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SOFT LANDING ON A PLANET: A TRAJECTORY PLANNING APPROACH FOR THE LIOUVILLIAN MODEL [1]

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Abstract

A feedback regulation scheme, based on an off-line trajectory planning, is proposed for the terminal descent trajectory of a vertically controlled spacecraft attempting a soft landing maneuver on the surface of a planet with no atmosphere. The approach is based on the "Liouvillian", character of the system, i.e., it has a *defect* regarding the *flatness* property of the spacecraft mass variable. This fact allows off-line trajectory planning controlled by a linear, time-varying, state feedback regulator complementing an ideal (nominal) open loop controller. Simulation results demonstrating the robustness of the approach are presented.

1 Introduction

In this article, a feedback regulation scheme is developed which allows for the regulation of a terminal descent maneuver of a vertically controlled spacecraft. The approach is based on an off-line trajectory planning which exploits the fact that the vertically controlled spacecraft model is "Liouvillian", i.e., it exhibits a *defect* regarding the flatness of the total spacecraft mass variable. In fact, the spacecraft height position dynamics is shown to satisfy a scalar linear time-varying differential equation whose defining parameter (i.e., eigenvalue) is constituted by a differential function of the flat variable. In other words, the spacecraft position dynamics is expressible in terms of *quadratures* of the flat output and of a finite number of its time derivatives. This fact considerably facilitates the feedback controller design task by allowing an off-line computation of the ideal open loop control which regulates the spacecraft towards a constant hovering equilibrium position. As usual, from such a small height hovering position the final touchdown maneuver can be safely accomplished with a shut-off of the main thruster. The approach therefore results in a linear, time varying, state feedback regulator complementing the ideal, off-line computed, open loop controller.

Liouvillian systems constitute the simplest extension of *differentially flat systems* (see the work of Prof. M. Fliess and his colleages [6]) into the area of systems which are not linearizable by means of endogenous feedback. The class of Liouvillian systems constitutes a subclass of non-flat systems with an identifiable flat subsystem of maximal dimension i.e., they are nonflat systems of lowest *defect*. A nonflat system is said to be Liouvillian, or *integrable by quadratures*, if the variables not belonging to the flat subsystem are expressible as elementary integrations of the flat outputs and a finite number of their time derivatives. This class of systems has been recently introduced by Chelouah in [4], from the perspective of Differential Galois theory in the context of Piccard-Vessiot extensions of differentially flat fields.

Section 2 is devoted to present the vertically controlled spacecraft model and the corresponding analysis depicting the difficulties inherent in the regulation of such a system. In this section we also demonstrate the Liouvillian character of such a controlled system. Section 3 derives the feedback control scheme and proposes the linearization-based feedback regulator accomplishing a smooth landing maneuver stably guiding the spacecraft towards the final hovering position. Section 4 presents some simulation results testing the robustness of the proposed controller.

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2 A Landing Model for a Vertically Controlled Spacecraft

2.1 A non-differentially flat system

Consider the nonlinear model describing the motion and mass behaviour, of a thrust controlled vehicle attempting a vertically regulated landing on the surface of a planet of constant gravity acceleration g and negligible atmospheric resistance (see [7], [5] and [1]).

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g - \frac{\sigma\alpha}{x_3} u \\ \dot{x}_3 &= -\sigma u \end{aligned} \tag{1}$$

where x_1 is the position (heigth) on the vertical axis, positively oriented downwards (i.e., $x_1 < 0$ for actual positive heigth), x_2 is the downwards velocity and x_3 represents the combined mass of the vehicle and the residual fuel (See Figure 1). The control input is represented by the controlled rate of ejection per unit time, denoted by u. The parameter σ is the relative ejection velocity. The constant α is a positive parameter such that the product $\sigma\alpha$ is the maximum thrust of the braking engine. The control input u is restricted to take values on the interval $[0, 1]$. This means that the spacecraft cannot accelerate *towards* the surface of the planet and the maximum downward acceleration is represented by the free fall condition $u = 0$.

It is quite easy to show that the above system is not linearizable by means of static state feedback and hence, according to the results of Charlet *et al*, [3], it is not linearizable by means of *dynamical* state feedback either. As a result, the system is not differentially flat [6].

