Trajectory planning in the regulation of a stepping motor: A combined sliding mode and flatness approach*

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Abstract

A sliding mode controller, which suitably incorporates the flatness property of the system, is proposed for the effective equilibrium-to-equilibrium feedback regulation of the angular position in a permanent magnet (PM) stepping motor described in traditional $a - b$ coordinates. The controller is devised to accomplish off-line planned trajectory tracking for the system's flat outputs, indirectly accomplishing a controlled transfer from the initial to the final desired equilibrium positions of all system variables.

1 Introduction

The conceptually appealing "hidden linear controllable" features of a large class of nonlinear systems can be efficiently exploited at the controller design stage while benefitting from their intrinsically simple controller design specification task. Thus, linearizability properties, although frequently resulting in rather complex algebraic manipulations and a consequential lack of parametric and unmodelled external signal robustness, should not be under-estimated, or entirely neglected. Even though it has been little recognized, differential flatness, unfairly tied to feedback linearization, constitutes a valuable asset in feedback controller design, in system analysis and off-line system performance evaluation. Some of the linearity features provide irreplaceable conceptual characteristics and design options, such as: closed loop simplicity, robustness with respect to *internal* instability problems (i.e., no zero dynamics problems), design flexibility (a large number of controller design techniques to which it can be effectively combined), as well as a complete portrait of the equally important: "inverse physics", as "seen" from the system's most internal properties and limitations towards the imposed designer demands. This saddle issue is usually neglected in place of an alluded "respect" for the system's *direct* physical structure that, strangely enough, is not at all respected at the final controller specification stage, when inversion of the control inputs is also carried out.

The theoretical features, and structural possibilities, have been elegantly bestowed into a single and ubiquitous property: *differential flatness* (see the far reaching theoretical contributions, and interesting applications examples, developed by Prof. M. Fliess and his colleagues in [4]-[5]).

In this article, a nonlinear feedback controller is proposed which effectively combines the differential flatness property of the nonlinear multivariable stepping motor model with the robustness of sliding mode controller performance. These two important methods are shown to be combined in the context of a sliding mode based feedback controller for the position regulation of the stepping motor system.

Section 2 discusses the flatness property of the stepping motor system, already established in [8], from a slightly different viewpoint. The proposed sliding mode feedback controller is then obtained in terms of the off-line planned trajectories for the flat outputs which effectively stabilize the entire vector of state variables around a stable equilibrium. Section 3 presents the simulation results. Section 4 is devoted to present some conclusions.

2 A Flatness based Sliding Model Controller for the PM stepping Motor

The PM stepping motor model used in this article is directly taken from the work of Zribi and Chiasson [9]. Further developments of nonlinear state and output feedback control techniques can be found in the articles by Bodson *et al* [1] and Chiasson *et al* [3]. An actual experimental sliding mode control implementation, based on flatness considerations, was reported in an article by Zribi *et al* [10]. The reader is referred to the book of Leohnard [6] for background in the area and a complete derivation of the model, starting from fundamental considerations.

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2.1 A Nonlinear model for the permanent magnet stepping motor

Consider the following nonlinear model of a permanent magnet (PM) stepping motor

$$
\begin{aligned}
\frac{di_a}{dt} &= \frac{1}{L}\left(v_a - Ri_a + K_m\omega\sin(N_r\theta)\right)\\
\frac{di_b}{dt} &= \frac{1}{L}\left(v_b - Ri_b - K_m\omega\cos(N_r\theta)\right)\\
\frac{d\omega}{dt} &= \frac{1}{J}\Big(-K_m i_a\sin(N_r\theta) + K_m i_b\cos(N_r\theta) \\
&\qquad -B\omega - \tau\Big)\\
\frac{d\theta}{dt} &= \omega
\end{aligned} \tag{1}
$$

where i_a represents the current in phase A of the motor, i_b is the current in the phase B of the motor, θ is the angular displacement of the shaft of the motor, v_a and v_b, stand, respectively, for the voltage applied on the windings of the phase A and phase B. The parameters R and L, the resistance and self inductances in each of the phase windings, are constant and assumed to be perfectly known. Similarly the number of rotor teeth N_r, the torque constant of the motor K_m, the rotor load inertia J and the viscous friction B are assumed known and constant. The load torque perturbation, denoted by τ, is, for all analysis purposes, assumed to be zero.

