen to converge towards their corresponding equilibrium points when the different control schemes are used.
It can been seen from these figures that the angular position of the shaft of the motor converges towards its desired value (1.8°=0.03142 radians) in about 0.1 sec. In addition, the response of the angular position of the shaft of the motor is critically damped. The currents in coils A and B, the voltages v_a, v_b, as well as the sliding surfaces are also shown in figure 4 (for the static controller) and figure 6 (for the dynamic controller). It should be noted that because of the hardware constraints, the input voltages v_a and v_b are restricted to the range of 0 to 12 volts. when the static controller is used, the control inputs v_a and v_b are seen to exhibit a discontinuous behavior of the bang-bang type caused by the existence, in finite time, of a sliding regime on the intersection of the proposed stabilizing sliding surfaces $s_1 = 0$ and $s_2 = 0$. The controlled angular position trajectory x_4, is seen to be quite smooth, due to the fact that a third order integration separates the bang-bang control input u_2 from the angular position x_4. However, chattering is quite strong on the regulated motor currents, since only one order of integration separates the bang-bang control input u_1 from the regulated variable x_1. When the dynamic sliding mode controller is used, the chattering is very much reduced.

It turns out that the best performance is obtained when the dynamic sliding mode controller is used. This is the case because the chattering is tremendously reduced. The performance of the system is quite good when the feedback linearization controller is used. However, the feedback linearization controller has the inherent problem of not being very robust to changes in the parameters of the system.

Figures 8 and 9 show the state variables of the system with the sliding mode controllers for the multi-stepping case. In the implementation, the shaft of the motor is commanded to move by 10 steps. It can be seen, from these two figures that the sliding mode controller gives a faster response than the dynamic sliding mode controller. However, the chattering is reduced when the dynamic controller is used.

7 Conclusions

In this article we have used multivariable sliding mode control to control a PM stepper motor. Sliding mode controller design was shown to be greatly facilitated by resorting to the differential flatness of the system. The design of a static and a dynamic sliding mode controllers for a PM stepper motor is discussed. The results of the implementation of the proposed control schemes on an experimental setup are outlined. These results shows that the proposed controllers work very well for the single step and the multi-stepping cases.

References

[1] Utkin, V. I., *Sliding Modes in Control and Optimization*. Springer-Verlag, Berlin, 1992.

[2] Zinober, A. S. I., *Variable Structure and Lyapunov Control*, Lecture Notes in Control and Information Sciences, 193, Springer-Verlag, New York, 1994.

[3] Sira-Ramirez, H., *Differential geometric methods in variable structure control*. International J. of Control, vol. 48, pp. 1359-1391, 1988.

[4] Fliess, M. and Sira-Ramirez, H., *A module theoretic approach to sliding mode control of linear systems*, Proceedings of the 32nd IEEE Conference on Decision and Control, pp. 1322-1323, San Antonio, Texas, USA, 1993.

[5] Sira-Ramirez, H., *On the sliding mode control of multivariable nonlinear systems*. International J. of Control, vol. 64, no. 4, pp. 745-765, 1996.

[6] Fliess, M. , Levine, J., Martin, P., and Rouchon, P., *On differentially flat nonlinear systems*. In Nonlinear control systems design, edited by M. Fliess, pergamon Press, 1992.

[7] Fliess, M. , Levine, J., Martin, P., and Rouchon, P., *Flatness and defect of nonlinear systems: introductory theory and examples*. International J. of Control, vol. 61, no. 6, pp. 1327-1362, 1995.

[8] Levine, J., Lottin, J., and Ponsart, J-C., *Control of Magnetic Bearings. Flatness with Constraints*. In the Proceedings of the 13th Triennial World Congress, San Francisco, USA, 1996.

[9] Martin, P., and Rouchon, P., *Flatness and Sampling Control of Induction Motors*. In the Proc. of the 13th Triennial World Congress, San Francisco, USA, 1996.

[10] Rothfub, R., Rudolph, J. and Zeitz, M. , *Control of a chemical reactor model using its*. In the Proc. of the 13th Triennial World Congress, San Francisco, USA, 1996.

[11] Kenjo, T. and A. Sugawara, *Stepping Motors and their Microprocessor Controls*, Second Edition, Clarendon Press, Oxford, 1994.

[12] Zribi, M. and J. Chiasson, *Position Control of a PM Stepper Motor by Exact Linearization*, IEEE Transactions on Automatic Control, Vol. 36, No. 5, 1991.

