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ON THE REGULATION OF A DOUBLE EFFECT EVAPORATOR: A TRAJECTORY PLANNING AND PASSIVITY APPROACH

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Abstract: A dynamical feedback controller is proposed for the regulation of a simplified model of a double effect evaporator. The controller is based on feedback passivity considerations, complemented with a trajectory planning scheme in terms of the systems passive output. A drift vector field decomposition greately facilitates the direct application of the "energy shapping plus damping injection" feedback controller design methodology. The controller performance is enhanced with the aid of a trajectory planning scheme which effectively avoids singularity arcs in the state space, related to a loss of the relative degree of the passive output, while allowing for the satisfaction of physically meaningful state space and input space restrictions. The performance of the obtained controller design is evaluated by means of computer simulations. *Copyright © 1999 IFAC*

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1 INTRODUCTION

Passivity based control of nonlinear systems is rapidly gaining well deserved popularity due to the several advantages related to controller simplicity, robustness and the physically appealing features of the approach. General developments of passivity based control, into the realm of nonlinear affine systems, have been carried over relatively recently. The seminal contribution is that of Willems (1972) in the context of *dissipative systems*. The articles by Hill (1976) constitute also a general approach with emphasis on conditions for stability of feedback interconnected systems. A geometric approach to feedback equivalence of nonlinear passive systems was contributed by the work of Byrnes *et al* (1991). Recent contributions, within the context of adaptive systems, have been given by Serón *et al* (1995). A full perspective of the area is found in the book by Sepulchre *et al* (1997) and the book by Ortega *et al* (1998).

Section 2 presents some generalities regarding a canonical form for passivity based control presented in Sira-Ramírez (1998). Section 3 is devoted to derive a passivity based controller for a simplified double effect evaporator model. The performance of the designed controller is also illustrated by means of digital computer simulations. Section 4 contains the conclusions and suggestions for further research.

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2 A CANONICAL FORM FOR PASSIVITY BASED CONTROL

2.1 Background Results

Consider the class of nonlinear single-input single-output systems described by

$$\begin{aligned} \dot{x}(t) &= f(x) + g(x)u,\ x \in \mathcal{X} \subset \mathbf{R}^n,\ u \in \mathcal{U} \subset \mathbf{R} \\ y &= h(x),\ y \in \mathcal{Y} \subset \mathbf{R} \end{aligned} \tag{1}$$

where $\mathcal{X}$ denotes the *operating region* of the system, constituted by a sufficiently large open set containing a continuum of equilibrium points, possibly parametrized by a constant control input value $u = U \in \mathcal{U}$, of the form $x = \overline{x}(U)$ and given by the solution of $f(\overline{x}) + g(\overline{x})U = 0$. In particular, for $u = 0$, we assume $f(\overline{x}) = 0$ implies $\overline{x} = 0$. However, motivated by a large class of real life systems, we are specifically interested in *nonzero* constant state equilibrium points $x = \overline{x}$, obtained by nonzero constant control inputs $u = U$.

We assume that a C^1 positive definite *storage function* $V : \mathbf{R}^n \to \mathbf{R}^+$ is given such that $V(0) = 0$.

By $\partial V/\partial x$ we denote the *column* vector field with components $\partial V/\partial x_i$ $i = 1, \ldots, n$. The transpose of this gradient field, $(\partial V/\partial x)^T$, is denoted by the *row* vector $\partial V/\partial x^T$. Let $L_g V(x)$ denote the directional derivative of the scalar function $V(x)$ with respect to the control input vector field $g(x)$ at the point x. We assume throughout the entire article that the following assumption holds valid:

$$L_g V(x) = \frac{\partial V}{\partial x^T} g(x) \neq 0 \quad \forall\, x \in \mathcal{X} \tag{2}$$

This last condition is usually known as the *transversality condition* and simply establishes that the vector field $g(x)$ is not orthogonal to the gradient of $V(x)$ at any point x in $\mathcal{X}$. In other words, the control vector field $g(x)$ is not tangential, at each x, to the storage function level sets, defined in the state space of the system as, $\{x \in \mathcal{X}\ :\ V(x) = constant\}$. This condition is quite familiar in sliding mode control of nonlinear systems (see Sira-Ramirez, 1988) and it amounts to having a storage function which is locally *relative degree* one in $\mathcal{X}$.

