Flatness and trajectory tracking in sliding mode based regulation of dc-to-ac conversion schemes ¹

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Abstract

A DC-to-AC power conversion scheme is presented which uses conventional DC-to-DC switched power converters. The proposed feedback controller is based on an indirect sliding mode inductor current tracking scheme for a suitable off-line generated reference trajectory, thus circumventing non-minimum phase problems associated with the regulation of the output capacitor voltage variable. The inductor current reference trajectory is generated by means of an iterative functional approximation algorithm placing the required input current in terms of the desired output voltage trajectory and an indefinite number of its time derivatives.

1 Introduction

The most popular devices achieving DC-to-DC power conversion are known as: the "buck" converter, the "boost" converter, the "buck-boost" converter, the "sepic" and the "Cúk" converter (see Severns and Bloom [6]). The simplicity of the DC-to-DC switched-mode power converter topologies joined to the robustness of its discontinuous feedback control and their transportability make them highly desirable as potential candidates for DC-to-AC conversion schemes.

Traditional approaches to DC-to-AC power conversion include among other options: PWM commanded switch based *inverters*, fed by Un-interruptible Power Supplies (UPS) or rectified voltage sources; Various combinations of Series-Resonant DC/AC inverters and, more recently, the so called Zero-Voltage-Switching (ZVS) PWM commutation cells linking constant voltage sources and "buck" converters (see, among an im-

mense wealth of articles in these areas, the works of Mendes de Seixas [4], [5], García and Barbi [1], Jung and Tzou [3], Hsieh et al [2] and the many references therein). In [9] DC-to-AC power conversion is tackled using a Fourier series solution of an Abel type of differential equation in combination with a backstepping controller.

In this artice, we propose a systematic approach for yielding sliding mode control based DC-to-AC power conversion scheme using the traditional "boost" and "buck-boost" converters. We propose an off-line iterative computational scheme which formally generates an infinite order differential parametrization of the inductor current reference trajectory in terms of the desired AC capacitor voltage reference signal. However, a finite number of iterations of the algorithm (typically one or two) approximately expresses a suitable inductor current reference trajectory as a differential function, of finite order, of the desired capacitor voltage. An indirect sliding mode control approach is then pursued which tracks the candidate inductor current reference signal thus circumventing the well known nonminimum phase character of the voltage variable as a system output.

Section 2 revisits the feasibility of an indirect sliding mode control scheme based on current signal tracking, as opossed to voltage signal tracking, for the "boost" converter. We also revisit the differential flatness of the "boost" converter and proceed to develop an off-line computational scheme for the generation of a suitable inductor current reference signal. Simulation results depict the rapidly convergent nature of the proposed off-line computational algorithm and the precision with which a desired AC output voltage signal is tracked by the indirect control scheme. Section 3 presents the corresponding developments and simulations for the "buck-boost" converter circuit. Section 4 is devoted to present some conclusions.

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2 DC-to-AC Power Conversion using the "boost" converter

The switched model of a "boost" DC-to-DC power converter is given by

$$L\frac{d}{dt}x_1 = -ux_2 + E \tag{1}$$

$$C\frac{d}{dt}x_2 = ux_1 - \frac{x_2}{R} \tag{2}$$

where x_1 represents the (input) inductor current and x_2 is the (output) capacitor voltage. The switch position is represented by the control variable u and it takes values on the discrete set $\mathcal{U} = \{0,1\}$. The circuit parameters L, C and R are assumed to be perfectly known

2.1 Problem formulation

It is desired to devise a discontinuous feedback control law for u, such that the capacitor voltage, x_2 , tracks a given desired voltage signal $x_2^*(t)$. This signal is assumed to be bounded and sufficiently differentiable. In fact, we assume that $x_2^*(t)$ is smooth, i.e., infinitely differentiably. Specifically, we are interested in generating an output voltage of the form $x_2^*(t) = A + (B/2) \sin \omega t$ with A, ω > 0 and B being a constant of arbitrary sign.

2.2 Some of the difficulties

The "boost" converter state variables are known to exhibit the following properties (see [8]).