[13] Bodson, M., J. N. Chiasson, R. T. Novotnak and R. B. Rekowski, *High-Performance Nonlinear Feedback Control of a Permanent Magnet Stepper Motor*, IEEE Trans. on Control Systems Tech., Vol. 1, No. 1, 1993.

[14] Speagle, R. C. and D. M. Dawson, *Adaptive Tracking Control of a Permanent Magnet Stepper Motor Driving a Mechanical Load*, Proc. of Southeastern Conf., 1993.

[15] Chiasson, J. and R. T. Novotnal, *Nonlinear Speed Observer for the PM Stepper Motor*, IEEE Trans. on Automatic Control, Vol 38, No. 10, 1993.

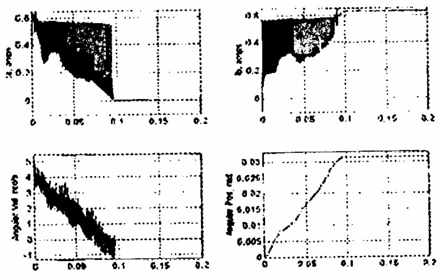

Fig. 3 Response of the motor when the static sliding control is used (single step case).

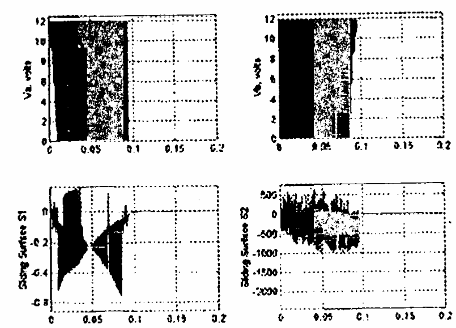

Fig. 4 Switching surfaces and control actions when the static sliding mode controller is used (single step case).

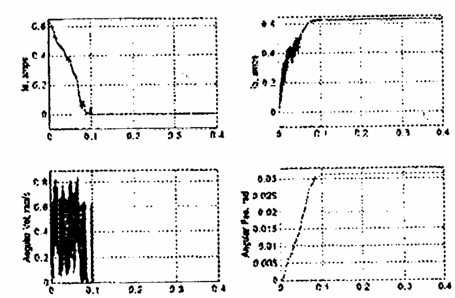

Fig. 5 Response of the motor when the dynamic sliding mode control is used (single step case).

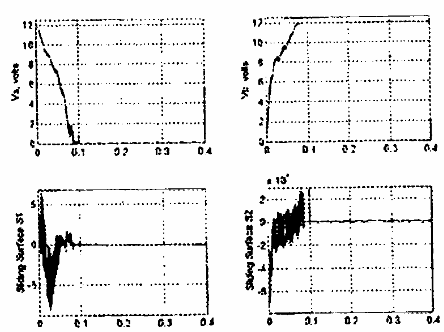

Fig. 6 Switching surfaces and control actions when the dynamic Sliding mode control is used (single step case).

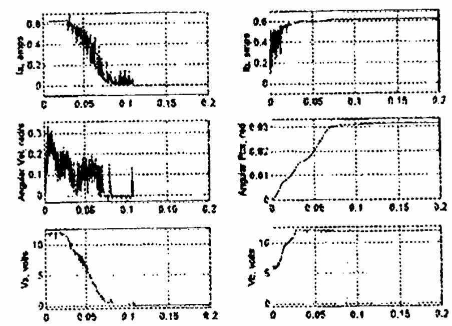

Fig. 7 Response of the motor when the feedback linearization controller is used (single step case).

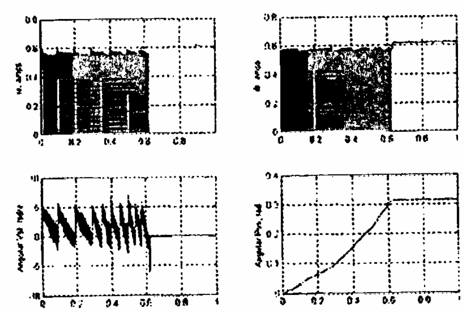

Fig. 8 Response of the motor when the static sliding mode control is used (multi-step case).

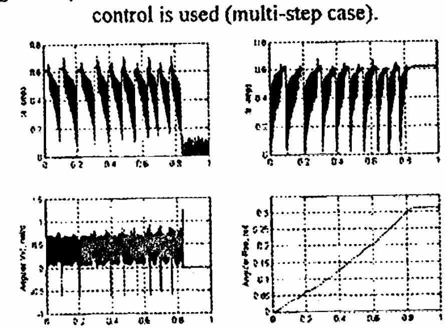

Fig. 9 Response of the motor when the dynamic sliding mode control is used (multi-step case).