ON DC TO AC POWER CONVERSION: A DIFFERENTIAL FLATNESS APPROACH [1]

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Abstract

A DC-to-AC power conversion scheme is presented which uses a conventional DC-to-DC switched power converter of the "boost" type. The control scheme is constituted by a sliding mode controller which indirectly tracks a suitable inductor current reference trajectory resulting in a corresponding tight approximation to the desired AC capacitor voltage as the corresponding trajectory for the ideal sliding dynamics. The differential flatness property of the "boost" converter allows one to approximately express the (input) inductor current reference trajectory in terms of a static nonlinear *differential function* of the (output) desired capacitor voltage AC signal. The AC signal generation features of the approach are demonstrated on a biased sinusoidal signal but, in fact, any sufficiently biased differentiable voltage reference signal is achievable in principle. This approximate differential relation between the reference current and the desired voltage is obtained as the outcome of a rapidly convergent offline iterative procedure devised as part of a trajectory planning task (only one or two iterations are sufficient to obtain a remarkable precision). Simulation results are presented for the first few sliding surface candidates obtained from the proposed algorithm.

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1 Introduction

Switchmode DC-to-DC Power conversion represents a vast sub-field of the area of Power Electronics which exploits electrical circuit theory and nonlinear automatic control in a non-traditional manner (See Severns and Bloom [1], Kassakian *et al* [2], Rashid [3] etc.). The most popular devices achieving DC-to-DC power conversion are known as: the "buck" converter, the "boost" converter, the "buck-boost" converter and the "Cúk" converter. Generally speaking, the topologies of the various converters differ in accordance with the switching possibilities between an input circuit, which drains energy from the constant input source and temporarily stores it in the magnetic field of an inductor branch, and an output circuit which converts this magnetic energy into a potential energy using an output low pass filter constituted by an R-C circuit. While the "buck" converter is a "step down" converter, in the sense that it yields as an output voltage a fraction of the constant source input voltage, the "boost" converter "steps up", or amplifies, the source voltage value at the output. The "buck-boost" and the "Ćuk" converters are capable of both amplifying and reducing the constant input voltage, modulo a voltage inversion. The simplicity of the DC-to-DC switchmode power converter topologies joined to the far reaching possibilities of discontinuous feedback control, their transportability and widespread commercial availability makes them highly desirable as potential candidates for DC-to-AC conversion schemes, provided the underlying controller design problem is sensibly solved for the reference signal tracking task.

Traditional approaches to DC-to-AC power conversion include, among other options, PWM commanded switch based *inverters*, fed by Uninterruptible Power Supplies (UPS) or rectified voltage sources; Various combinations of Series-Resonant DC/AC inverters, and more recently, the so called Zero-Voltage-Switching

(ZVS) PWM commutation cells linking constant voltage sources and buck converters (see, among an immense wealth of articles, the works of Mendes de Seixas [9], [10], García and Barbi [5], Jung and Tzou [8], Hsieh *et al* [7] and the many references therein).

DC-to-AC power conversion using the traditional DC-to-DC power converter topologies constitutes a relatively recent sub-area of power electronics and it has proven to constitute a challenging area from the automatic control viewpoint, specially for schemes using converters other than the "buck" converter (see the work of Cáceres and Barbi [4] and, more recently, the article by Fossas *et al* [6]).

In this artice, we propose a systematic approach for yielding an efficient *sliding mode* control based DC-to-AC power conversion. We use a traditional "boost" converter as a working example. The approach, however, has also been successfully extended to include the "buck-boost" converter and the "Ćuk" converter. The differential flatness of the boost converter is shown to allow for the development of an efficient, rapidly convergent, iterative off-line computation scheme which yields a suitable *finite differential parametrization* of the inductor current reference trajectory in terms of the desired AC capacitor voltage reference trajectory and a finite number of its time derivatives. This *indirect* sliding mode control approach which tracks an inductor current reference signal, rather than the desired capacitor voltage signal, seems to be unavoidable, as it is shown that a direct sliding mode option results in an unfeasible unstable closed loop behaviour.

The off-line obtained reference inductor current is used then to devise a time-varying sliding surface for the converter dynamics on which the sliding mode existence conditions can be easily inspected. The frequency and amplitude limitations for the desired AC output voltage signal naturally emerge as a consequence of the well known sliding mode existence conditions (Utkin [12]).

Section 2 establishes the differential flatness of the "boost" converter and the generation of a suitable reference signal for the converters inductor current on which a sliding mode tracking strategy can be proposed for efficient AC signal generation as the output voltage. Section 3 presents various simulation results which depict the rapidly convergent nature of the proposed off-line computational algorithm and the remarkable precision with which a desired AC output voltage signal is tracked by the proposed indirect sliding mode control scheme. This section also presents some of the robustness features of the proposed approach when the derived inverter is subject to unmodelled load variations. Section 4 is devoted to present the conclusions and suggestions for further research.