For each $x \in \mathcal{X}$, we define a *projection operator, along the span of the control vector field $g(x)$ onto the tangent space to the constant level sets of the energy function* $V(x)$, as the matrix $M(x)$ given by

$$M(x) = \left[I - \frac{1}{L_g V(x)} g(x) \frac{\partial V}{\partial x^T} \right] \tag{3}$$

The following are some properties of the matrix $M(x)$ which further justify the given name of "projection operator" (see Sira-Ramírez, 1998)

Proposition 1 *The matrix $M(x)$ enjoys the following properties:*

$$\begin{gathered} g(x) \in \mathcal{K}er M(x) \\ \frac{\partial V}{\partial x} \in \mathcal{K}er M^T(x) \\ M(x)\left[I - M(x)\right] = 0. \end{gathered}$$

The following proposition depicts further properties of the projection matrix $M(x)$.

Proposition 2 *Let $f(x)$ be a smooth vector field, then the vector $M(x)f(x)$ can be written as*

$$M(x)f(x) = \tilde{\mathcal{J}}(x) \frac{\partial V}{\partial x}$$

where $\tilde{\mathcal{J}}(x)$ is a skew-symmetric matrix, i.e., $\tilde{\mathcal{J}}(x) + \tilde{\mathcal{J}}^T(x) = 0$. On the other hand the vector field $[I - M(x)]f(x)$ can be written as

$$[I - M(x)]\, f(x) = -\frac{1}{2} \tilde{\mathcal{J}}(x) \frac{\partial V}{\partial x} + \mathcal{S}(x) \frac{\partial V}{\partial x}$$

where $\mathcal{S}(x)$ is a symmetric matrix, i.e., $\mathcal{S}(x) = \mathcal{S}^T(x)$.

2.2 Vector field decompositions through projection operators

As a consequence of the above propositions and definitions we have the following decomposition of drift vector fields

Proposition 3 *A vector field $f(x)$ can be naturally decomposed in the following sum*

$$\begin{aligned} f(x) &= M(x)f(x) + (I - M(x))f(x) \\ &= \mathcal{J}(x)\frac{\partial V}{\partial x} + \mathcal{S}(x)\frac{\partial V}{\partial x} \end{aligned} \tag{4}$$

where $\mathcal{J}(x)$ is a skew-symmetric matrix and $\mathcal{S}(x)$ is a symmetric matrix.

2.3 A canonical form for nonlinear systems

As a corollary to the above results, a nonlinear system of the form (1), with a positive definite storage function $V(x)$ which also satisfies the transversality condition $L_g V(x) \neq 0$, can always be rewritten as

$$\begin{aligned} \dot{x}(t) &= \mathcal{J}(x)\frac{\partial V}{\partial x} + \mathcal{S}_p(x)\frac{\partial V}{\partial x} \\ &\quad + \mathcal{S}_n(x)\frac{\partial V}{\partial x} + g(x)u \end{aligned} \tag{5}$$

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with $\mathcal{J}(x)$ being skew-symmetric, $\mathcal{S}_p(x)$ being positive semi-definite and $\mathcal{S}_n(x)$ being negative semi-definite. However, if $\mathcal{S}_p(x)$ is positive definite, then $\mathcal{S}_n(x)$ is zero and conversely if $\mathcal{S}_n(x)$ is negative definite then $\mathcal{S}_p(x)$ is zero.