2.2 Minimum phase properties of the PM stepping motor outputs

The equilibrium points $(\bar{i}_a, \bar{i}_b, \overline{\omega}, \overline{\theta})$ of the system, for given constant values of the voltages $v_a = \overline{v}_a$ and $v_b = \overline{v}_b$, are given by

$$
\begin{aligned}
\bar{i}_a &= \frac{\overline{v}_a}{R},\ \bar{i}_b = \frac{\overline{v}_b}{R},\ \overline{\omega} = 0,\ \overline{\theta} = \frac{1}{N_r}\arctan\left(\frac{\overline{v}_b}{\overline{v}_a}\right)\\
&= \frac{1}{N_r}\arctan\left(\frac{\bar{i}_b}{\bar{i}_a}\right)
\end{aligned} \tag{2}
$$

Suppose, for a moment, that the, vector relative degree $\{1,1\}$, outputs i_a and i_b are held *constant* at some nonzero value $(i_a, i_b) = (\bar{i}_a, \bar{i}_b)$. Then the *zero dynamics* corresponding to this set of values is given by the nonlinear system

$$
\frac{d\theta}{dt} = \omega\ ;\ J\frac{d\omega}{dt} = -K_m\bar{i}_a\sin(N_r\theta) + K_m\bar{i}_b\cos(N_r\theta) - B\omega \tag{3}
$$

A simple algebraic manipulation, which involves the nonzero quantity $\overline{\rho} = \sqrt{\bar{i}_a^2 + \bar{i}_b^2}$, yields the zero-dynamics in the form

$$
\begin{aligned}
\dot{\theta} &= \omega\\
J\dot{\omega} &= K_m\overline{\rho}\cos(N_r\theta + \overline{\phi}) - B\omega
\end{aligned} \tag{4}
$$

with $\overline{\phi} = \arctan(\bar{i}_a/\bar{i}_b)$. The zero dynamics (4) exhibits an infinite set of critical points located on the $\omega = 0$ axis, of the (θ, ω) phase plane, at

$$
\overline{\omega} = 0,\quad \overline{\theta}(k) = \frac{1}{N_r}\left((2k+1)\frac{\pi}{2} - \overline{\phi}\right)
$$

for $k = 0, \pm 1, \pm 2, \ldots$

Proposition 2.1 *The zero dynamics (4) is locally asymptotically stable towards the equilibrium points*

$$
\overline{\theta}_s(k) = \frac{1}{N_r}\left((4j+1)\frac{\pi}{2} - \overline{\phi}\right) \tag{5}
$$

with $j = 0, \pm 1, \pm 2, \ldots$

Proof

To prove this proposition consider the Jacobian linearization of the zero dynamics around an arbitrary critical point $(\overline{\theta}(k), \overline{\omega}) = \left(\frac{1}{N_r}\left((2k+1)\frac{\pi}{2} - \overline{\phi}, 0\right)\right.$. For this, define $\theta_\delta = \theta - \overline{\theta}(k)$ and $\omega_\delta = \omega$

$$
\begin{aligned}
\frac{d}{dt}\theta_\delta &= \omega_\delta\\
\frac{d}{dt}\omega_\delta &= -\left[K_m N_r\overline{\rho}\sin\left(N_r\overline{\theta}(k) + \overline{\phi}\right)\right]\theta_\delta - B\omega_\delta\\
&= -\left[K_m N_r\overline{\rho}\sin\left((2k+1)\frac{\pi}{2}\right)\right]\theta_\delta - B\omega_\delta
\end{aligned} \tag{6}
$$

The linearized system (6) is clearly globally asymptotically stable to the origin of incremental coordinates for those values of k which render the constant factor term $\sin((2k+1)\frac{\pi}{2})$ strictly positive, i.e., for $k = 0, \pm 2, \pm 4, \ldots$. The rest of the equilibrium points, $k = 1, \pm 3, \pm 5, \ldots$, clearly yield an unstable linearized system with two real eigenvalues of different sign, i.e. the corresponding equilibrium point is of the saddle type.

□

The system outputs, i_a and i_b are, thus locally *minimum phase*. Since they are also vector relative degree $\{1,1\}$, then they conform, according to the definitions in [2], a set of *passive* outputs.

