A HAMILTONIAN SYSTEMS APPROACH TO CHAOTIC SYSTEMS SYNCHRONIZATION

Hebertt Sira-Ramírez

Dept. Ing. Eléctrica
Secc. Mecatrónica
CINVESTAV-IPN
México, D.F., México
email: hsira@mail.cinvestav.mx

ABSTRACT

A new approach to chaotic systems synchronization is presented from the perspective of state observer design in the context of Generalized Hamiltonian systems including dissipation and de-stabilizing vector fields. For a number of chaotic systems, the approach reduces the synchronization problem to a linear problem intimately related to the concepts of observability, or, detectability associated with constant maps.

INTRODUCTION

In this article, we are concerned with the synchronization of chaotic systems from the perspective of Generalized Hamiltonian systems including non-conservative terms. It turns out that the great majority of chaotic systems can be placed in such a Generalized Hamiltonian canonical form, from where the reconstructibility of the state vector, from a defined output signal, may be assessed from the observability or, in its absence, the detectability of a pair of *constant* matrices. The Generalized Hamiltonian structure of most known chaotic systems allows one to clearly decide on the nature of the synchronizing (output) signal on the basis of the system dissipation and conservative energy managing structure and a need for elimination, at the receiver end, of the locally, or globally, de-stabilizing vector field. For an extensive bibliography about chaotic systems, in general, and the synchronization problem, in particular, the reader is referred to the collection of references gathered by professor Chen [1].

Section 2 briefly describes a class of Generalized Hamiltonian Systems and proposes a state observer construction. The class of system comprises nearly all of the best known chaotic systems addressed in the literature. Section 3 analyses the synchronization problem, from the perspective of the obtained results, for three standard chaotic system examples. The last section is devoted to some conclusions and suggestions for further work.

NONLINEAR OBSERVER DESIGN FOR A CLASS OF SYSTEMS IN GENERALIZED HAMILTONIAN FORM

We consider a special class of Generalized Hamiltonian systems with de-stabilizing vector fields and linear output map , y, given by

$$\begin{aligned} \dot{x} &= \mathcal{J}(y)\frac{\partial H}{\partial x} + (\mathcal{I}+\mathcal{S})\frac{\partial H}{\partial x} + \mathcal{F}(y), \\ y &= \mathcal{C}\frac{\partial H}{\partial x}, \;\; x \in R^n, \;\; y \in R^m \end{aligned} \tag{1}$$

where $\mathcal{S}$ is a constant symmetric matrix, not necessarily of definite sign. The matrix $\mathcal{I}$ is a constant skew symmetric matrix. The vector variable y is referred to as the system *output*. The matrix $\mathcal{C}$ is a constant matrix.

We denote the *estimate* of the state vector x by ξ, and consider the Hamiltonian energy function $H(\xi)$ to be the particularisation of H in terms of ξ. Similarly, we denote by η the estimated output, computed in terms of the estimated state ξ. The gradient vector $\partial H(\xi)/\partial\xi$ is, naturally, of the form $\mathcal{M}\xi$ with $\mathcal{M}$ being a, constant, symmetric positive definite matrix.

A dynamic nonlinear state observer for the system (1) is readily obtained as

$$\begin{aligned} \dot{\xi} &= \mathcal{J}(y)\frac{\partial H}{\partial \xi} + (\mathcal{I}+\mathcal{S})\frac{\partial H}{\partial \xi} + \mathcal{F}(y) \\ &\quad + K(y-\eta) \\ \eta &= \mathcal{C}\frac{\partial H}{\partial \xi} \end{aligned} \tag{2}$$

where K is a constant vector, known as the *observer gain*.

The state estimation error, defined as $e = x - \xi$ and the output estimation error, defined as $e_y = y - \eta$, are governed by

$$\begin{aligned} \dot{e} &= \mathcal{J}(y)\frac{\partial H}{\partial e} + [\mathcal{I}+\mathcal{S}-\mathcal{K}\mathcal{C}]\frac{\partial H}{\partial e}, \\ e_y &= \mathcal{C}\frac{\partial H}{\partial e}, \;\; e \in R^n, \;\; e_y \in R^m \end{aligned}$$

where the vector, $\partial H/\partial e$ actually stands, with some abuse of notation, for the gradient vector of the *modified* energy function,

$$\begin{aligned}\partial H(e)/\partial e &= \partial H/\partial x - \partial H/\partial \xi \\ &= \mathcal{M}(x-\xi) = \mathcal{M}e\end{aligned}$$

Below, we set, when needed, $\mathcal{I} + \mathcal{S} = \mathcal{W}$.