Indeed, consider the following locally invertible state coordinate transformation, valid away from the singularity line $x_3 = 0$,

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= x_2 \\ z_3 &= x_2 - \alpha \ln x_3 \end{aligned} \quad ; \quad \begin{aligned} x_1 &= z_1 \\ x_2 &= z_2 \\ x_3 &= \exp\left(\frac{z_2 - z_3}{\alpha}\right) \end{aligned} \tag{2}$$

The transformed system clearly exhibits an uncontrollable coordinate given by z_3.

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= g - \sigma\alpha \exp\left(\frac{z_3 - z_2}{\alpha}\right) u \\ \dot{z}_3 &= g \end{aligned} \tag{3}$$

2.2 The soft landing spacecraft as a Liouvillian system

The system (1)therefore has a defect. In fact, the largest flat subsystem is represented by the variable x_3.

The flat output of the largest flat subsystem of the vertically controlled spacecraft is, therefore, given by the variable mass coordinate x_3, denoted by$F$. The height variable x_1, that we denote by W, and the downwards velocity x_2, represent the non-flat outputs. However, the height coordinate W may be represented by a quadrature of a differential function of the flat output F.

The following integro-differential parametrization is readily obtained from the system equations (1), and the flatness of the mass variable F.

$$\begin{aligned} x_3 &= F \ ; \ u = -\frac{\dot{F}}{\sigma} \ ; \ \ddot{W} = g + \alpha\frac{\dot{F}}{F} \ ; \\ & x_1 = W \ ; \ x_2 = \dot{W} \end{aligned} \tag{4}$$

The system is Liouvillian since the non-flat output W is expressibe in terms of quadratures of a differential function of F, specifically, a function involving F and $\dot{F}$. Notice, however, that in this particular case, the flat variable F can also be obtained by an elementary quadrature of the non-flat variable W

$$\dot{F} = -\frac{1}{\alpha}\left(g - \ddot{W}\right) F \tag{5}$$

2.3 Analysis of the diferential-integral parametrization

The preceeding integro-differential parametrization (4) contains useful information regarding the relations between the possible static equilibrium values for some of the variables and it also contains the properties of the system variables.

For instance, if $W = \overline{W}$ is a given nonzero constant, which is the case of a hovering condition above the ground, then $\dot{W} = \ddot{W} = 0$ and from (4), the variable F is exponentially asymptotically stable to zero with eigenvalue $-g/\alpha < 0$. This means that the spacecraft consumes all the fuel mass, and its own "dead" mass tool, in order to keep the constant equilibrium condition of the height variable W. The system does not exhibit a physically meaningful equilibrium point for the flat variable F, when the non-flat output W is held constant with the descent velocity being identically zero.

A free fall condition is given by $\ddot{W} = g$. This implies that $\dot{F} = 0$ i.e., the spacecraft mass remains constant, and the control input is $u = 0$ as read from (4). On the other hand, notice that the gravity acceleration g is necessarily larger than $\ddot{W}$ in a controlled descent. Otherwise, the spacecraft *increases its mass* which is phisically impossible. For *persistently* controlled descents ($u(t) \neq 0$ on any open interval of time) one may even assume that the quantity $-\left(g - \ddot{W}(t)\right)/\alpha$ is bounded away from zero and that it is smaller than a strictly negative constant $-\mu$. As a result, the mass variable evolution $F(t)$ is strictly decreasing, and, moreover, from linear systems theory resluts (see Calier and Desoer [2], Ch. 7), it is *exponentially asymptotically stable to zero.* Notice, however, that, phsically speaking, much before $F(t)$ is close to zero, the fuel mass has been completely depleted and the spacecraft mass becomes constant. Thus, $F(t)$ has a physical lower bound represented by the "dead mass" of the spacecraft.