3 A PASSIVITY BASED FEEDBACK CONTROLLER DESIGN FOR THE DOUBLE EFFECT EVAPORATOR

Industrial evaporators are extensively used in the chemical and food industries to enhance final product concentrations from a given low concentration feed solution. Under standard assumptions, treated in Silva-Navarro and Alvarez-Gallegos (1995), a reasonable reduced order model for a double effect evaporator is given by,

$$\begin{aligned} \dot{x}_1 &= \delta_1 F_0(C_0 - x_1) + \delta_2 x_1 u \\ \dot{x}_2 &= \delta_3 F_0(x_1 - x_2) + (\delta_4 x_1 + \delta_5 x_2)u \\ y &= x_2 \end{aligned} \tag{6}$$

where x_1 and x_2 represent the weight fraction solution concentration in the first and the second constitutive parts of the evaporator, also called, for simplicity, the "first effect" and the "second effect" respectively. The state variables x_1, x_2 of the plant, naturally satisfy the following physically meaningful restrictions:

$$x \in \tilde{\mathcal{X}} = \{0 < C_0 \le x_1 \le x_2 < 1\} \tag{7}$$

These restrictions effectively bound the set of possible state trajectories to a triangular region in the two-dimensional state space (see Fig. 1).

The parameter C_0 is the constant weight fraction input concentration to the first effect, while F_0 is the solution feed flow rate expressed in Kg/min. The control input u is the steam feed flow rate, also expressed in Kg/min. The constants, $\delta_1, \ldots \delta_4$, are given by

$$\begin{aligned} \delta_1 &= \frac{1}{W_1},\ \delta_2 = \frac{k_1}{W_1},\ \delta_3 = \frac{1}{W_2} \\ \delta_4 &= -\frac{k_1}{W_2},\ \delta_5 = \frac{k_1(1+k_2)}{W_2} \end{aligned} \tag{8}$$

where W_1,W_2 are the hold-up masses in each one of the effects, respectively. These quantities are expressed in Kg and they satisfy $(0 < W_1 < W_2)$. The constant parameters k_1 and k_2 are positive constants denoting the ratio of the vapor flow rate produced with respect to the steam fed flow rate in both effects. They are such that the condition : $0 < k_2 < k_1 < 1$ holds valid. The control input variable u may be constrained by a set of the following form,

$$u \in \mathcal{U} = \{u \in R \ :\ U_{max} > u > 0\}$$

The family of equilibrium points, parametrized by a constant equilibrium value for u, set to be $\overline{u} = U$, is given by

$$\overline{x}_1 = \frac{F_0 C_0}{F_0 - k_1 U} \ ;\ \overline{x}_2 = \frac{F_0 C_0}{F_0 - k_1(1+k_2)U}$$

Eliminating the constant parameter U from the above relations, it readily follows that the family of physically meaningful equilibrium points, $(\overline{x}_1, \overline{x}_2)$, are all located on the first quadrant, in the corresponding branch of the following hyperbola,

$$x_2 = \frac{x_1 C_0}{(1+k_2)C_0 - k_2 x_1} \tag{9}$$

The manifold of physically valid equilibrium points is, necessarily, constrained to lay within the triangular region previously described by equation (7).

It is easy to verify that the output variable $y = x_2$ is a relative degree one output, as long as the singularity condition,

$$\delta_4 x_1 + \delta_5 x_2 = 0$$

is not valid. This singularity line is also given by,

$$x_2 = -\frac{\delta_4}{\delta_5} x_1 = \frac{1}{1+k_2} x_1 \tag{10}$$

The singularity line (10) is a straight line with positive slope crossing the origin and having two points in common with the manifold of equilibrium points, (9), one in the first quadrant and the second one in the third quadrant which is not physically meaningful. In the first quadrant the singularity line lies below the set of possible equilibria.

It has been shown in Silva-Navarro and Alvarez-Gallegos (1995) using Lyapunov stability theory, that the outupt variable x_2 is also a minimum phase output. Thus, the variable x_2 is a *passive output*. Stabilization of x_2 towards a desired constant or reference value can be achieved by means of system inversion, with a stable zero dynamics, as long as the resulting trajectory does not violate the relative degree one condition for this variable.

