BCC99

ON THE REGULATION OF A HELICOPTER SYSTEM: A TRAJECTORY PLANNING APPROACH FOR THE LIOUVILLIAN MODEL *

H. Sira-Ramírez and **R. Castro-Linares**

Centro de Investigación y de Estudios Avanzados del IPN
Departamento de Ingeniería Eléctrica,
Apartado Postal 14-740
07300 México, D.F., México
Fax: 52-5-747-7089, e-mail : {hsira@mail,rcastro@ctrl}.cinvestav.mx

Keywords : Helicopter, Liouvillian systems, trajectory planning

Abstract

A feedback regulation scheme is presented which allows for a wide range of displacement maneuvers on a simplified longitudinal model of a helicopter system which is underactuated and nondifferentially flat. The system, nevertheless, is shown to be of the "Liouvillian" type. An off-line, open loop, trajectory planning procedure is proposed, based on the solution of differential equations for the desired attitude angle reference signal, which precisely results in the desired nominal horizontal displacement trajectory. The performance of a proposed feedback controller, which exploits the fact that the linearized system has its coefficients on a Hardy filed, is evaluated through digital computer simulations including unmodelled mid course wind gusts and initial setting errors.

1 Introduction

In this article, a feedback regulation scheme is proposed which effectively accomplishes a wide range of horizontal displacement maneuvers tracking tasks for a simplified longitudinal dynamics of a helicopter system model developed in the work of [8]. The proposed feedback regulation method is based on the fact that the simplified model of the helicopter constitutes a ***Liouvillian system*** i.e. a non flat system with a flat subsystem of largest dimension. The flat output completely determines the nonflat subsystem variables, or the *defect* up to elementary quadratures. In the helicopter model adopted here, the flat subsystem is represented by the attitude angle kynematic variable, while the non-flat subsystem is represented by the horizontal displacement.

An off-line trajectory planning is carried out for the attitude angle which is computed on the basis of the the desired horizontal displacement. The calculation is performed by inversion of the horizontal displacement dynamics in conjunction with the desired flat output subsystem. The latter viewed as a dynamical system forced by the desired displacement reference trajectory and its time derivatives. This off-line procedure is certainly made possible by the Liouvillian character of the system. The nominal open loop controller, obtained from the off-line trajectory planning procedure, is complemented with an "outer loop" state feedback control option of the "proportional-plus-derivative" type, specified on the basis of the linearized attitude angular position and angular rate, as well as the horizontal displacement and horizontal velocity tracking error signals. The proposed control scheme provides the required robustness with respect to small inital setting errors and unexpected perturbations occuring during the flight maneuver execution.

Section 2 presents a simplified description of the helicopter dynamics together with the physically plausible adopted assumptions. Section 3 contains a brief introduction to Liouvillian systems. The proposed feedback control scheme, based on open loop trajectory planning and a Jacobian linearization-based controller is also presented in this section. Section 4 presents some simulations testing the performance of the closed loop system. These include initial state setting errors as well as, unmodelled, stochastic mid-air wind gusts perturbations. The conclusions are presented in the last section.

*This research was supported by the National Council for Scientific and Technological Research of Venezuela (CONICIT) under Research Grant S1-95-000886 and by the Center of Research and Advanced Studies of the National Polythecnic Institute (CINVESTAV-IPN), Mexico.

2 A Simplified Model for the Helicopter

We consider a simplified model of an helicopter, which has been fully reported in various works (see, for instace, [8]). The model only represents constrained flight, at a constant heigth, and it is conformed by the following set of controlled differential equations

$$\begin{aligned} \ddot{\theta} &= L\,u \\ \ddot{x} &= -g\tan\theta - \left[\frac{1}{M\cos\theta}\right]\,u \end{aligned} \qquad (1)$$

where x denotes the horizontal position of the helicopter, u is the control input, the variable θ denotes the attitude angular position of the main rotor. L is a known constant. It is worthwhile to notice that the model (1) is different from the one analyzed in [7], where the forward velocity dynamics, in straight level flight, reduces to $\ddot{x} = -g\tan(\theta)$.