- 1. The capacitor voltage x_2 is a nonminimum phase output variable. This means that if a particular value for x_2 , say $x_2^*(t)$ is imposed on the system behavior, the resulting inductor current is characterized by an unstable "remaining dynamics".
- 2. The inductor current x_1 is a minimum phase output variable. Then, if a particular value for x_1 , say $x_1^*(t)$ is perfectly tracked, the resulting capacitor voltage is represented as the trajectory of a globally stable "remaining dynamics"
- 3. It is not clear how to generate the suitable inductor current reference signal which has as a "remaining dynamics" solution, precisely, the desired output capacitor reference signal $x_2^{\bullet}(t)$.

2.3 A feasible indirect tracking approach

Suppose that a suitable smooth inductor current reference signal is given as $x_1^*(t)$, whose time derivative is, of course, also bounded. A discontinuous feedback controller which reaches and sustains a sliding motion on the time-varying surface $\sigma = x_1 - x_1^*(t)$ is given by

$$u = 0.5(1 + \operatorname{sign} \sigma) \tag{3}$$

Indeed, starting from zero initial conditions for x_1 and x_2 , we have that initially $x_1(t)$ is smaller than $x_1^*(t)$ (i.e., $\sigma < 0$). The switching strategy (3) sets u = 0and the inductor current x_1 grows with slope equals to E/L, while x_2 remains at zero. The sliding surface reaching condition is thus satisfied from "below", provided the reference signal $x_1^*(t)$ is designed with a time derivative which is bounded above by E/L. Clearly, under such assumptions, the quantity $\sigma\dot{\sigma}$ is negative and given by $\sigma(E/L - \dot{x}_1^*(t)) < 0$. When the sliding surface is reached and slightly overshot, the controller (3) starts to inject large positive current pulses to the output RC filter by letting u = 1. As a consequence, x_2 immediately starts to grow from zero, rapidly reaching the converters amplifying mode $x_2 > E$. Thus, while σ is positive, its time derivative, $\dot{\sigma} = (-x_2 + E)/L$, becomes negative. Hence, the sliding surface reaching condition $\sigma\dot{\sigma}<0$ is also satisfied from "above" after the circuit is found in its amplifying mode.

The corresponding "equivalent control" is now obtained as

$$u_{eq} = \frac{E - L\dot{x}_1^{\bullet}(t)}{x_2} \tag{4}$$

The necessary and sufficient conditions for the existence of a sliding regime, given by

$$0 < u_{eq} < 1 \tag{5}$$

imply that, at each instant, the following set of inequalities must be satisfied,

$$0 < E - L\dot{x}_1^*(t) < x_2 \tag{6}$$

The restriction $\dot{x}_1^*(t) < E/L$ implies, roughly speaking, a limitation on the amplitude and frequency of the desired reference signal. Specific tracking limitations of the sliding mode control approach have to be worked out, in detail, for each particular given reference signal waveform $x_1^*(t)$.

The ideal sliding dynamics corresponding to the sliding surface $\sigma = x_1 - x_1^*(t)$ is given by the following asymptotically stable time-varying nonlinear dynamics,

$$\dot{x}_2 = \left(\frac{E - L\dot{x}_1^*(t)}{Cx_2}\right) x_1^*(t) - \frac{x_2}{RC} \tag{7}$$

In order to establish the stability of (7) we define the variable $\rho=x_2^2$ which is easily seen to satisfy the following exponentially asymptotically stable *linear* differential equation subject to bounded perturbations input signals,

$$\dot{\rho} = -\frac{2}{RC} \left[\rho - R \left(E - L \dot{x}_1^*(t) \right) x_1^*(t) \right] \tag{8}$$

2.4 Differential flatness of the "boost" converter

As already demonstrated in [7], the boost converter is differentially flat. This implies the existence of a differential function of the state, termed the flat output, which completely differentially parametrizes, all system variables (i.e., states, outputs, as well as the input). The flat output for the "boost" converter has been established to be the total stored energy of the circuit. For the circuit considered here, the flat output is F_1 , as given by

$$F = \frac{1}{2} \left(Lx_1^2 + Cx_2^2 \right) \tag{9}$$

Since the time derivative of F is independent of the input u, both F and \dot{F} locally qualify as a new set of state variables. Indeed, the relations,

$$F = \frac{1}{2} (Lx_1^2 + Cx_2^2)$$

$$\dot{F} = Ex_1 - \frac{x_2^2}{R}$$
 (10)

represent a locally invertible state coordinate transformation capable of yielding a system in controllable form.