We recall the basic definitions of *detectability* and *observability* in linear systems

Definition 3.1 *Given a pair of constant matrices $(\mathcal{C}, \mathcal{A})$, respectively of dimensions $m \times n$ and $n \times n$. The pair is said to be detectable if the matrix*

$$\begin{bmatrix} \mathcal{C} \\ sI - \mathcal{A} \end{bmatrix}$$

has full rank n for all values of s in the open right half of the complex plane. The system is said to be observable if the above matrix is full rank for all values of s in the complex plane.

If the pair of matrices $(\mathcal{C}, \mathcal{W})$ (resp. $(\mathcal{C}, \mathcal{S})$) is either *observable*, or *detectable*, it is well known, from linear systems theory, that there exists a constant vector $\mathcal{K}$ such that all, or at least the *observable*, eigenvalues of the matrix $\mathcal{W} - \mathcal{KC}$ (resp. $(\mathcal{C}, \mathcal{S})$) are placeable, modulo symmetry with respect to the real line, at pre-specified locations of the open left half of the complex plane. The distinction made above regarding *observable eigenvalues* means that some eigenvalues of $(\mathcal{C}, \mathcal{W})$ (resp. $(\mathcal{C}, \mathcal{S})$) may be *fixed* and cannot be influenced by any value of $\mathcal{K}$. In the case of a *detectable* pair, those fixed unobservable eigenvalues already exhibit negative real parts. If the pair of matrices $(\mathcal{C}, \mathcal{W})$, (resp. $(\mathcal{C}, \mathcal{S})$) is *observable* it means that, modulo the mentioned symmetry, *all* eigenvalues of $\mathcal{W} - \mathcal{KC}$ (resp. $(\mathcal{C}, \mathcal{S})$) can be placed at will in the left half of the complex plane by suitable choice of the matrix $\mathcal{K}$. As a consequence, the matrix $(\mathcal{W} - \mathcal{KC})^T$ also exhibits eigenvalues with negative real parts. This also implies that the sum

$$\begin{aligned}[\mathcal{W} - \mathcal{KC}] &+ [\mathcal{W} - \mathcal{KC}]^T \\ &= [\mathcal{S} - \mathcal{KC}] + [\mathcal{S} - \mathcal{KC}]^T \\ &= 2\left[\mathcal{S} - \frac{1}{2}(\mathcal{KC} + \mathcal{C}^T\mathcal{K}^T)\right]\end{aligned}$$

is a symmetric matrix with negative (real) eigenvalues.

Notice that the matrix $\mathcal{W} - \mathcal{KC}$ is a square matrix, with no particular structure. We can *always* trivially replace such a matrix by the following sum

$$\begin{aligned}\mathcal{W} - \mathcal{KC} &= \left\{\mathcal{S} - \frac{1}{2}\left[\mathcal{KC} + \mathcal{C}^T\mathcal{K}^T\right]\right\} \\ &\quad + \left\{\mathcal{I} - \frac{1}{2}\left[\mathcal{KC} - \mathcal{C}^T\mathcal{K}^T\right]\right\}\end{aligned}$$

The first two summands clearly conform a symmetric negative definite matrix while the second two summands conform a skew-symmetric matrix.

The state estimation error system may then be written in the following form

$$\begin{aligned}\dot{e} &= \left[\mathcal{J}(y) + \mathcal{I} - \frac{1}{2}\left(\mathcal{KC} - \mathcal{C}^T\mathcal{K}^T\right)\right]\frac{\partial H}{\partial e} \\ &\quad + \left[\mathcal{S} - \frac{1}{2}\left(\mathcal{KC} + \mathcal{C}^T\mathcal{K}^T\right)\right]\frac{\partial H}{\partial e}\end{aligned}$$