Straightforward manipulations of equation (1) yield the following relationship:

$$g - \ddot{x}\tan\theta = \sec^2\theta\left[g + \frac{1}{M}u\sin\theta\right] \qquad (2)$$

The right hand side of equation (2) must always be positive for reasonable maneuvers that do not exceed an attitude angular displacement restriction $-\theta_{max} \leq \theta \leq \theta_{\max}$ with $\theta_{\max} < \pi/2$. It follows that the quantity $g - \ddot{x}\tan\theta$ can be assumed to be non-negative. In fact, we assume, that there exists a ***strictly positive*** scalar μ such that $g - \ddot{x}\tan\theta > \mu$. We adopt this assumption throughout.

3 Regulation of the Helicopter Model

3.1 Liouvillian systems

Differentially flat systems, or *flat* systems in short, were introduced by Professor M. Fliess, and his colleages, in a series of articles [3]-[4]. Flat systems are characterized by the fact that all variables, including the inputs, can be expressed in terms of ***differential functions of the flat outputs***, which is a set of independent differential functions of the state, possibly involving the inputs and a finite number of their time derivatives. The set of flat outputs has the same cardinality as the set of control inputs. Flat systems constitute a subclass of the set of controllable nonlinear systems which are equivalent to a linear system in Brunovsky's form by means of ***endogenous feedback***. A non flat system may still be controllable, but not all its variables can be expressed as differential functions of a particular set of independent outputs. The number of independent variables in the system, not expressible in terms of the flat outputs, is known as the ***defect*** of the nonflat system. Liouvillian systems have been recently introduced by Chelouah in [1] from the perspective of Differential Galois theory in the context of Piccard-Vessiot extensions of differentially flat fields. Liouvillian systems constitute a natural extension of the class of differentially flat systems characerized by the existence of an identifiable flat subsystem of maximal dimension. A non-flat system is said to be Liouvillian, or ***integrable by quadratures***, if the variables not belonging to the flat subsystem are expressible in terms of elementary integrations of the flat outputs and a finite number of their time derivatives. This new class of systems has also been shown to have interesting implications within the ralm of ***finitely discretizable*** nonlinear systems, as inferred from the work of Chelouah and Petitot [2].

3.2 The helicopter model as a Liouvillian system

One may easily verify that system (1) is not linearizable by means of static state feedback and, hence, it is also non linearizable by dynamic state feedback either. On the other hand, the simplified helicopter model (1) is clearly Liouvillian, with the flat subsystem being represented by the kinematics state variable θ. The subsystem flat output is given by the attitude angle, $F = \theta$ and, hence, the rest of the kinematics subsystem variables are expressible as differential functions of the attitude angle. Indeed, $\dot{\theta} = \dot{F}$ and $u = \ddot{F}/L$. The nonflat subsystem is characterized by the displacement variable x. This variable, and its rate of change, are expressible in terms of ***quadratures*** of the flat output F and its second order time derivative $\ddot{F}$. Indeed, from the last equation in (1) and the previous considerations about the flat subsystem, one obtains,

$$\begin{aligned} x &= \int_{t_0}^{t}\int_{t_0}^{\sigma}\left(g\tan(F) + \frac{1}{ML}\sec(F)\ddot{F}\right)d\sigma dt \\ &\quad + \dot{x}(t_0)(t-t_0) + x(t_0) \end{aligned} \qquad (3)$$

The following ***integro-differential parameterization*** of the system variables allows for some elementary equilibrium analysis and also establishes the main features of the system:

$$\begin{aligned} \theta &= F \;;\; u = \frac{\ddot{F}}{L} \\ \ddot{F} &= -LM\ddot{x}\cos F - LgM\sin F \end{aligned} \qquad (4)$$