2.5 An iterative procedure for generating a suitable inductor current reference

Consider then the set of relations (10). Evidently, one may "embbed" the such set of relations as the outcome of a convergent iterative procedure, aimed at elimination F, where the value of $x_1 = x_{1,\infty}$ has been computed exclusively in terms of a given fixed function x_2 , and, possibly, an *infinite* number of its time derivatives. In other words, x_1 , viewed as the outcome of such an iterative procedure, could be represented by

$$x_{1,k} = \frac{x_2^2}{RE} + \frac{\dot{F}_k}{E}$$

$$F_{k+1} = \frac{1}{2} \left(Lx_{1,k}^2 + Cx_2^2 \right)$$
 (11)

This algorithm sequentially yields an approximation of a static relationship between x_1 and x_2 , which only involves polynomial expression of x_2 and of its time derivatives. The algorithm of course should be "initialized" by an arbitrary but reasonable trajectory, $F_0(t)$, for the flat output F.

Starting from the natural equilibrium condition, $F_0(t) = constant$, one obtains a sequence $\{x_{1,k}\}$ of approximating expressions for the inductor current reference trajectory x_1 ,

$$x_{1,0} = \frac{x_2^2}{RE} \Longrightarrow F_1 = \frac{1}{2} \left(L \frac{x_2^4}{R^2 E^2} + C x_2^2 \right)$$
 (12)

$$\begin{array}{rcl} x_{1,1} & = & \frac{x_2^2}{RE} + \frac{C}{E} x_2 \dot{x}_2 \left(1 + \frac{2L}{R^2 C E^2} x_2^2 \right) \Longrightarrow \\ F_2 & = & \frac{1}{2} \left[L \left(\frac{x_2^2}{RE} + \frac{C}{E} x_2 \dot{x}_2 \left(1 + \frac{2L}{R^2 C E^2} x_2^2 \right) \right)^2 \right. \\ & & \left. + C x_2^2 \right] & (13) \\ x_{1,2} & = & \frac{x_2^2}{RE} + \left\{ \frac{C}{E} x_2 \dot{x}_2 + \frac{L}{E} \left[\frac{x_2^2}{RE} \right. \right. \\ & \left. - \left. - \left(\frac{C}{E} x_2 \dot{x}_2 + \frac{L}{E} x_2 \dot{x}_2 \right) \right] \times \right. \\ & \left. \left(\frac{2x_2 \dot{x}_2}{RE} + \frac{C}{E} \dot{x}_2^2 + \frac{6L}{R^2 E^3} \dot{x}_2^2 x_2^2 + \frac{C}{E} x_2 \ddot{x}_2 \right. \\ & \left. + \frac{2L}{R^2 E^3} x_2^3 \dot{x}_2 \right) \right\} \end{array}$$

$$\begin{array}{ccc} : & \\ x_{1,\infty} & = & \psi(x_2, \dot{x}_2, x_2^{(2)}, \dots, x_2^{(k)}, \dots) \end{array}$$
 (15)

Section 3 below presents the numerical assessment of the above computational algorithm in the context of a biased sinusoidal output voltage wave generation. The convergence properties are seen to be remarkably fast.

