Trajectory planning in the regulation of a PM stepper motor: A combined passivity and flatness approach [1]

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Abstract

A passivity based controller, which suitably incorporates the flatness property of the system, is proposed for the effective equilibrium-to-equilibrium feedback regulation of the angular position in a permanent magnet (PM) stepping motor described in traditional $a-b$ coordinates.

Keywords: Passivity Based Control, Differential Flatness

1 Introduction

In this article, a nonlinear feedback controller is proposed which effectively combines the natural energy dissipation properties of the PM stepping motor system with its differential flatness property (see Fliess *et al* [1]). These two important structural properties of the system can be combined in the context of a dynamic passivity based feedback controller. The proposed controller naturally arises from "energy modification and damping injection" considerations achievable on the basis of identifying, and exploiting, the natural "conservative and dissipation" structure of the nonlinear system dynamics and a complete identification and advantageous use of the hidden linear controllable features of the system, represented by the flat output. The passivity based controller translates into an efficient control scheme which allows for an equilibrium-to-equilibrium stabilization task, based on off-line planned trajectories prescriptions and on-line feedback trajectory tracking.

An energy shaping plus damping injection based dynamic feedback controller is synthesized in Section 2 which requires knowledge of the passive outputs reference trajectories achieving the desired equilibrium to equilibrium stabilization. The flatness property of the stepping motor system, already established in [2], is further discussed in Section 2 the context of passivity (see [3]). The passive outputs trajectories, which, due to flatness, are parameterized in terms of flat outputs trajectories, are then used in the feedback controller expression. Section 3 presents the simulation results. Section 4 is devoted to some conclusions.

2 A Passivity plus Flatness based Controller for the PM Stepper Motor

The PM stepper motor model used in this article is directly taken from the work of Zribi and Chiasson [4]. An actual experimental sliding mode control implementation, based on flatness considerations, was reported in an article by Zribi *et al* [5].

2.1 A Nonlinear model for the permanent magnet stepper motor

Consider the following nonlinear model of a permanent magnet (PM) stepper motor

$$\begin{aligned}\frac{di_a}{dt} &= \frac{1}{L}\left(v_a - Ri_a + K_m\omega\sin(N_r\theta)\right)\\ \frac{di_b}{dt} &= \frac{1}{L}\left(v_b - Ri_b - K_m\omega\cos(N_r\theta)\right)\\ \frac{d\omega}{dt} &= \frac{1}{J}\left(-K_m i_a\sin(N_r\theta) + K_m i_b\cos(N_r\theta)\right.\\ &\qquad \left.-B\omega - \tau\right)\\ \frac{d\theta}{dt} &= \omega \end{aligned} \quad (2.1)$$

where i_a represents the current in phase A of the motor, i_b is the current in the phase B of the motor, θ is the angular displacement of the shaft of the motor, v_a and v_b, stand, respectively, for the voltage applied on the windings of the phase A and phase B. The parameters

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R and L, the resistance and self inductances in each of the phase windings, are constant and assumed to be perfectly known. Similarly the number of rotor teeth N_r, the torque constant of the motor K_m, the rotor load inertia J and the viscous friction B are assumed known and constant. The load torque perturbation, denoted by τ, is, for all analysis purposes, assumed to be zero.

2.2 Passivity properties of the PM stepper motor

The equilibrium points $(\bar{i}_a, \bar{i}_b, \bar{\omega}, \bar{\theta})$ of the system, for given constant values of the voltages $v_a = \bar{v}_a$ and $v_b = \bar{v}_b$, are given by

$$\begin{aligned} \bar{i}_a &= \frac{\bar{v}_a}{R}, \;\; \bar{i}_b = \frac{\bar{v}_b}{R}, \;\; \bar{\omega} = 0, \\ \bar{\theta} &= \frac{1}{N_r}\arctan\left(\frac{\bar{v}_b}{\bar{v}_a}\right) = \frac{1}{N_r}\arctan\left(\frac{\bar{i}_b}{\bar{i}_a}\right) \end{aligned} \quad (2.2)$$

Proposition 2.1 *The zero dynamics associated to constant equilibrium values of the outputs i_a and i_b is locally asymptotically stable towards the equilibrium points,*

$$\bar{\theta}_s(k) = \frac{1}{N_r}\left((4j+1)\frac{\pi}{2} - \bar{\phi}\right)$$

with $j = 0, \pm 1, \pm 2, \ldots$

The system outputs, i_a and i_b are, thus locally *minimum phase*. Since they are also vector relative degree $\{1,1\}$, then they conform a set of *passive* outputs.

Consider the following positive definite energy *storage function*

$$V(i_a, i_b, \omega, \theta) = \frac{1}{2}\left[L(i_a^2 + i_b^2) + J\omega^2 + \gamma\theta^2\right]$$

with $\gamma > 0$. The time derivative of V, along the controlled motions satisfies

$$\dot{V} \leq \left(v_a + \gamma\frac{\theta\omega}{i_a}\right) i_a + v_b i_b$$

The system is thus passive between the modified inputs ϑ_a, ϑ_b

$$\vartheta_a = v_a + \gamma\frac{\theta\omega}{i_a}, \quad \vartheta_b = v_b$$

and the outputs i_a and i_b.

2.3 The regulation problem via trajectory tracking

The control objective is to drive the system from a given initial equilibrium value towards a final equilibrium value achieving, as a result, a desired final value for the position variable θ.