Thus, the ***zero dynamics***, or ***remaining dynamics***, corresponding to a resting hovering position, characterized by $x = constant$, $\dot{x} = 0$ and $\ddot{x} = 0$, is given, according to (4), by the following dynamics

$$\ddot{F} = -LgM\sin F \qquad (5)$$

The zero dynamics is represented by the locally stable oscillatory system (5) with equilibria located at the origin and, also, at the attitude angles of the form, $F = \pm k\pi$ $k = 1, 2, ...$ The system is, hence, ***weakly minimum*** *phase* around the origin, with respect to the horizontal position coordinate x, taken as a system output (See Figure 1).

3.3 Off-line trajectory planning

Suppose that a desired displacement maneuver is specified as a sufficiently smooth trajectory $x^*(t)$ for the horizontal position variable x. The desired maneuver is to take place over a given, finite, time interval $[t_0, T]$. The specified trajectory is supposed to take the helicopter from the initial equilibrium hovering position, located at $x(t_0)$, towards a final hovering horizontal coordinate value, specified as $x(T)$. The displacement maneuver may include an arbitrary number of intermediate resting equilibria, or hovering positions, as well as "backing-ups" and advancements of arbitrary lengths. The time evolution of the "off-line" planned trajectory $x^*(t)$ is also assumed to start with a sufficient number of zero initial and final time derivatives (with similar features for intermediate resting positions). This last requirement guarantees smooth departures from initial or intermediate resting points as well as smooth arrivals at intermediate equilibria, or at the final resting position.

3.4 A trajectory tracking feedback controller based on Pole Placement

The differential flatness property of the θ subsystem allows one to express the control input u as the quantity $\ddot{F}/L$. Therefore, if a desired displacement is given by $x^*(t)$, then, the corresponding flat output trajectory may be computed by finding the solution $F^*(t)$ of the following nonlinear second order differential equation, obtained from the displacement equation,

$$\ddot{F}^* = -LM\ddot{x}^*(t)\cos F^* - LgM\sin F^* \qquad (6)$$

with initial conditions given by the ideal hovering condition $F^*(t_0) = 0$, $\dot{F}^*(t_0) = 0$.

The "off-line" attitude angle trajectory $F^*(t)$, generated by the differential equation (6) is to be used as part of an "on-line" feedback controller, obtained from the following approximate Jacobian linearization scheme. For this, let us define the following state variables tracking errors and control input error,

$$\begin{aligned} x_{1\delta} &= \theta - F^*(t) \ ; \ x_{2\delta} = \dot{\theta} - \dot{F}^*(t) \ ; \ x_{3\delta} = x - x^*(t) \\ x_{4\delta} &= \dot{x} - \dot{x}^*(t) \ ; \ u = u_\delta + \frac{\ddot{F}^*(t)}{L} \end{aligned} \qquad (7)$$

The linearized dynamics, under the assumption of small deviations from the planned trajectories, is given by

$$\begin{aligned} \dot{x}_{1\delta} &= x_{2\delta} \ ; \ \dot{x}_{2\delta} = Lu_\delta \\ \dot{x}_{3\delta} &= x_{4\delta} \\ \dot{x}_{4\delta} &= -[g - \ddot{x}^*\tan F^*]x_{1\delta} - \frac{1}{M\cos F^*}u_\delta \end{aligned} \qquad (8)$$

The fourth order, time-varying, linear system (8), which is of the form $\dot{x}_\delta = A(t)x_\delta + b(t)u_\delta$, is found to be controllable by standard tests,

$$\text{rank}\left[b(t), \left(A - \frac{d}{dt}\right)b(t), ..., \left(A - \frac{d}{dt}\right)^3 b(t)\right] = 4 \quad (9)$$

We expoit here the fact that the coefficients of the linearized model (8) belong to a *Hardy Field*, i.e., to one in which the largest comparability class, among functions, is constituted by the class of exponentials. Using well known results obtained from the work of Fliess and Rudolph, [5], we proceed to place the poles of the linearized system, at constant locations, sufficiently deep into the left hand portion of the complex plane, by means of time-varying linear feedback.