2.6 Simulation Results for the "Boost Converter"

A typical "boost" converter was chosen, with circuit parameters L=20 mH, C=1 $\mu\mathrm{F}$, R=50 Ω and E=15 Volts. We took as sliding surfaces, $\sigma_k=x_1-x_{1,k}^*(t)$ for k=0,1, with $x_{1,0}^*(t)$ and $x_{1,1}^*(t)$ as given by (12)-(13), respectively. As a desired output capacitor voltage signal, we chose, $x_2^*(t)=A+B/2$ $\sin\omega t$. The constants A>0, B and ω were set so that the sliding mode existence conditions (5) were satisfied. The parameters of the desired sinusoidal voltage reference signal, $A+(B/2)\sin(\omega t)$, were set to be A=22.5, B=6, $\omega=\sqrt{2}\times10^2$ [rad/s], i.e, which correspond to a sinusoidal voltage of the form

$$x_2^*(t) = 22.5 + 6\sin(\sqrt{2} \times 10^2 t)$$
 Volts

In accordance with a possible realistic implementation, the underlying sampling frequency used in the simulations of the sliding mode control was taken to be $70.71\,\mathrm{KHz}$

Figure 1 shows the closed loop output voltage responses of the proposed sliding mode tracking controller for various sliding surface candidates of the form, $\sigma_k = x_1 - x_{1,k}^*(t)$, with k = 0, 1. The simulated output voltage responses are shown, for comparison purposes, along with the desired output capacitor voltage signal $x_2^*(t)$. These signals can hardly be distinguished from each other in both cases. Figure 1 also shows the trajectories of the off-line generated inductor current signals,

along with the actual inductor current responses. As it can be seen, for k=0 we already have an excellent agreement between the generated sinusoidal signal x_2 and the desired reference signal $x_2^*(t)$. This agreement is not substantially improved, in the next iteration when k=1. After sliding starts, the equivalent control trajectories are bounded by the closed interval [0,1].

3 The "buck-boost" converter

Consider the switched model of a "buck-boost" DC-to-DC power converter

$$L\frac{d}{dt}x_1 = (1-u)x_2 + uE (16)$$

$$C\frac{d}{dt}x_2 = -(1-u)x_1 - \frac{x_2}{R}$$
 (17)

where x_1 represents the (input) inductor current and x_2 is the (output) capacitor voltage. The switch position is represented by $u \in \mathcal{U} = \{0,1\}$. We take as the differentialy flat output F, the energy like quantity

$$F = \frac{1}{2} \left(Lx_1^2 + C(x_2 - E)^2 \right) \tag{18}$$

As before, it is desired to devise a discontinuous feedback control law for u, such that the capacitor voltage, x_2 , tracks a given biased smooth desired voltage signal $x_2^*(t)$.

The capacitor voltage x_2 of the "buck-boost" converter is known to be a nonminimum phase output variable, while the inductor current x_1 is a minimum phase output (see [8]). As in the "boost" converter case it is not clear how to generate a suitable inductor current reference signal which has as a "remaining dynamics" solution, precisely, the desired output capacitor reference signal $x_2^*(t)$.

3.1 An indirect sliding mode tracking solution Suppose that a suitable smooth inductor current reference signal is given as $x_1^*(t)$. A discontinuous feedback controller which reaches and sustains a sliding motion on the time-varying surface $\sigma = x_1 - x_1^*(t)$ is given by

$$u = 0.5(1 - \operatorname{sign} \sigma) \tag{19}$$

Indeed, starting from zero initial conditions for x_1 and x_2 , we have that initially $x_1(t)$ is smaller than $x_1^*(t)$ (i.e., $\sigma < 0$). The switching strategy (19) sets u = 1 and the inductor current x_1 grows with slope equals to E/L, while x_2 remains at zero. The sliding surface reaching condition is thus satisfied from "below", provided the reference signal $x_1^*(t)$ is designed with a time derivative which is bounded above by E/L. Clearly,

under such assumptions, the quantity $\sigma\dot{\sigma}$ is negative and given by $\sigma(E/L-\dot{x}_1^*(t))<0$. When the sliding surface is reached and slightly overshot, the controller (19) starts to inject large negative current pulses to the output RC filter by letting u=0. As a consequence, x_2 immediately starts to decrease from zero, rapidly reaching the converters "reverse" amplifying mode, $x_2<-E$. Thus, while σ is positive, its time derivative, $\dot{\sigma}=(x_2)/L$, is negative. Hence, the sliding surface reaching condition $\sigma\dot{\sigma}<0$ is also satisfied from "above" the surface just after the circuit is found in its reverse amplifying mode.