3 A Feedback Controller based on Off-line Trajectory Planning and Flatness

3.1 Trajectory planning and the open loop controller

We assume that the spacecraft is initially located at a certain hovering height, $W(t_0)$, in a landing site exploration maneuver. Our control problem consists in achieving a controlled descent that softly brings the spacecraft from the intial heigth $W(t_0) = W_0$ to a small final heigth $W(T) = W_T < 0$, in a finite amount of time $T - t_0$. The control input should not saturate to any of its extremal values, 0 or 1, since this means either free fall or an undesired vertical ascent moving the spacecraft away from the target equilibrium position. The initial mass of the spacecraft, at the instant t_0, is assumed to be known and given by $F(t_0) = F_0$.

We propose a suitable planned trajectory for the non-flat output W, which we denote by $W^*(t)$ satisfying the initial and final conditions, $W^*(t_0) = W_0$ and $W^*(T) = W_T$ and our previous assumption $-(g - \ddot{W}^*(t))/\alpha < -\mu$. This may be achieved by specifying a suitable, sufficiently smooth, polynomial spline function $\psi(t, t_0, T)$, satisfying the conditions

$$\left\{ \begin{array}{lcl} \psi(t_0, t_0, T) & = & 0 \quad ; \frac{d^j}{dt^j}\psi(t, t_0, T)|_{t=t_0} \\ \psi(T, t_0, T) & = & 1 \quad ; \quad \frac{d^i}{dt^i}\psi(t, t_0, T)|_{t=T} \end{array} \right.$$
$$j = 1, 2, \ldots\text{finite} \; ; \; i = 1, 2, \ldots\text{finite}$$
$$\left(\frac{\ddot{\psi}(t, t_0, T) - g}{\alpha} \right) < -\mu \; \forall t \in [t_0, T] \qquad (6)$$

The planned trajectory (6) imposes a finite number of initial and final time derivatives of the prescribed "polynomial spline" $\psi(t, t_0, T)$. These conditions guarantee a sufficiently smooth departure from the initial hovering equilibrium and a sufficently smooth arrival at the final hovering position. The required planned trajectory would then be given by,

$$W^*(t) = F_0 + \psi(t, t_0, T)\,(W_T - W_0) \qquad (7)$$

The planned trajectory (7) is used in solving the following linear time-varying ordinary differential equation for the flat mass trajectory $F^*(t)$

$$\dot{F}^*(t) = -\frac{1}{\alpha}\left(g - \ddot{W}^*(t)\right) F^*(t) \quad ; \quad F(t_0) = F_0 \qquad (8)$$

The solution of (7) is next used in the off-line computation of the ideal (open-loop) control input $u^*(t)$ achieving the desired heigth transfer under ideal conditions. The open loop control, according to (4), is given by

$$u^*(t) = \frac{1}{\sigma\alpha}\left(g - \ddot{W}^*(t)\right) F^*(t) \qquad (9)$$

3.2 A feedback controller based on approximate linearization

Eviently the open loop controller (9) cannot be used alone in an actual descent maneuver due to its lack of robustness with respect to initial and on-line perturbations. The traditional solution idea is then to compensate for the small deviations around the ideal trajectories $F^*(t), W^*(t)$. This is accompished on the basis of linear (time-varying) feedback computed from an approximately linearized model.

We define, $x_{1\delta} = x_1 - W^*(t)$, $x_{2\delta} = x_2 - \dot{W}^*(t)$ and $x_{3\delta} = x_3 - F^*(t)$. The incremental control input is defined by the relation $u = u^*(t) + u_\delta$.

A Jacobian linearization of the system (1), around the planned trajectories, is given by

$$\begin{array}{rcl} \dot{x}_{1\delta} & = & x_{2\delta} \\ \dot{x}_{2\delta} & = & \left(\dfrac{g - \ddot{W}^*(t)}{F^*(t)} \right) x_{3\delta} - \left(\dfrac{\sigma\alpha}{F^*(t)} \right) u_\delta \\ \dot{x}_{3\delta} & = & -\sigma u_\delta \end{array} \qquad (10)$$

A suitable linear time-varying feedback controller for the linearized system (10) would be given by

$$u_\delta = \frac{F^*(t)}{\sigma\alpha}\left[\left(\frac{g - \ddot{W}^*(t)}{F^*(t)} \right) x_{3\delta} + 2\zeta\omega_n x_{2\delta} + \omega_n^2 x_{1\delta} + \lambda x_{3\delta} \right] \qquad (11)$$

where ζ and ω_n and λ are positive design constants representing the positive damping coefficient and the natural frequency of the time-invariant closed loop linearized dynamics of the incremental heigth and velocity variables.