$$u_\delta = k_1(t)x_{1\delta} + k_2(t)x_{2\delta} + k_3(t)x_{3\delta} + k_4(t)x_{4\delta} \qquad (10)$$

with

$$\begin{aligned} k_1(t) &= \frac{b_2}{L} + \frac{b_0}{L^2M\cos F^*(t)\,(g - \tan F^*(t)\ddot{x}^*(t))} \\ k_2(t) &= \frac{b_3}{L} + \frac{b_1}{L^2M\cos F^*(t)\,(g - \tan F^*(t)\ddot{x}^*(t))} \\ k_3(t) &= \frac{b_0}{L\,(g - \tan F^*(t)\ddot{x}^*(t))} \\ k_4(t) &= \frac{b_1}{L\,(g - \tan F^*(t)\ddot{x}^*(t))} \end{aligned} \qquad (11)$$

where the set of constants $\{b_0, b_1, b_2, b_3\}$ constitute a set of constant *Hurwitz coefficients*, defining the following polynomial in the complex variable λ,

$$p(\lambda) = \lambda^4 + b_3\lambda^3 + b_2\lambda^2 + b_1\lambda + b_0 \qquad (12)$$

The roots of this polynomial are assumed to be located at fixed locations on the left portion of the complex plane.

A full feedback controller for the helicopter model, based on the above considerations, is thus given by a combination of the nominal and the incremental controller as follows,

$$u = \frac{1}{L}\left[\ddot{F}^*(t)\right] + u_\delta \qquad (13)$$

with u_δ given by (10),(11).

4 Simulation Results

4.1 Off-line trajectory planning and nominal controller

Figure 2 shows the a typical horizontal displacement maneuver specified by the following polynomial spline,

$$x^*(t) = \begin{cases} x(t_0) & \text{for } t < t_0 \\ x(t_0) + (x(T) - x(t_0))\psi(t, t_0, T) & \text{for } t_0 \leq t \leq T \\ x(T) & \text{for } t > T \end{cases} \qquad (14)$$

with

$$\psi(t,t_0,T) = \left[\frac{t-t_0}{T-t_0}\right]^5 \left\{ r_1 - r_2\frac{t-t_0}{T-t_0} + r_3\left(\frac{t-t_0}{T-t_0}\right)^2 - r_4\left(\frac{t-t_0}{T-t_0}\right)^3 + r_5\left(\frac{t-t_0}{T-t_0}\right)^4 - r_6\left(\frac{t-t_0}{T-t_0}\right)^5 \right\} \quad (15)$$

$$r_1 = 252 \; ; \; r_2 = 1050 \; ; \; r_3 = 1800 \; ; \; r_4 = 1575$$

$$r_5 = 700 \; ; \; r_6 = 126$$

The inital and terminal times, as well as the initial and terminal points on the horizontal line coordinate were taken to be

$$x(t_0) = 100 \text{ m} \; ; \; x(T) = 300 \text{ m} \; ; \; t_0 = 30 \text{ s} \; ; \; T = 60 \text{ s}$$

The above prescribed trajectory $x^*(t)$ for the helicopter exhibits four time derivatives equal to zero at the initial instant t_0, at which the maneuver is started, and also at the final instant T, at which the maneuver is ended. The horizontal position smoothly increases from the value $x(t_0)$ towards the final position value, $x(T)$, reaching the final position at time T. The helicopter indefinitely settles there after time T.

Figure 2 also shows the off-line computed reference atttitude angle trajectory $F^*(t)$ as obtained from the solution of the differential equation (6) with $x^*(t)$ specified by (14) and initial conditions taken to exactly coincide with the ideal hovering conditions. The corresponding nominal control input u^*, is given, according to the flatness property of the kinematics subsystem, by $u^* = \ddot{F}^*/L$.