2 A Flatness Approach to DC-to-AC Power Conversion

The following developments summarize the contents of the article. Many statements are presented without proofs. These are provided in the full version of the article.

Consider a normalized model of the "boost" converter, shown in Figure 1, given by

$$\begin{aligned} \dot{z}_1 &= -uz_2 + 1 \\ \dot{z}_2 &= uz_1 - \frac{z_2}{Q} \end{aligned} \qquad (1)$$

where z_1 is the normalized inductor current, z_2 is the normalized capacitor voltage and u is the switch position function acting as a control input taking values on the discrete set $\mathcal{U} = \{0, 1\}$. The constant parameter Q is known as the circuit *quality*. The "dot" notation stands for derivation with respect to the normalized time variable τ.

2.1 Problem formulation

It is desired to devise a discontinuous feedback control law such that the normalized capacitor voltage, z_2, tracks a given desired voltage signal $z_2^*(\tau)$. This signal is assumed to be bounded and sufficiently differentiable. Specifically, we are interested in generating a normalized output voltage of the form $z_2(t) = A + (B/2)\sin \omega\tau$ with $A > 0$ and B being a constant of arbitrary sign.

2.2 Some of the difficulties

The problem with the above output signal tracking task is that the normalized output capacitor voltage variable, z_2, is *non-minimum phase*. This renders any direct control scheme based on system inversion plainly unfeasible due to closed loop instability. On the other hand, an indirect scheme which makes z_1 track a suitable inductor current reference signal $z_1^*(t)$ results in a stable closed loop system due to the minimum phase character of the state variable z_1.

The second difficulty associated with the AC signal generation problem is that if an indirect approach is adopted, it is not clear how to generate the suitable inductor current reference signal which has as a "remaining dynamics" solution precisely the desired output capacitor reference signal $z_2^*(\tau)$.

The normalized model (1) of the "boost" converter is known to be *differentially flat*, (see Sira-Ramírez, [11]) which in the single input case at hand means that it is exactly linearizable by means of static feedback. However, due to the discrete nature of the control input values, such an exact linearization is not feasible except in an average sense.

2.3 Finding a suitable inductor current reference signal through flatness

We exploit the flatness of the system in a manner different to that of exact linearization. Thus, consider the flat output, $F = 0.5(z_1^2 + z_2^2)$ and its time derivative $\dot{F} = z_1 - z_2^2/Q$. We rewrite these two expressions in terms of reference signal values as follows

$$z_1^*(\tau) = \dot{F}^* + \frac{z_2^*(\tau)}{Q} \;\; ; \;\; F^* = \frac{1}{2}\left((z_1^*(\tau))^2 + (z_2^*(\tau))^2\right) \tag{2}$$

These expressions may be *embeded* in an iterative computational alorithm of which they represent the converged solution. Consider then the following expressions

$$\begin{aligned} z_{1,k}^*(\tau) &= \dot{F}_k^* + \frac{z_2^*(\tau)}{Q} \\ F_{k+1}^* &= \frac{1}{2}\left((z_{1,k}^*(\tau))^2 + (z_2^*(\tau))^2\right) \end{aligned} \tag{3}$$

Evidently, if the computations converges to, say, $z_{1\infty}^*(\tau)$, $F_\infty^*(\tau)$ for the given signal $z_2^*(\tau)$, then these quantities precisely satisfy the set of equations (2).

The first few reference signal candidates generated by the above algorithm are given by

$$z_{1,0} = \frac{z_2^2}{Q} \Longrightarrow F_1 = \frac{1}{2}\frac{z_2^4}{Q^2} + \frac{1}{2}z_2^2 \tag{4}$$

$$\begin{aligned} z_{1,1} &= \frac{z_2^2}{Q} + z_2\dot{z}_2\left(1 + \frac{2}{Q^2}z_2^2\right) \Longrightarrow \\ F_2 &= \frac{1}{2}\left[\frac{z_2^2}{Q} + z_2\dot{z}_2\left(1 + \frac{2}{Q^2}z_2^2\right)\right]^2 + \frac{1}{2}z_2^2 \end{aligned} \tag{5}$$

$$\begin{aligned} z_{1,2} &= \frac{z_2^2}{Q} + \left(\frac{z_2^2}{Q} + z_2\dot{z}_2 + \frac{2z_2^3}{Q^2}\dot{z}_2\right) \times \\ &\left(\frac{2}{Q}z_2\dot{z}_2 + (\dot{z}_2)^2 + z_2\ddot{z}_2 + \frac{6z_2^2}{Q^2}(\dot{z}_2)^2 + \frac{2z_2^3}{Q^2}\ddot{z}_2\right) \\ &+ z_2\dot{z}_2 \end{aligned} \tag{6}$$

where we have dropped the asterisks and the time arguments for the sake of simplicity.