Then, taking as a modified Hamiltonian energy function the positive definite function $H(e)$, it is readily found that the time derivative of this function, along the trajectories of the observation error system, satisfies

$$\begin{aligned}\dot{H}(e) &= \frac{\partial H(e)}{\partial e^T}\dot{e} \\ &= \frac{\partial H(e)}{\partial e^T}\left[\mathcal{S} - \frac{1}{2}\left(\mathcal{KC} + \mathcal{C}^T\mathcal{K}^T\right)\right]\frac{\partial H(e)}{\partial e} \leq 0\end{aligned}$$

with $\dot{H}(e) = 0$ if and only if $e = 0$. In fact, it is not difficult to show that the stability of the error space origin $e = 0$ is *exponentially* asymptotically stable for an energy function of the form $H(e) = \frac{1}{2}e^T\mathcal{M}e$. In this case we have

$$\begin{aligned}\dot{H}(e) &= e^T\mathcal{M}^T\left[\mathcal{S} - \frac{1}{2}\left(\mathcal{KC} + \mathcal{C}^T\mathcal{K}^T\right)\right]\mathcal{M}e \\ &\leq -\frac{1}{2}\alpha e^T\mathcal{M}e = -\alpha H(e)\end{aligned}$$

with α being a suitable scalar constant. We have then proven the following result

Theorem 3.2 *The state x of the nonlinear system (1) can be globally exponentially asymptotically estimated by the state ξ of an observer of the form (2), if the pair of matrices $(\mathcal{C}, \mathcal{W})$, or the pair $(\mathcal{C}, \mathcal{S})$, is either observable or, at least, detectable.*

The observability condition on the either the pair $(\mathcal{C}, \mathcal{W})$, or the pair $(\mathcal{C}, \mathcal{S})$, is clearly a *sufficient* but not necessary condition for asymptotic state reconstruction. The following simple example readily demonstrates this issue.

Example 3.3 *The pair of matrices*

$$\mathcal{S} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

constitutes a non-observable pair, although it is a detectable pair. Nevertheless, setting $\mathcal{K} = 0$ already renders the sum,

$$2\left[\mathcal{S} - \frac{1}{2}(\mathcal{KC} + \mathcal{C}^T\mathcal{K}^T)\right] = 2\mathcal{S}$$

a negative definite *matrix.*

A necessary and sufficient condition for global asymptotic stability to zero of the state estimation error is given by the following theorem.

Theorem 3.4 *The state x of the nonlinear system (1) can be globally exponentially asymptotically estimated, by the state ξ of the observer (2) if and only if there exists a constant matrix $\mathcal{K}$ such that the symmetric matrix*

$$\begin{aligned}[\mathcal{W} - \mathcal{KC}] \; + \; & [\mathcal{W} - \mathcal{KC}]^T \\ = \; & 2\left[\mathcal{S} - \frac{1}{2}\left(\mathcal{KC} + \mathcal{C}^T\mathcal{K}^T\right)\right]\end{aligned}$$

is negative definite.

APPLICATIONS TO SYNCHRONI SATION OF CHAOTIC CIRCUITS

The Lorenz system

Consider the Lorenz system [2]

$$\begin{aligned}\dot{x}_1 &= \sigma(x_2 - x_1) \\ \dot{x}_2 &= rx_1 - x_2 - x_1x_3 \\ \dot{x}_3 &= x_1x_2 - bx_3\end{aligned}$$

The system can be easily written in Generalized Hamiltonian form, taking as the Hamiltonian energy function the scalar function

$$H(x) = \frac{1}{2}\left[\frac{1}{\sigma}x_1^2 + x_2^2 + x_3^2\right]$$

This yields, according to the previous formulation,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\sigma & 0 \\ -\frac{1}{2}\sigma & 0 & -x_1 \\ 0 & x_1 & 0 \end{bmatrix} \frac{\partial H}{\partial x} + \begin{bmatrix} -\sigma^2 & \frac{1}{2}\sigma & 0 \\ \frac{1}{2}\sigma & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \frac{\partial H}{\partial x} + \begin{bmatrix} 0 \\ rx_1 \\ 0 \end{bmatrix}$$