We next consider the storage function $V(x)$ given by,

$$V = \frac{1}{2}\left(x_1^2 + x_2^2\right)$$

The storage function directional derivative, $L_g V(x)$, along the control input vector field

$$g(x) = [\delta_2 x_1 \quad (\delta_4 x_1 + \delta_5 x_2)]^T$$

is given by,

$$L_g V(x) = \delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2$$

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The transversality condition is valid everywhere in the region $\tilde{\mathcal{X}}$. and it fails to be valid on the set,

$$\{x \in R^2 : L_g V(x) = 0\} = \{x \in R^2 : \delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2 = 0\} \quad (11)$$

This set actually represents the origin of coordinates (0,0) in R^2 and, therefore, it is not a meaningful restriction to the proposed solution.

A control problem involving a transfer between two equilibria, or a maneuver involving a trajectory tracking task, must be defined in such a way that the transversality condition $L_g V(x) \neq 0$, is always satisfied by each and every point of the prescribed state trajectory in R^2.

The operating region, free of singularities, is constituted by the following set,

$$\mathcal{X} = \{x \in R^2 : 0 < C_0 \leq x_1 \leq x_2 < 1 ; \; x_2 - \frac{1}{1+k_2} x_1 \neq 0\} \quad (12)$$

This set is shown in Fig. 1, along with the manifold of equilibrium points and the line of singularity for the relative degree 1 condition of the passive output x_2. Note that on $\mathcal{X}$ the quantity $L_g V(x)$ is strictly positive.

The given system may be rewritten in the form,

$$\dot{x} = [\mathcal{J}(x) + \mathcal{S}(x)] \frac{\partial V}{\partial x} + g(x)u \quad (13)$$

where,

$$\mathcal{J}(x) = \alpha(x) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\alpha(x) = \frac{\delta_1 F_0 (C_0 - x_1)(\delta_4 x_1 + \delta_5 x_2) - \delta_2 \delta_3 F_0 x_1 (x_1 - x_2)}{2[\delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2]}$$

and

$$\mathcal{S}(x) = \beta(x) \begin{bmatrix} S_{11}(x) & S_{12}(x) \\ S_{12}(x) & S_{22}(x) \end{bmatrix} \quad (14)$$

with

$$\begin{aligned} S_{11}(x) &= 2\delta_1 \delta_2 F_0 (C_0 - x_1) x_1 \\ S_{12}(x) &= \delta_1 F_0 (C_0 - x_1)(\delta_4 x_1 + \delta_5 x_2) + \delta_2 \delta_3 F_0 x_1 (x_1 - x_2) \\ S_{22}(x) &= 2\delta_3 F_0 (x_1 - x_2)(\delta_4 x_1 + \delta_5 x_2) \\ \beta(x) &= \frac{1}{2[\delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2]} \end{aligned} \quad (15)$$

The symmetric matrix $\mathcal{S}(x)$, with the state x constrained to the operating region $\mathcal{X}$, can be further decomposed into the sum of a positive semi-definite matrix $\mathcal{S}_p(x)$ and a negative semi-definite matrix $\mathcal{S}_n(x)$ The following input coordinate transformation makes invariant the positive term appearing in the contribution to $\dot{V}$. Its effect is that of neutralizing the corresponding vector field by making it tangent to the stored energy level sets. This procedure renders the system output passive, from the new input v towards the original output x_2, with respect to the proposed storage function $V(x)$.