The corresponding "equivalent control" is now obtained as

$$u_{eq} = \frac{x_2 - L\dot{x}_1^*(t)}{x_2 - E} \tag{20}$$

The necessary and sufficient conditions for the existence of a sliding regime, given by

$$0 < u_{eq} < 1 \tag{21}$$

imply that, at each instant, the following set of inequalities must be satisfied,

$$0 > x_2 - L\dot{x}_1^*(t) > x_2 - E \tag{22}$$

where the inequality signs have been reversed due to the negative character of the quantity $x_2 - E$.

The restriction $\dot{x}_1^*(t) < E/L$ implies a limitation on the amplitude and frequency of the desired reference signal, while $x_2 < L\dot{x}_1^*(t)$ represents a time-varying constraint which forces the nominal desired voltage across the inductor to be larger than the capacitor voltage response, at all times.

The ideal sliding dynamics corresponding to the sliding surface $\sigma = x_1 - x_1^*(t)$ is given by the following stable time-varying nonlinear dynamics,

$$\dot{x}_2 = \left(\frac{E - L\dot{x}_1^*(t)}{C(x_2 - E)}\right) x_1^*(t) - \frac{x_2}{RC} \tag{23}$$

The asymptotic stability of (23) is determined by defining $\rho = x_2(x_2 - 2E)$, which is easily seen to satisfy the following exponentially asymptotically stable *linear* differential equation subject to bounded perturbations input signals,

$$\dot{\rho} = -\frac{2}{RC} \left[\rho - R \left(E - L \dot{x}_1^*(t) \right) x_1^*(t) \right] \tag{24}$$

3.2 Differential flatness of the "buck-boost" converter

The flat output for the "boost" converter has been established to be the quantity (see [7]),

$$F = \frac{1}{2} \left(Lx_1^2 + C(x_2 - E)^2 \right) \tag{25}$$

Consider the flat output F and its time derivative, written in the following form

$$x_1 = \frac{x_2(x_2 - E)}{RE} + \frac{\dot{F}}{E}$$

$$F = \frac{1}{2} \left(Lx_1^2 + Cx_2(x_2 - E) \right)$$
 (26)

The set of relations (26) motivate the following iterative procedure, which sequentially yields an approximation of a static relationship between x_1 and x_2 , involving only polynomial expression of x_2 and of its time deriva-

$$x_{1,k} = \frac{x_2(x_2 - E)}{RE} + \frac{\dot{F}_k}{E}$$

$$F_{k+1} = \frac{1}{2} \left(L x_{1,k}^2 + C x_2(x_2 - E) \right)$$
 (27)

Starting from the equilibrium condition, $F_0(t) =$ constant, one obtains a sequence $\{x_{1,k}\}$ of approximating expressions for the inductor current reference trajectory x_1 ,

$$x_{1,0} = \frac{x_2(x_2 - E)}{RE} \Longrightarrow$$

$$F_1 = \frac{1}{2} \left(L \frac{x_2^2(x_2 - E)^2}{R^2 E^2} + C(x_2 - E)^2 \right) \quad (28)$$

$$x_{1,1} = \frac{x_2(x_2 - E)}{RE} + \frac{C}{E}(x_2 - E)\dot{x}_2 \times \left(1 + \frac{L}{R^2 C E^2} x_2 (2x_2 - E) \right) \Longrightarrow$$

$$F_2 = \frac{1}{2} \left[L(x_2 - E)^2 \left(\frac{x_2}{RE} + \frac{C}{E} \dot{x}_2 \left(1 + \frac{L}{R^2 C E^2} x_2 (2x_2 - E) \right) \right)^2 + C(x_2 - E)^2 \right] \quad (29)$$