2.3 The regulation problem via trajectory tracking

The control objective is to drive the system from a given initial equilibrium value towards a final equilibrium value achieving, as a result, a desired final value for the position variable θ.

We are given a pair of state equilibrium points, denoted by $\overline{x}^1$ and $\overline{x}^2$ specified , respectively, by $\overline{x}^1 = (\bar{i}_a^1, \bar{i}_b^1, \overline{\omega}^1, \overline{\theta}^1)$ and $\overline{x}^2 = (\bar{i}_a^2, \bar{i}_b^2, \overline{\omega}^2, \overline{\theta}^2)$with, $\overline{\omega}^1 = \overline{\omega}^2 = 0$.

The regulation problem we address in this article consists in achieving, by means of a sliding mode based controller which suitably exploits the flatness property of the stepping motor model, an equilibrium to equilibrium transfer, $\overline{x}^1 \to \overline{x}^2$, in the state space, while accomplishing the tracking of an off-line prescribed state trajectory joining the given state equilibrium points. The state trajectory is completely determined once the flat output trajectories are specified.

2.4 Differential flatness of the system

Consider the following invertible partial state coordinate transformation to be performed on system (1) and replacing the *euclidian* representation of the currents, i_a and i_b, by their corresponding *polar* representation,

$$\begin{aligned} \rho &= \sqrt{i_a^2 + i_b^2} \ ; \ \ \phi = \arctan\left(\frac{i_a}{i_b}\right) \\ i_a &= \rho \sin\phi \ ; \ \ i_b = \rho\cos\phi \end{aligned} \tag{7}$$

The transformed system is, therefore, given by

$$\begin{aligned} L\frac{d}{dt}\rho &= -R\rho - K_m\omega\cos(N_r\theta+\phi) + v_b\cos\phi \\ &\qquad + v_a\sin\phi \\ L\rho\frac{d}{dt}\phi &= K_m\omega\sin(N_r\theta+\phi) - v_b\sin\phi + v_a\cos\phi \\ J\frac{d}{dt}\omega &= K_m\rho\cos(N_r\theta+\phi) - B\omega \\ \frac{d}{dt}\theta &= \omega \end{aligned} \tag{8}$$

The model (8) of the PM stepping motor clearly exhibits the *differential flatness* property of the system, since all its state variables can be completely parameterized in terms of *differential* functions of the two independent (flat) outputs, constituted by the norm $\rho = \sqrt{i_a^2 + i_b^2}$ of the vector of phase currents, $[i_a, i_b]^T$, and by their angular position variable, θ. Notice that the transformed state variable $\phi = \arctan(i_a/i_b)$ and the angular position, θ, also qualify as flat outputs. A similar result has also been established for the induction motor in the interesting work of Martín and Rouchon [7]. For the flatness of the simpler "d-q coordinates model" of the permanent magnet stepping motor, the reader is referred to the articles by [9], [8] and [10].

The flat outputs, denoted by $F = (F_1, F_2) = (\rho, \theta)$, yield, the following *complete* differential parameterization of the transformed system variables,

$$\begin{aligned} \rho &= F_1 \\ \theta &= F_2 \\ \omega &= \dot F_2, \\ \phi &= \arccos\left(\frac{J\ddot F_2 + B\dot F_2}{K_m F_1}\right) - N_r F_2 \\ \begin{bmatrix} v_a \\ v_b \end{bmatrix} &= \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & cos\phi \end{bmatrix}^{-1} \times \end{aligned} \tag{9}$$

$$\begin{bmatrix} L\dot F_1 + RF_1 + K_m\dot F_2\cos(N_rF_2+\phi) \\ LF_1\left(\frac{\left(JF_2^{(3)}+B\ddot F_2\right)F_1 - \left(J\ddot F_2 + B\dot F_2\right)\dot F_1}{\sqrt{K_m^2F_1^2 - \left(J\ddot F_2+B\dot F_2\right)^2}} - N_r\dot F_2\right) \\ -K_m\dot F_2\sin(N_rF_2+\phi) \end{bmatrix}$$

All state variables properties are already reflected in the above complete differential parameterization, as it can be easily verified.