2.4 A sliding mode controller for tracking the reference candidates

Suppose that the off-line computed expression for the inductor current reference signal, $z_{1,k}^*(\tau)$, is found to be suitable, for some finite k, for the proposed sinusoidal capacitor voltage generating task. Then, an indirect sliding mode control approach to the tracking problem requires the use of a time-varying sliding surface of the form

$$\sigma_k = z_1 - z_{1,k}^*(\tau) \tag{7}$$

It is easy to show that the switching law

$$u = \frac{1}{2}(1 + \operatorname{sign}\sigma_k) \tag{8}$$

produces a state trajectory response which reaches the time-varying sliding surface $\sigma_k = 0$ in finite time and, under mild conditions, it is capable of sustaining a sliding regime thereafter.

Indeed, while $z_1(\tau)$ is smaller than $z_{1,k}^*(\tau)$ (i.e., $\sigma_k < 0$), the switching strategy (8) sets $u = 0$ and the sliding surface reaching condition, $\sigma_k\dot{\sigma}_k < 0$, is satisfied since the quantity $\sigma_k\dot{\sigma}_k$ is given by $\sigma_k(-uz_2 + 1) = \sigma_k < 0$. Similarly, when $z_1(\tau)$ is greater than $z_{1,k}^*(\tau)$ then σ_k is positive and $\dot{\sigma}_k = -z_2 + 1$. Hence, as long as $z_2 > 1$ (which is the nonminalamplifying character of the "boost" circuit), the reaching condition is satisfied.

The corresponding "equivalent control" is now obtained as

$$u_{eq} = \frac{1 - \dot{z}_{1,k}^*(\tau)}{z_2} \tag{9}$$

The necessary and sufficient conditions for the existance of a sliding regime $0 < u_{eq} < 1$ imply that, at each instant, the sign of z_2 should coincide with the sign of the signal $1 - \dot{z}_{1,k}^*(\tau)$. Assume that z_2 is to remain positive. This implies that the following conditions must be satisfied,

$$\dot{z}_{1,k}^*(\tau) < 1 \;\; ; \;\; 1 - \dot{z}_{1,k}^*(\tau) < z_2 \tag{10}$$

For a particular k, the above set of inequalities lead, after some algebraic manipulations, to a time-independent relationship between the magnitude of the amplitudes A, B and the frequency ω of the sinusoidal voltage reference signal z_2^*. Generally speaking, such derived constraints represent *sufficient conditions* for the existence of a sliding regime on the proposed sliding surface.

The corresponding *ideal sliding dynamics* is given by the following stable time-varying nonlinear dynamics,

$$\dot{z}_2 = \left(\frac{1 - \dot{z}_{1,k}^*(\tau)}{z_2}\right) z_{1,k}^*(\tau) - \frac{z_2}{Q} \tag{11}$$

3 Simulation Results

A normalized boost converter with circuit quality $Q = 2$ was chosen for illustration purposes. The parameters of the desired sinusoidal voltage reference signal, $A + (B/2)\sin(\omega\tau)$, were set to be $A = 1.5$, $B = 1$, $\omega = 0.02$, i.e,

$$z_2^*(\tau) = 1.5 + 0.5\sin(0.02\tau)$$

Figure 2 shows the closed loop output voltage responses of the proposed sliding mode tracking controller for various sliding surface candidates of the form $\sigma_k = z_1 - z_{1,k}^*(\tau)$ with $k = 0, 1, 2$. The simulated time responses are shown, for comparison purposes, along with the desired output capacitor voltage signal $z_2^*(\tau)$.

As it can be seen for $k = 0$ we already have a good agreement of the generated sinusoidal signal with the desired one.

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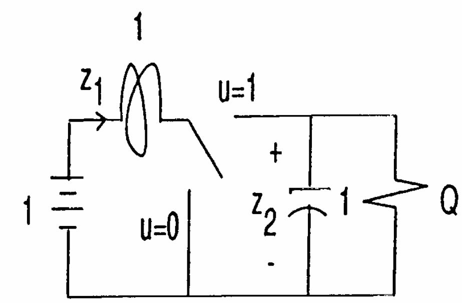

Figure 1: Normalized "boost" converter circuit