The closed loop system is then given by

$$\begin{aligned}
\dot{x}_{1\delta} &= x_{2\delta} \\
\dot{x}_{2\delta} &= -2\zeta\omega_n x_{2\delta} - \omega_n^2 x_{1\delta} - \lambda x_{3\delta} \\
\dot{x}_{3\delta} &= -\frac{1}{\alpha}\left(\lambda F^*(t) + g - \ddot{W}^*(t)\right) x_{3\delta} \\
&\quad - \left[\frac{F^*(t)}{\alpha}\right]\left(2\zeta\omega_n x_{2\delta} + \omega_n^2 x_{1\delta}\right) \qquad (12)
\end{aligned}$$

The incremental variables $x_{1\delta}$ and $x_{2\delta}$ are the state variables of a time-invariant linear system with eigenvalues placed at will in the open left portion of the complex plane. This system is excited by the state variable $x_{3\delta}$, which we may show is an $\mathcal{L}_2$ signal which converges to zero. This implies that $x_{1\delta}$ and $x_{2\delta}$ are asymptotically stable to zero. In order to show that $x_{3\delta}$ is an $\mathcal{L}_2$ signal we proceed as follows. According to the assumption that $-(g - \ddot{W}^*(t))/\alpha < -\mu$, and the fact that λ is a positive constant, it follows that the quantity $-(\lambda F^*(t) + g - \ddot{W}^*(t))/\alpha$ is also stricity smaller than $-\mu < 0$, for all t. Since the quantity $F^*(t)$ is exponentially stable to zero, the second summand in the last of equation (12) represents a forcing input signal which decays to zero as $t \to \infty$. It follows from linear systems theory (see [2]) that the incremental variable $x_{3\delta}$ is an $\mathcal{L}_2$ signal which is also asymptotically stable to zero. So, in fact, the linear system describing the linearized flat subsystem $(x_{1\delta}, x_{2\delta})$ is excited by an $\mathcal{L}_2$ signal which decays to zero. The linearized closed loop system (12) is then asymptotically stable to zero for any given set of incremental initial conditions.

The proposed feedback controller is given by

$$\begin{aligned}
u &= u^*(t) + u_\delta(t) = \left(\frac{g - \ddot{W}^*(t)}{\sigma\alpha}\right)(F^*(t) + x_{3\delta}) \\
&\quad + \frac{F^*(t)}{\sigma\alpha}\left[2\zeta\omega_n x_{2\delta} + \omega_n^2 x_{1\delta} + \lambda x_{3\delta}\right] \qquad (13)
\end{aligned}$$

4 Simulation Results

Simulations were performed to test the effectiveness, and robustness, of the proposed feedback regulation scheme (13).

We prescribed the planned trajectory $W^*(t)$, for the non-flat output x_1, by means of the following polynomial spline,

$$W^*(t) = W_0 + \psi(t, t_0, T)(W_T - W_0) \qquad (14)$$

with

$$\begin{aligned}
\psi(t, t_0, T) &= \left(\frac{t - t_0}{T - t_0}\right)^5 \left[r_1 - r_2\left(\frac{t - t_0}{T - t_0}\right)\right. \\
&\quad + r_3\left(\frac{t - t_0}{T - t_0}\right)^2 - r_4\left(\frac{t - t_0}{T - t_0}\right)^3 \\
&\quad \left. + r_5\left(\frac{t - t_0}{T - t_0}\right)^4 - r_6\left(\frac{t - t_0}{T - t_0}\right)^5\right] \quad (15)
\end{aligned}$$

with

$$r_1 = 252\ ;\ r_2 = 1050\ ;\ r_3 = 1800\ ;\ r_4 = 1575\ ;$$
$$r_5 = 700\ ;\ r_6 = 126$$