For instance, the differential parameterization (9) allows one to express the phase A and phase B currents, in terms of the flat outputs. From (7) and (9) we obtain,

$$\begin{aligned} i_a &= F_1\sin\left[\arccos\left(\frac{J\ddot F_2 + B\dot F_2}{K_mF_1}\right) - N_rF_2\right] \\ i_b &= F_1\cos\left[\arccos\left(\frac{J\ddot F_2 + B\dot F_2}{K_mF_1}\right) - N_rF_2\right] \end{aligned} \tag{10}$$

From (9) is readily seen that i_a and i_b are *passive* outputs. For this, let i_a and i_b be arbitrary constants, say, $\bar i_a$, $\bar i_b$. Then, it follows, from the expressions of i_a and i_b that,

$$J\ddot F_2 = K_m\left(\sqrt{\bar i_a^2 + \bar i_b^2}\right)\cos\left[N_rF_2 + \arctan(\frac{\bar i_a}{\bar i_b})\right] - B\dot F_2 \tag{11}$$

which is the same locally asymptotically stable zero dynamics (4), studied in the previous section.

Other important properties such as *constant equilibrium state detectability*, which is specially useful in seeking output feedback regulation schemes based on Lyapunov stability theory, can also be assessed from the differential parameterization provided by flatness. This issue, however, is not pursued in this article.

2.5 A sliding mode controller based on flatness

The idea of combining sliding mode control and differential flatness arises from the fact that the sliding mode controller while exhibiting a remarkable degree of robustness, it is also quite easy and natural to implement in many electrical and electro-mechanical systems.

While flatness allows one to asses the convenience, or inconvenience, of some desired reference trajectories candidates by exploiting the invertibility of the system, it is generally conceded that that the linearizability features, inherent in the flatness approach, are not robust with respect to external or parametric perturbations and uncertainties. Thus, the flatness property can be advantageously used with sliding mode control in order to bestow the desired robustness to a suitable feedback linearization scheme based on flatness.

Secondly, the flat outputs are fundamental system outputs which are devoid of internal dynamics and precisely correspond to the linear decoupled multivariable controllability properties of the system. Hence, indirectly forcing these outputs to track pre-specified trajectories does not, *per se*, yield any internal stability problems due to the presence of some (undesired) zero dynamics.

Consider the following set of decoupled sliding surface coordinates,

$$\begin{aligned} s_1 &= F_1 - F_1^*(t), \\ s_2 &= \ddot F_2 - \ddot F^*(t) + \alpha_2(\dot F_2 - \dot F_2^*(t)) + \alpha_1(F_2 - F_2^*(t)) \end{aligned} \tag{12}$$

Evidently, if s_1 and s_2 are forced to converge to zero in finite time, and control actions guarantee that the sliding surface coordinates evolution are forcefully kept at zero for all times, then the tracking errors $e_1 = F_1 - F_1^*(t)$ and

$e_2 = F_2 - F^*(t)$ evolve according to the following asymptotically stable dynamics

$$e_1(t) = 0, \quad \ddot{e}_2(t) + \alpha_2 \dot{e}_2(t) + \alpha_1 e_2(t) = 0$$

This is achieved by imposing the following decoupled discontinuous dynamics on the sliding surface expressions,

$$\dot{s}_1 = -W_1 \text{ sign } s_1, \quad \dot{s}_2 = -W_2 \text{ sign } s_2$$

which immediately lead to the following controllers,

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & cos\phi \end{bmatrix}^{-1} \begin{bmatrix} L\Gamma_1 + RF_1 + K_m \dot{F}_2 \cos(N_r F_2 + \phi) \\ LF_1 \left(\frac{(J\Gamma_2 + B\ddot{F}_2)F_1 - (J\ddot{F}_2 + B\dot{F}_2)\Gamma_1}{\sqrt{K_m^2 F_1^2 - (J\ddot{F}_2 + B\dot{F}_2)^2}} - N_r \dot{F}_2 \right) \\ -K_m \dot{F}_2 \sin(N_r F_2 + \phi) \end{bmatrix} \tag{13}$$

where

$$\begin{aligned} \Gamma_1 &= \dot{F}_1^*(t) - W_1 \text{ sign } (F_1 - F_1^*(t)) \\ \Gamma_2 &= [F_2^*(t)]^{(3)} - \alpha_2(\ddot{F}_2 - \ddot{F}_2^*(t)) - \alpha_1(\dot{F}_2 - \dot{F}_2^*(t)) \\ &\quad -W_2 \text{ sign}\Big(\ddot{F}_2 - \ddot{F}_2^*(t) + \alpha_2(\dot{F}_2 - \dot{F}_2^*(t)) \\ &\quad + \alpha_1(F_2 - F_2^*(t)) \Big) \end{aligned} \tag{14}$$