For the simulations shown in this article, the following values were assigned to the system parameters:

$$M = 4313\text{Kg} \; ; \; g = 9.8\text{m/s}^2 \; ; \; L = 1.0456 \times 10^{-4}\text{N} - \text{m/s}^2$$

4.2 Closed loop feedback controller performance

The performance of the proposed controller (10),(11),(13) was tested in two typical desired horizontal displacement maneuvers, including initial states perturbation errors and unmodelled disturbance gusts taking place in mid-air, during the execution of the prescribed maneuver. Figure 3 depicts the performance of the closed loop system for the attitude angular displacement, the attitude rate, as well as the horizontal displacement and horizontal velocity variables corresponding to a prescribed trajectory of the form (15). The simulation included an initial position error with respect to the planned trajectory as well as initial discrepancies from the ideal hovering conditions. The proposed feedback controller manages to effectively correct all initial discrepancies and achieve satisfactroy tracking of, both, the computed attitude reference trajectory $F^*(t)$ and the originally given horizontal position maneuver $x^*(t)$. Tracking is achieved with unnoticeable discrepancies.

In order to test the recovery features of the prescribed controller to unknown disturbances, an unmodelled wind gust disturbance was simulated which affected the maneuver around the mid point between the intial and final horizontal locations of the displacement maneuver. The unmodelled gust disturbance is specified, for simulation purposes, as an unexpected force disturbance in the attitude dynamics occuring around a certain fixed horizontal location of the maneuver path. The disturbance magnitude is assumed to have a shape similar to a "gaussian probability distribution" curve. Such a force disturbance is modelled in the following form:

$$r(x) = R\exp\left[-\left(\frac{x - x_{dl}}{\Sigma}\right)^2\right] \quad (16)$$

where R is the maximum magnitude of the force disturbance, occuring at the unknown location x_{dl}. The wind gust disturbance vanishes away from the maximum force location in accordance with the value of the constant parameter Σ.

The simulation results, for a rather strong wind disturbance of maximal amplitude of $R = 6000\text{Kgm/sec}^2$, are shown in Figure 4. The horizontal tracking is for all practical purposes unaffected by the disturbance while the attitude angle trajectory is seen to be severly perturbed by the perturbation. The controller however retakes the attitude tracking task, right after the disturbance ceases.

Finally a more complex maneuver, involving an intermediate rest point with a "backing up" requirement, was also prescribed as indicated next. Let the function $\phi(t,\tau,\sigma)$, for $\sigma \geq t \geq \tau$, be defined in a similar fashion to the previously defined function $\psi(t,t_0,T)$ in (15). Then, a possible realization of the trajectory described above is given by

$$x^*(t) = \begin{cases} x(t_0) & \text{for } t < t_0 \\ x(t_0) + (x(t_1) - x(t_0))\phi(t,t_0,t_1) & \text{for } t_0 \leq t \leq t_1 \\ x(t_1) & \text{for } t_1 \leq t \leq t_2 \\ x(t_3) + (x(t_3) - x(t_2))\phi(t,t_2,t_3) & \text{for } t_2 \leq t \leq t_3 \\ x(t_3) & \text{for } t_3 \leq t \leq t_4 \\ x(T) + (x(T) - x(t_4))\phi(t,t_4,T) & \text{for } t_4 \leq t \leq T \\ x(T) & \text{for } t > T \end{cases} \quad (17)$$

The simulation results of the closed loop system responses are shown in Figure 5. The simulation test includes, again, a wind gust disturbance located around the horizontal position $x = 200$ m. The helicopter is commanded to advance from $x = 100$ m, to $x = 300$ m, through the wind gust, located around $x = 200$ m. The helicopter is then forced to "back up" from $x = 300$ m, to $x = 200$ m, and stand in a hovering position, precisely, at the point where the wind gust disturbance exhibits its maximum value. Again, the horizontal displacement reference trajectory $x^*(t)$ is followed by the closed loop system with no alteration whatsoever while the tracking of the attitude reference signal is temporarily lost in the forward phase of the maneuver. Notice, however, that the prescribed atitude trajectory tracking is lost during the "backing up" stage of the maneuver while the helicopter is being placed in the middle of the wind gust disturbance. Nevertheless, since the second order derivative of the perturbed attitude trajectory still has the ideal steady state value needed for tracking, the horizontal displacement tracking features are unaffected by the wind gust perturbation in the steady state final hovering position.