The initial surveying hovering heigth was set to be $W_0 = -700$ m and the final hovering equilibrium $W_T = -1$ m. The prescribed trajectory $W^*(t)$ has its first four time derivatives equal to zero at the initial time $t = t_0$ and its first five time derivatives equal to zero at the final time $t = T$. This guarantees a landing maneuver with a sufficiently smooth departure and arrival features. The initial mass was set to be $F_0 = 1500$ Kg. The parameters defining the system model were set to be

$$\sigma = 50 \text{ Kg/s} \ ; \ \alpha = 200 \text{ m/s} \ ; \ g = 1.63 \text{ m/s}^2$$

Figure 2 displays the results of the off-line computations represented by the solution of the differential equation (8) for the flat mass variable $F^*(t)$ and the calculation of the open loop control. The ideal non-flat output trajectory $W^*(t)$, together with the computed nonminal mass trajectory $F^*(t)$, in turn allows for the computation of the ideal open loop control policy $u^*(t)$ from the expression (9). The time evolution of all these ideal descent maneuver variable are shown in Figure 2, along with the corresponding ideal vertical velocity $\dot{W}^*(t)$ and ideal vertical acceleration $\ddot{W}^*(t)$.

The controller parameters were set so as to obtain real closed loop eigenvalues of the controlled system

$$\zeta = 1 \ ; \ \omega_n = 0.4 \ ; \ \lambda = 0.8$$

Figure 3 depicts the performance of the feedback control policy (13) when significant initial setting errors of the initial surveying hovering height are included. As depicted in the simulations, the controller manages to reset the spacecraft position and downward velocity to the prescribed initial values of the planned landing maneuver.

5 Conclusions

A trajectory planning approach, has been proposed for the feedback regulation of a soft landing maneuver in a partially differentially flat vertically controlled spacecraft system. The approach is allowed by the Liouvillian character of the controlled model. This feature allows for an off-line computation of all relevant signals required for the trajectory planning control scheme. The off-line computations include the calculation of the non flat variable evolution which is in correspondance with the given flat output trajectory. This requires the solution of a linear time-varying differential equation with appropriate initial data. The off-line computations also include that calculation of the open loop control input signal in terms of the planned flat output trajectory and the computed non-flat variable trajectory, represented by the spacecraft height. The open loop control would perform a smooth descent under ideal, unperturbed, flight conditions and exact initial settings. The proposed feedbak controller uses the off-line computed open loop control signal complemented with a linearization based feedback controller providing the necessary on-line correction maneuvers. The incremental control input policy, which is just a proportional derivative feedback controller with time-varying compensation terms, was shown to asymptotically stabilize the resulting linearized model describing the deviations from the off-line computed ideal descent trajectory. The performance of the controller was satisfactorily tested using digital computer simulations which included initial errors with respect to the planned trajectory initial setting values.

Several extensions are possible regarding the proposed approach. The first one would be to include a more general spacecraft model considering lateral and forward motions over the landing horizontal plane. Another possibility is represented by suitably combining the presented approach with an optimal path scheme guided by an optimal fuel expenditure, or equivalently by a minimum time descent, requirement.

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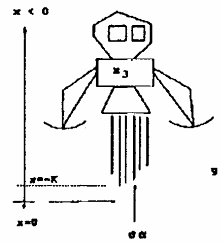

Figure 1: A vertically controlled spacecraft.

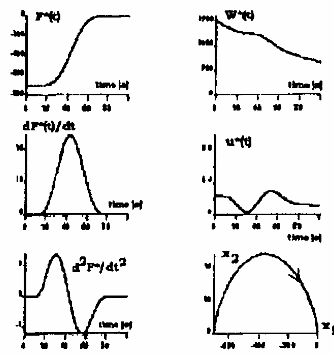

Figure 2: Open loop trajectory planning signals

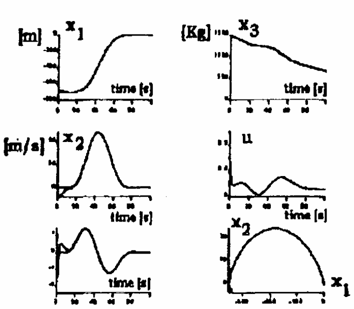

Figure 3: Closed loop performance of the off-line computed plus linearization-based controller.