3 Simulation Results

We consider a PM stepping motor characterized by the following set of parameters

$$R = 8.4\,\Omega \;\; L = 0.010\,\text{H}, \;\; K_m = 0.05\,\text{V} - \text{s/rad}$$

$$J = 3.6 \times 10^{-6}\,\text{N} - \text{m} - \text{s}^2/\text{rad},$$

$$B = 1 \times 10^{-4}\,\text{N} - \text{m} - \text{s/rad}, \;\; N_r = 50$$

It was desired to transfer the angular position, θ, from the initial value of $\overline{\theta}^1$ [rad], towards the final value, $\overline{\theta}^2$ [rad], following a trajectory specified by means of an interpolating time polynomial of the form $\psi(t, t_0, t_f)$ satisfying,

$$\psi(t_0, t_0, t_f) = 0, \quad \psi(t_f, t_0, t_f) = 1 \tag{15}$$

Thus,

$$\theta^*(t) = \overline{\theta}^1 + \psi(t, t_0, t_f)\left[\overline{\theta}^2 - \overline{\theta}^1\right] \tag{16}$$

One such possible expression for $\psi(t, t_0, t_f)$, is given by

$$\begin{aligned} \theta^*(t) &= \theta_0 + \left(\frac{t - t_0}{t_f - t_0}\right)^5 \left[r_1 - r_2 \left(\frac{t - t_0}{t_f - t_0}\right) \right. \\ &\quad \left. + r_3 \left(\frac{t - t_0}{t_f - t_0}\right)^2 - \ldots - r_6 \left(\frac{t - t_0}{t_f - t_0}\right)^5 \right] \\ &\quad \times (\theta_F - \theta_0) \end{aligned} \tag{17}$$

with

$$r_1 = 252, \; r_2 = 1050, \; r_3 = 1800, \; r_4 = 1575,$$

$$r_5 = 700, \; r_6 = 126$$

and $t_0 = 0.01$ [s], $t_f = 0.02$ [s].

The flat output variable, ρ, was also made to follow a similar time trajectory $\rho^*(t)$, taking this coordinate from the initial value $\rho(t_0) = \overline{\rho}^1$, towards the final value $\rho(t_f) = \overline{\rho}^2$, during the same time interval, $[t_0, t_f]$, used for the angular position change. In other words, we specified $\rho^*(t)$ as

$$\rho^*(t) = \overline{\rho}^1 + \psi(t, t_0, t_f)\,(\overline{\rho}^2 - \overline{\rho}^1) \tag{18}$$

The initial and final values for the motor shaft angular position were taken to be $\overline{\theta}^1 = 0$ [rad] and $\overline{\theta}^2 = 0.02$ [rad]. The proposed angular position transfer makes the phase angle, ϕ, take the initial and final values $\overline{\phi}^1 = \pi/2 = 1.5707$ [rad] and $\overline{\phi}_2 = \pi/2 - N_r\overline{\theta}^2 = 0.5707$ [rad]. This planning helps in avoiding the condition $\rho = 0$, which is required in order to avoid a singularity in the controller (13).

The nominal initial value of θ, chosen as $F_2^*(t_0) = \overline{\theta}^1 = 0$ implies, according to (2), that $i_b(t_0) = \overline{i}_b^1 = 0$ with $\overline{i}_a^1$ being arbitrary. We choose, just for convenience, the initial phase A current to be strictly positive ($\overline{i}_a^1 = 0.4$ A). The planned trajectory for $F_1^*(t) = \rho^*(t)$ must also evade the condition $i_a(t) = 0$, at any time $t \in [t_0, t_f]$. We choose the following initial value, $\overline{\rho}^1$ for $\rho^*(t)$,

$$F_1^*(t_0) = \overline{\rho}^1 = \overline{i}_a^1 = 0.4A, \;\; \overline{i}_b^1 = 0 \tag{19}$$