5 Conclusions

In this article, we have proposed a simple linearization based controller, complemented with a nominal off-line computed control signal related to a trajectory planning scheme, for the effective regulation of the horizontal displacement and attitude dynamics of a simplified nonlinear model of a helicopter system. The approach is based on exploting the fact that the system belongs to a particular class of non differentially flat systems, called Liouvillian systems. This last property allows for an off-line trajectory planning of the flat output in terms of a required, displacement, or defect, trajectory. The proposed controller has been tested through digital computer simulations with very encouraging results. A wide range of longitudinal maneuvers including initial state and mid course unknown perturbations were efficiently handled by the proposed controller. A more complete study is being pursued, using the full nonlinear model of the helicopter derived in [8]. The scheme here proposed seems to go through rather well for the multivariable case. A challenging task is represented by the development of feedback control strategies for the "toycopter" model, developed by Mallhaupt and his colleages [6]. Our proposed feedback control scheme, in that case, works suitably well, provided only attitude restrictions are imposed with no additional orientation prescriptions.

References

[1] Chelouah, A., "Extensions of differential flat fields and Liouvillian systems" in *Proceedings of the 36th Conference on Decision and Control*, San Diego, California, USA., pp. 4268-4273 (1997).

[2] Chelouah, A., and Petitot, M., "Finitely discretizable nonlinear systems: Concepts and definitions" in *Proc. of the 34th IEEE Conference on Decision and Control*, New Orleans, Louisiana, USA, December 1995, pp. 19-24.

[3] Fliess, M., Lévine, J., Martín, Ph., and Rouchon, P., "Sur les systèmes nonlinéaires différentiellement plats", *C. R. Acad. Sci. Paris*, **315** Série I, pp. 619-624, (1992).

[4] Fliess, M., Lévine, J., Martín, Ph., and Rouchon, P., "Flatness and defect of nonlinear systems: Introductory theory and examples ", *International Journal of Control*, **61**, pp. 1327-1361, (1995).

[5] Fliess, M., and Rudolph, J., "Corps de Hardy et observateurs asymptotiques locaux pour systèmes différentiellement plats" *C. R. Acad. Sci. Paris*, **324**, Série II b, pp. 513-519, (1997).

[6] Mallhaupt, P., Srinivasan, B., Levine, J., and Bonvin, D., " A toy more difficult to control than the real thing", *Proceedings 1997 European Control Conference, ECC'97*, (CD-Rom version), Brussels, Belgium, (1997).

[7] van Nieuwstadt, M., and Murray, R. M., "Outer Flatness: Trajectory Generation for a Model Helicopter," *Proceedings of the 1997 European Control Conference, ECC'97*,(CD-Rom version), Brussels, Belgium, (1997).

[8] Thomson, D. G., and Bradley, R., "Prediction of the Dynamic Characteristics of Helicopters in Constrained Flight," *Aeronautica Journal of The Aeronautical Royal Society*, (1990).

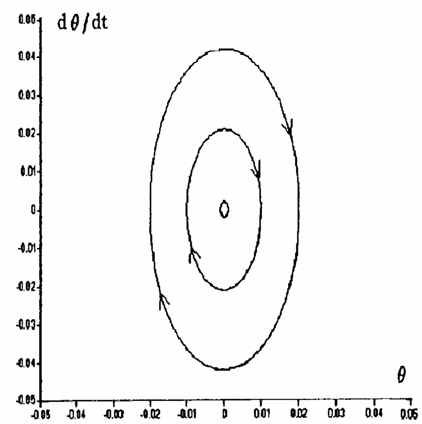

Figure 1: Zero dynamics corresponding to constant horizontal displacement.