The output signal to be transmitted should be the state $y = x_1 = [\sigma \;\; 0 \;\; 0]\partial H/\partial x$. The matrices $\mathcal{C}$, $\mathcal{S}$ and $\mathcal{I}$, are given by

$$\mathcal{C} = [\sigma \;\; 0 \;\; 0], \quad \mathcal{S} = \begin{bmatrix} -\sigma^2 & \frac{1}{2}\sigma & 0 \\ \frac{1}{2}\sigma & -1 & 0 \\ 0 & 0 & -b \end{bmatrix},$$

$$\mathcal{I} = \begin{bmatrix} 0 & \frac{1}{2}\sigma & 0 \\ -\frac{1}{2}\sigma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The pair of matrices $(\mathcal{C}, \mathcal{S})$ already constitute a pair of detectable, but non observable, matrices. Even though the addition of the matrix $\mathcal{I}$ to $\mathcal{S}$ does not improve the lack of observability, the pair $(\mathcal{C}, \mathcal{W}) = (\mathcal{C}, \mathcal{S} + \mathcal{I})$ remains, nevertheless, detectable. In this case, the dissipative structure of the system is fully "damped" due to the negative definiteness of the matrix $\mathcal{S}$. Then, there is no need for an output estimation error injection for complementing, or enhancing, the system's natural dissipative structure. The receptor is designed as

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\sigma & 0 \\ -\frac{1}{2}\sigma & 0 & -y \\ 0 & y & 0 \end{bmatrix} \frac{\partial H}{\partial \xi} + \begin{bmatrix} -\sigma^2 & \frac{1}{2}\sigma & 0 \\ \frac{1}{2}\sigma & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \frac{\partial H}{\partial \xi} + \begin{bmatrix} 0 \\ ry \\ 0 \end{bmatrix}$$

and the synchronization error is therefore governed by the globally asymptotically stable system

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\sigma & 0 \\ -\frac{1}{2}\sigma & 0 & -y \\ 0 & y & 0 \end{bmatrix} \frac{\partial H}{\partial e} + \begin{bmatrix} -\sigma^2 & \frac{1}{2}\sigma & 0 \\ \frac{1}{2}\sigma & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \frac{\partial H}{\partial e}$$

Figure 2 shows the simulations of the Lorenz system and the receiver's state tracking abilities for large initial deviations. The system parameters were set to be

$$\sigma = 10, \quad r = 28, \quad b = 8/3,$$

Chua's Circuit

Consider Chua's circuit, [4] shown in Figure 1. This circuit is described by the following set of differential equations

$$\begin{aligned}C_1\dot{x}_1 &= G(x_2 - x_1) - F(x_1) \\ C_2\dot{x}_2 &= G(x_1 - x_2) + x_3 \\ L\dot{x}_3 &= -x_2\end{aligned}$$

where $F(x_1)$ is a voltage -dependent nonlinear resistance of the form

$$F(x_1) = ax_1 + \frac{1}{2}(b-a)\left(|1+x_1| - |1-x_1|\right),$$

with a, $b < 0$, which clearly plays the role of a *negative* resistor.

Consider, as a Hamiltonian energy function, the total stored energy in the circuit, given by

$$H(x) = \frac{1}{2}\left[C_1 x_1^2 + C_2 x_2^2 + L x_2^2\right]$$

whose gradient vector is readily obtained as

$$\frac{\partial H}{\partial x} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & L \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} C_1 x_1 \\ C_2 x_2 \\ L x_3 \end{bmatrix}$$

The system may be written in Generalized Hamiltonian Canonical form, with a destabilizing vector field, as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{LC_2} \\ 0 & -\frac{1}{LC_2} & 0 \end{bmatrix} \frac{\partial H}{\partial x} + \begin{bmatrix} -\frac{G}{C_1^2} & \frac{G}{C_1 C_2} & 0 \\ \frac{G}{C_1 C_2} & -\frac{G}{C_2^2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{\partial H}{\partial x} + \begin{bmatrix} -\frac{1}{C_1}F(x_1) \\ 0 \\ 0 \end{bmatrix}$$

The de-stabilizing vector field evidently calls for x_1 to be used as the output, y, of the transmitter. The matrices $\mathcal{C}$, $\mathcal{S}$, and $\mathcal{I}$ are found to be

$$\mathcal{C} = \begin{bmatrix} \frac{1}{C_1} & 0 & 0 \end{bmatrix},$$

$$\mathcal{S} = \begin{bmatrix} -\frac{G}{C_1^2} & \frac{G}{C_1 C_2} & 0 \\ \frac{G}{C_1 C_2} & -\frac{G}{C_2^2} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{I} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{LC_2} \\ 0 & -\frac{1}{LC_2} & 0 \end{bmatrix}$$