The final value $\overline{\rho}^2$ of ρ can be deduced from the following equilibrium relations

$$\tan\left(N_r\overline{\theta}^2\right) = \overline{i}_b^2/\overline{i}_a^2 \;\; ; \;\; \overline{\rho}^2 = \sqrt{(\overline{i}_a^2)^2 + (\overline{i}_b^2)^2} \tag{20}$$

which yield

$$\overline{\rho}^2 = \overline{i}_a^2 \sec\left(N_r\overline{\theta}^2\right) \tag{21}$$

Choosing $\overline{i}_a^2 = 0.21588$ A, the final value of $\overline{F}_1^*(t)$ at time t_f is found to be, $\overline{F}_1^*(t_f) = \overline{\rho}^2 = 0.4$ A, and the singularity condition is thus avoided. The initial and terminal times for the equilibrium transfer were set to be $t_0 = 0.02$ s and $t_f = 0.04$ s.

In order to avoid the bang-bang behavior of the control inputs with its associated "chattering" motions of the controlled responses we used a well-known high gain substitution of the *signum* functions in the sliding mode controller. This was accomplished by using:

$$\text{sign } s_1 \hookrightarrow \frac{s_1}{|\, s_1 \,| + \epsilon}, \;\; \text{sign } s_2 \hookrightarrow \frac{s_2}{|\, s_2 \,| + \epsilon}$$

The controller design constants W_1, W_2, ϵ, $\alpha_2 = 2\xi\omega_n$ and $\alpha_1 = \omega_n^2$, were set to be

$$W_1 = 100, \;\; W_2 = 100, \;\; \epsilon = 0.005, \;\; \xi = 0.8, \;\; \omega_n = 10$$

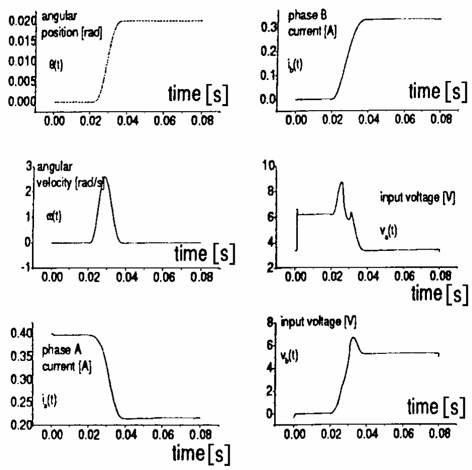

Figure 1: PM stepping motor ideal closed loop response to Sliding + Flatness based controller (a-b coordinates)

Figure 1 shows the simulations of the ideal closed loop performance of the stepping motor mechanical and electrical variables, in the $a - b$ coordinates, commanded by the designed sliding mode plus flatness based controller with reference trajectories planned in terms of the flat outputs.

Figure 2 shows the performance of the sliding plus flatness based controller in the presence of constant, but unknown, load torque perturbations. The load torque amplitude was taken to be 10^{-5} N-m. In order to have the sliding mode controller (implemented with the high gain substitution) track the planned trajectories we had to increase the parameter gains W_1 and W_2 to a value of 300. With these values the performance of the controller is practically the same as the previous ideal one, except that the control input voltage amplitudes are now reasonably increased.

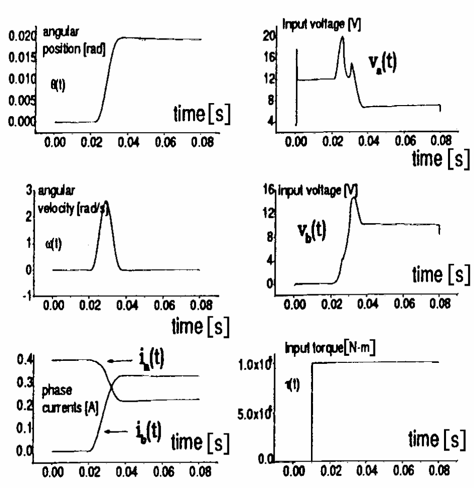

Figure 2: PM stepping motor closed loop response to Sliding + Flatness based controller including load torque perturbation

4 Conclusions

In this article, we have proposed a combination of "sliding and flatness" for the feedback regulation of a (nontrivial) nonlinear multi-variable system constituted by the PM stepping motor. The sliding based considerations lead to a natural feedback controller that takes advantage of the linearizable structure of the system, while creating suitable stabilizing feedback control actions.

The controlled system comfortably tracks the described trajectories, thanks to their intimate relation with the hidden linear controllability properties of the system.

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