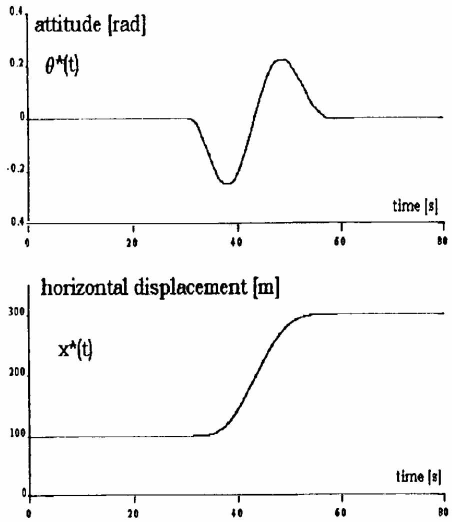

Figure 2: Ideal open loop horizontal displacement maneuver via computed attitude trajectory planning.

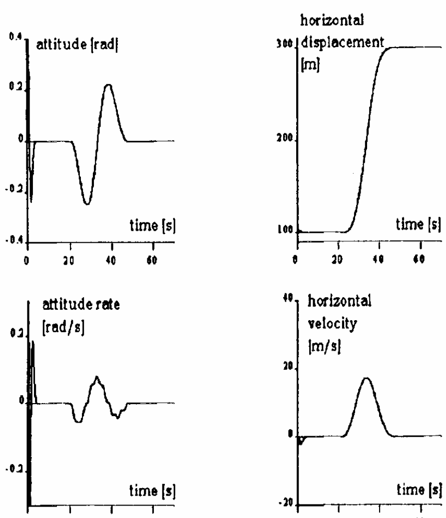

Figure 3: Closed loop response in a hovering point-to-hovering point maneuver with initial state setting errors.

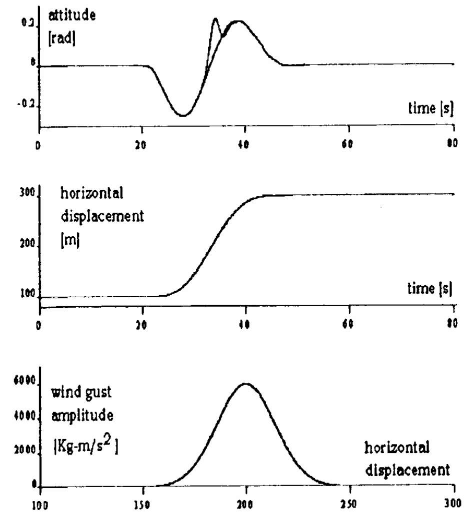

Figure 4: Closed loop response with, unmodelled, mid-air wind gust perturbation.

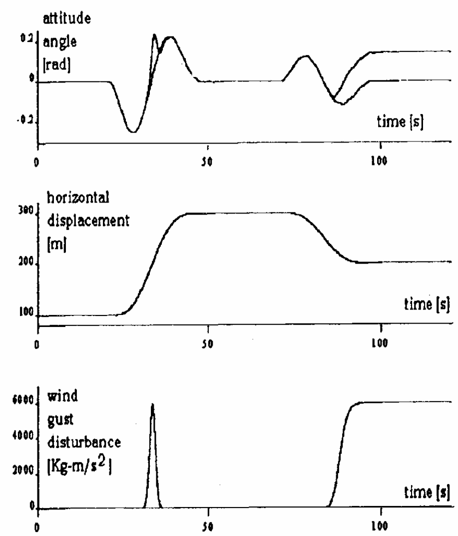

Figure 5: Forward and backing maneuver with unmodelled wind gust perturbation and initial setting errors.