The pair $(\mathcal{C}, \mathcal{S})$ is neither observable nor detectable. However, the pair $(\mathcal{C}, \mathcal{W})$ is observable. The system lacks damping in the x_3 variable, and either in the x_1 or the x_2 variable as inferred from the negative semi-definite nature of the dissipation structure matrix, $\mathcal{S}$. If x_1 is used as an output, then the output error injection term can enhance the dissipation in the error state dynamics. The receiver is designed as

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{LC_2} \\ 0 & -\frac{1}{LC_2} & 0 \end{bmatrix} \frac{\partial H}{\partial \xi} + \begin{bmatrix} -\frac{G}{C_1^2} & \frac{G}{C_1 C_2} & 0 \\ \frac{G}{C_1 C_2} & -\frac{G}{C_2^2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{\partial H}{\partial \xi} + \begin{bmatrix} -\frac{1}{C_1}F(y) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} e_y$$

The choice of K_1, K_2 and K_3 as arbitrary strictly positive constants suffices to guarantee the asymptotic exponential stability to zero of the synchronization error.

The synchronization error dynamics is governed by

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{K_2}{2C_1C_2} & \frac{K_3}{2LC_1} \\ -\frac{K_2}{2C_1C_2} & 0 & \frac{1}{LC_2} \\ -\frac{K_3}{2LC_1} & -\frac{1}{LC_2} & 0 \end{bmatrix} \frac{\partial H}{\partial e} + \begin{bmatrix} -\frac{G+C_1K_1}{C_1^2} & \frac{2G-K_2}{2C_1C_2} & -\frac{K_3}{2LC_1} \\ \frac{2G-K_2}{2C_1C_2} & -\frac{G}{C_2^2} & 0 \\ -\frac{K_3}{2LC_1} & 0 & 0 \end{bmatrix} \frac{\partial H}{\partial e}$$

Figure 3 depicts the simulations of Chua's chaotic circuit state trajectories with the corresponding receiver responses. To ease the simulations we resorted to the following normalized version of the circuit (see Huijberts et al [?])

$$\begin{aligned} \dot{x}_1 &= \beta(-x_1 + x_2 - \phi(y)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\gamma x_2 \end{aligned}$$

with $\phi(y) = ay + \frac{1}{2}(b-a)\left[|\,1+y\,| + |\,1-y\,|\right]$ and

$$a = -\frac{5}{7}, \quad b = -\frac{8}{7}, \quad \beta = 15.6, \quad \gamma = 27$$

The parameter gains for the receiver were chosen to be

$$K_1 = 2, \quad K_2 = 3, \quad K_3 = 3$$

The Rössler System

Consider the following chaotic system, known as the Rössler system [3]

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3 \\ \dot{x}_2 &= x_1 + ax_2 \\ \dot{x}_3 &= b + x_3(x_1 - c) \end{aligned}$$

With the energy function,

$$H = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$$

we immediately obtain the system equations in the form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1/2 \\ 1 & 0 & 0 \\ 1/2 & 0 & 0 \end{bmatrix} \frac{\partial H}{\partial x}$$

$$+\begin{bmatrix} 0 & 0 & -1/2 \\ 0 & a & 0 \\ -1/2 & 0 & -c \end{bmatrix} \frac{\partial H}{\partial x}$$

$$+\begin{bmatrix} 0 \\ 0 \\ b + x_1 x_3 \end{bmatrix}$$

The de-stabilizing field is a function of x_1 and x_3. Thus, the outputs should be taken as $y_1 = x_1$ and $y_2 = x_3$. Notice, however, that the pair of matrices

$$\mathcal{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad \mathcal{W} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ 0 & 0 & -c \end{bmatrix}$$

is detectable and observable. This allows us to perform an eigenvalue placement using only injections of the synchronization error $e_1 = y_1 - \xi_1$ and, thus the multivariable pole placement is evaded.