Substituting the feedback expression

$$u = \frac{1}{\delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2} \times [x_2 v - F_0 x_1 (\delta_1 C_0 + \delta_3 x_2)] \quad (16)$$

in equation (13) and after some straightforward algebraic manipulations and some simplifications, the partially closed loop system in PBCCF is seen to be expressed as,

$$\dot{x} = m(x) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -\delta_1 F_0 & 0 \\ 0 & -\delta_3 F_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{\delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2} \begin{bmatrix} \delta_2 x_1 x_2 \\ (\delta_4 x_1 + \delta_5 x_2) x_2 \end{bmatrix} v \quad (17)$$

$$y = \frac{1}{\delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2} \times [\delta_2 x_1 x_2 \quad (\delta_4 x_1 + \delta_5 x_2) x_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \quad (18)$$

with

$$m(x) = \frac{\delta_1 F_0 C_0 (\delta_4 x_1 + \delta_5 x_2) - \delta_2 \delta_3 F_0 x_1^2}{\delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2}$$

The "Energy Shapping plus Damping Injection" controller design methodology (see Ortega *et al*, 1998) yields the following dynamical feedback controller,

$$v = \frac{\delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2}{x_2 (\delta_4 x_1 + \delta_5 x_2)} \times \quad (19)$$

$$\left\{ \dot{x}_{2d}^* - \frac{[\delta_2 \delta_3 F_0 x_1^2 - \delta_1 F_0 C_0 (\delta_4 x_1 + \delta_5 x_2)]}{\delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2} \xi + \delta_3 F_0 x_{2d}^* - R_2 (x_2 - x_{2d}^*) \right\} \quad (20)$$

with ξ given by the solution of

$$\dot{\xi} = \frac{[-\delta_2 \delta_3 F_0 x_1^2 + \delta_1 F_0 C_0 (\delta_4 x_1 + \delta_5 x_2)]}{\delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2} x_{2d}^* - \delta_1 F_0 \xi + \frac{\delta_2 x_1 x_2}{\delta_2 x_1^2 + \delta_4 x_1 x_2 + \delta_5 x_2^2} v + R_1 (x_1 - \xi) \quad (21)$$

The specification of a suitable trajectory for $x_{2d}^*(t)$ and its corresponding time derivative $\dot{x}_{2d}^*(t)$ is carried out so that the systems response evolves away

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from the singular line in R^2 where $L_gV(x) = 0$. The simulation example, presented next, illustrates this point in detail.

3.1 Simulation results

We let, the system parameters to be given by

$$\begin{aligned} F_0 &= 2.525 \text{ Kg/min} \\ C_0 &= 0.04 \text{ Kg.sugar/Kg.water} \\ W_1 &= 95.0 \text{ Kg. } ; \; W_2 = 105.0 \text{ Kg.} \\ k_1 &= 0.808458 \; ; \; k_2 = 0.338461 \end{aligned} \quad (22)$$

A controlled transfer from an initial equilibrium point, given by $\overline{x}_1(t_0) = 0.070$ Kg.sugar/Kg.water and $\overline{x}_2(t_0) = 0.093954$ Kg.sugar/Kg.water, towards a second equilibrium point given by $\overline{x}_1(t_0 + T) = 0.09549$ Kg.sugar/Kg.water and $\overline{x}_2(t_0+T) = 0.1800$ Kg.sugar/Kg.water was attempted by prescribing a desired reference trajectory, $x^*_{2d}(t)$, for the derived dynamical feedback controller. The initial and final equilibrium points are located well within the operating region. The transition time between the equilibrium points was set to be achieved in $T = 250$ minutes, with $t_0 = 300$ min and $t_0 + T = 550$ min.

The following open loop trajectory was prescribed for $x^*_{2d}(t)$,

$$x^*_{2d}(t) = \begin{cases} \overline{x}_2(t_0) \text{ for } t < t_0 \\ \overline{x}_2(t_0) + (\overline{x}_2(t_0+T) - \overline{x}_2(t_0)) \times \\ \frac{(t-t_0)^5}{(T)^5}\left[21 - 35\frac{t-t_0}{T} + 15\frac{(t-t_0)^2}{(T)^2}\right] \\ \qquad \text{for } t_0 \le t \le t_0 + T \\ \overline{x}_2(t_0+T) \text{ for } t > T \end{cases}$$