$$x_{1,2} = \frac{x_2(x_2 - E)}{RE} + \frac{C}{E} \dot{x}_2 (x_2 - E) + \left\{ \frac{L}{E} \left[(x_2 - E) \left(\frac{x_2}{RE} + \frac{C}{E} \dot{x}_2 \times \left(1 + \frac{L}{R^2 C E^2} x_2 (2x_2 - E) \right) \right] \times \left[\frac{\dot{x}_2(2x_2 - E)}{RE} + \frac{C}{E} \left(\dot{x}_2^2 \left(1 + \frac{L}{R^2 C E^2} x_2 (x_2 - E) \right) + (x_2 - E) \left[\ddot{x}_2 \left(1 + \frac{L}{R^2 C E^2} x_2 (x_2 - E) \right) + \dot{x}_2^2 \frac{L}{R^2 C E^2} (4x_2 - E) \right] \right] \right\}$$

$$\vdots$$

$$x_{1,\infty} = \psi(x_2, \dot{x}_2, x_2^{(2)}, \dots, x_2^{(k)}, \dots) \quad (30)$$

(30)

3.3 Simulation Results for the "Buck-Boost" Converter

Digital computer simulations were carried out on a "buck-boost" converter with the same circuit parameters as in the "boost" example: L = 20 mH, $C = 1 \mu F$, $R=50~\Omega$ and E=15 Volts. The tested sliding surfaces candidates are of the form, $\sigma_k = x_1 - x_{1,k}^*(t)$, corresponded with k = 0, 1, with $x_{1,0}^*(t)$ and $x_{1,1}^*(t)$ given by (28)-(29), respectively. As a desired output capacitor voltage signal, we chose, $x_2^*(t) = -A - B/2 \sin \omega t$. The constants A, B and ω were set so that the sliding mode existence conditions (21) were satisfied. The parameters of the desired sinusoidal voltage reference signal, were set to be $A = 22.5, B = 6, \omega = 353.55 \text{ [rad/s]}$, which correspond to a sinusoidal voltage of the form

$$x_2^*(t) = -22.5 - 6\sin(353.55 t)$$
 Volts

Figure 4 shows the closed loop output voltage responses of the proposed sliding mode tracking controller for the two sliding surface candidates, $\sigma_k = x_1 - x_{1,k}^*(t)$, with k = 0, 1. Figure 4 also shows the trajectories of the inductor current reference signal, and the actual inductor current responses. For k = 0 we already have an excellent agreement between the sinusoidal signal response x_2 and the desired reference signal $x_2^*(t)$. The tracking error for x_2 is so small that no noticeable discrepancy exists between the desired signal and the converter's response. The equivalent control trajectories are also shown to be bounded signals constrained by the closed interval [0, 1].

4 Conclusions

In this article we have proposed a new approach for the generation of biased sinusoidal AC voltage signals in the output of a DC-to-DC power converter circuit of the "boost" and "buck-boost" types. The proposed control scheme is based on an indirect sliding mode reference inductor current trajectory tracking task for a suitable reference trajectory. The reference inductor current signal is obtained in an off-line fashion from a simple iterative recursive algorithm which sequentially yields approximating finite order differential parametrizations of the inductor currents in terms of the capacitor voltages. Such an off-line procedure is made possible thanks to the differential flatness of the "boost" and "buck-boost" circuits.

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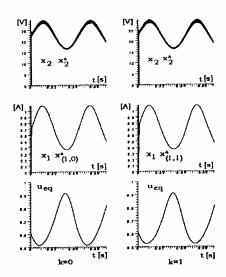


Figure 1: "Boost" converter closed loop response using two generated inductor current reference trajectories, $x_{1,0}^*$ (left column) and $x_{1,1}^*$, (right column), $\omega = 141.42$ [rad/s].

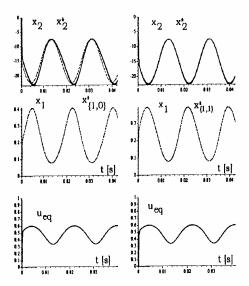


Figure 2: "Buck-boost" closed loop response using two generated inductor current reference trajectories, $x_{1,0}^{\star}$ (left column) and $x_{1,1}^{\star}$ (right column), $\omega = 353.5$ [rad/s].