The receiver may then be designed as

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1/2 \\ 1 & 0 & 0 \\ 1/2 & 0 & 0 \end{bmatrix} \frac{\partial H}{\partial \xi}$$

$$+\begin{bmatrix} 0 & 0 & -1/2 \\ 0 & a & 0 \\ -1/2 & 0 & -c \end{bmatrix} \frac{\partial H}{\partial \xi}$$

$$+\begin{bmatrix} 0 \\ 0 \\ b + y_1 y_2 \end{bmatrix} + \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} \begin{bmatrix} y_1 - \xi_1 \end{bmatrix}$$

Figure 4 shows the state trajectories of the Rössler system along with those of the synchronizing system. The parameters for the system, and for the observer gains, used in the simulation were taken as,

$$a = 0.4, \quad b = 2, \quad c = -4, \quad K_1 = 2.4,$$

$$K_2 = -2.1418, \quad K_3 = -1.8182$$

CONCLUSIONS

In this article, we have approached the problem of synchronization of chaotic systems from the perspective of Generalized Hamiltonian systems including dissipation and destabilizing terms. The approach allows to give a simple design procedure for the receiver system and clarifies the issue of deciding on the nature of the output signal to be transmitted. This may be accomplished on the basis of a simple linear detectability or observability test. Several chaotic systems were analyzed from this new perspective and their possibilities for synchronization were either confirmed, in the case of already obtained positive results, or it was explained in those cases where there is a known lack of synchronization.

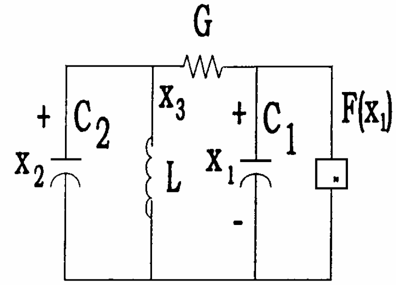

Figure 1: Chua's circuit

The Generalized Hamiltonian nature of a many chaotic systems definitely helps in the study of robust synchronization, under the addition of masked transmitted signals seen as perturbations of the state reconstruction error dynamics. More importantly, given the clear energy managing structure of Generalized Hamiltonian systems, the approach definitely helps in the study, via passivity based techniques, of linear and nonlinear feedback control strategies for chaotic systems. These will be the issues of a forthcoming publication.

REFERENCES

[1] G. Chen, *Control and Synchronization of Chaos, a Bibliography*, Department of Electrical Engineering. University of Houston, Houston TX (also available via ftp at ftp:uhoop.uh.edu/pub/chaos.tex), 1997.

[2] E. N. Lorenz, " Deterministic nonperiodic flow, " *Journal of Atmospheric Science*, Vol. 20, pp. 130-141, 1963.

[3] L. M. Pecora and T. L. Carroll, "Driving systems with chaotic signals", *Physical Review A*, Vol. 44, No. 4, pp. 2374-2383.

[4] C. W. Wu and L. Chua, " A simple way to synchronize chaotic systems with applications to secure communication systems" *Int. J. Biffurcation and Chaos*, Vol. 3 No. 6, pp. 1619-1627, 1993.

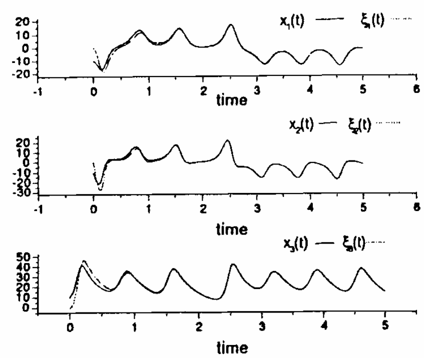

Figure 2: Lorenz system trajectories and synchronized receiver trajectories

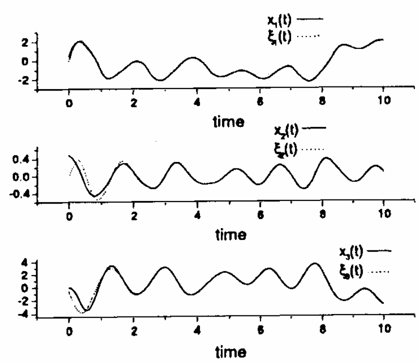

Figure 3: Chua's chaotic circuit state trajectories and synchronized receiver trajectories

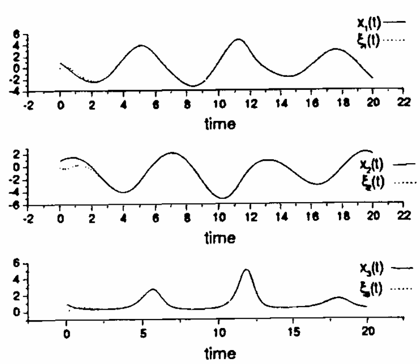

Figure 4: Rössler chaotic system state trajectories and synchronized receiver trajectories