The time derivative of the desired open loop trajectory is simply obtained as

$$\dot{x}^*_{2d}(t) = \begin{cases} 0 \text{ for } t < t_0 \\ (\overline{x}_2(T) - \overline{x}_2(t_0))\frac{(t-t_0)^4}{(T-t_0)^5} \times \\ \left[105 - 210\frac{t-t_0}{T-t_0} + 105\frac{(t-t_0)^2}{(T-t_0)^2}\right] \\ \qquad \text{for } t_0 \le t \le t_0 + T \\ 0 \text{ for } t > t_0 + T \end{cases}$$

The planned trajectory guarantees a sufficiently smooth departure from the initial equilibrium point (the first four time derivatives being zero at such initial point) and a smooth arrival at the second equilibrium point (the first two time derivatives being also zero at the arrival time). The control input is maintained within reasonable bounds, which are physically acceptable (see Fig. 2). Indeed, typically, the maximum allowed value for the control input is $U_{max} = 3.4$ Kg/min. The simulations show that the controller maximum effort is well below this limiting input value. In the x_2-x_1 coordinates, the proposed open loop trajectory follows a curve joining the equilibrium points $(\overline{x}_1(t_0), \overline{x}_2(t_0)) = (0.070, 0.093954)$ and $(\overline{x}_1(t_0+T), \overline{x}_2(t_0+T)) = (0.095541, 0.18)$. The controlled trajectory, in fact, moves away from the singularity line and from the origin and it lies entirely inside the operating region $\mathcal{X}$ (see Fig. 3).

Other equilibrium transfers are also possible without crossing the singularity arc. However, using special trajectory planning techniques, transfers including crossings of the singularity arc are also possible.

4 CONCLUSIONS

In this article a passivity based control design option was complemented with a trajectory planning approach in the context of a double effect evaporator controller design example. The passivity based controller was based on the "energy shapping -plus- damping injection" methodology which thus far has been unnecessarily restricted to Euler-Lagrange type of systems. The advantages of the proposed approach exploit the natural phisically oriented features of passivity while at the same time allows for efficient circumvention of singularities and other forbidden regions of the state space thanks to the trajectory planning features of the approach.

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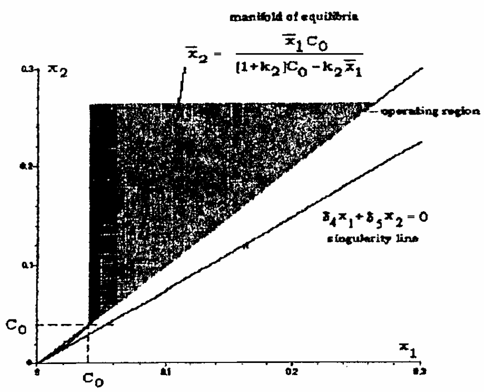

Figure 1: Singularity line, manifold of equilibria, and operating region in the state space.

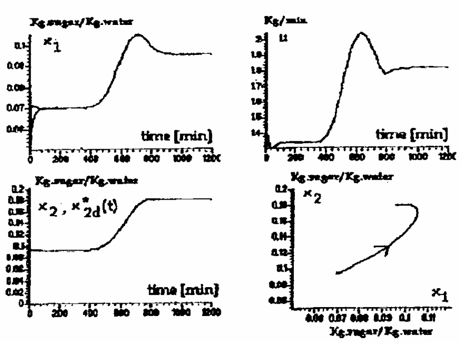

Figure 2: Simulations of passivity based control of the double effect evaporator system with output concentration trajectory planning.

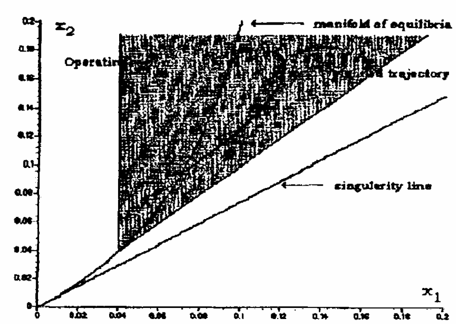

Figure 3: Planned state trajectory, the singularity line and the operating region.

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