We are given a pair of state equilibrium points, denoted by $\bar{x}^1$ and $\bar{x}^2$ specified , respectively, by $\bar{x}^1 = (\bar{i}_a^1, \bar{i}_b^1, \bar{\omega}^1, \bar{\theta}^1)$ and $\bar{x}^2 = (\bar{i}_a^2, \bar{i}_b^2, \bar{\omega}^2, \bar{\theta}^2)$with, $\bar{\omega}^1 = \bar{\omega}^2 = 0$ and $\bar{i}_a^j \neq 0$ for $j = 1, 2$. The regulation problem we address in this article consists in achieving, by means of a passivity based controller which suitably exploits the flatness property of the stepping motor model, an equilibrium to equilibrium transfer: $\bar{x}^1 \to \bar{x}^2$ in the state space, while accomplishing the tracking of an off-line prescribed state trajectory joining the given state equilibrium points.

2.4 A feedback controller based on "energy shaping plus damping injection"

The "energy shaping plus damping injection" dynamic feedback controller design method, extensively treated in [3], yields, the following dynamical feedback controller specification,

$$\begin{aligned} \vartheta_a &= L\frac{d}{dt}i_a^*(t) - K_m\zeta_1\sin(N_r\theta) + Ri_a^*(t) + \gamma\frac{\omega}{i_a}\zeta_2 \\ \vartheta_b &= L\frac{d}{dt}i_b^*(t) + K_m\zeta_1\cos(N_r\theta) + Ri_b^*(t) \end{aligned} \quad (2.3)$$

with ζ_1 and ζ_2, satisfying,

$$\begin{aligned} J\dot{\zeta}_1 &= -B\zeta_1 - K_m i_a^*(t)\sin(N_r\theta) \\ &+ K_m i_b^*(t)\cos(N_r\theta) + R_B(\omega - \zeta_1) \\ \gamma\dot{\zeta}_2 &= \gamma\frac{\omega}{i_a}i_a^*(t) + R_\theta(\theta - \zeta_2) \end{aligned} \quad (2.4)$$

where R_B, R_θ are strictly positive design constants enhancing the closed loop system damping structure.

The original control inputs to the system are determined from the equalities,

$$v_a = \vartheta_a - \gamma\frac{\theta\omega}{i_a} \quad ; \quad v_b = \vartheta_b \quad (2.5)$$

Proposition 2.2 *The passivity based dynamic feedback controller yields a state vector tracking error dynamics, described by the vector, $e = [i_a - i_a^*(t), i_b - i_b^*(t), \omega - \zeta_1, \theta - \zeta_2]$, which is globally exponentially asymptotically stable to zero.*

Proof Substituting the control input expressions (2.3) into the a-b coordinates system model (2.1), and using the set of differential equations (2.4), we obtain, after rearrangement,

$$\begin{aligned} L\dot{e}_1 &= K_m\sin(N_r\theta)e_3 - \gamma\frac{\omega}{i_a}e_4 - Re_1 \\ L\dot{e}_2 &= -K_m\cos(N_r\theta)e_3 - Re_2 \\ J\dot{e}_3 &= -K_m\sin(N_r\theta)e_1 + K_m\cos(N_r\theta)e_2 \\ &\quad -(B + R_B)e_3 \\ \gamma\dot{e}_4 &= \gamma\frac{\omega}{i_a}e_1 - R_\theta e_4 = 0 \end{aligned}$$

Using the modified energy function $V(e) = \frac{1}{2}\left(Le_1^2 + Le_2^2 + Je_3^2 + \gamma e_4^2\right)$, one establishes that

$$\begin{aligned} \dot{V}(e) &= -R(e_1^2+e_2^2) - (B+R_B)e_3^2 - R_\theta e_4^2 \\ &\leq -\frac{2\min\{R, B+R_B, R_\theta\}}{\max\{L,J,\gamma\}} V(e) \end{aligned} \quad (2.6)$$

i.e., $\dot{V}(e) \leq -\alpha V(e)$, with α being a strictly positive constant dependent upon the system parameters L,J, R and B and the design parameters, R_B, R_θ, and γ.

The tracking error is globally exponentially asymptotically stable to zero, i.e.

$$i_a \rightarrow i_a^*(t), \quad i_b \rightarrow i_b^*(t), \quad \omega \rightarrow \zeta_1, \quad \theta \rightarrow \zeta_2 \quad (2.7)$$

In the absence of load perturbations, τ, the desired currents $i_b^*(t)$ and $i_a^*(t)$ are made to converge to suitable constant values. As a consequence, i_a and i_b also converge to the prescribed constant values. The outputs i_a and i_b were shown to be passive, hence, the angular velocity, ω, asymptotically converges to zero. It follows that the auxiliary variable, ζ_1, also converges to zero. The angle θ, and the auxiliary state ζ_2, both converge to a constant value, to be established later. The flatness property allows to completely parametrize, in an invertible manner, the passive outputs trajectories, $i_a^*(t)$ and $i_b^*(t)$, in terms of desired trajectories for the angular position θ and the norm of the vector of phase currents, (i_a, i_b).

2.5 Differential flatness of the system

Consider the following invertible partial state coordinate transformation to be performed on system (2.1),

$$\begin{aligned} \rho &= \sqrt{i_a^2 + i_b^2}\ ;\ \phi = \arctan\left(\frac{i_a}{i_b}\right) \\ i_a &= \rho \sin\phi\ ;\ i_b = \rho\cos\phi \end{aligned} \quad (2.8)$$

The transformed system is given by

$$\begin{aligned} L\frac{d}{dt}\rho &= -R\rho - K_m\omega\cos(N_r\theta+\phi) + v_b\cos\phi \\ &\quad + v_a\sin\phi \\ L\rho\frac{d}{dt}\phi &= K_m\omega\sin(N_r\theta+\phi) - v_b\sin\phi \\ &\quad + v_a\cos\phi \\ J\frac{d}{dt}\omega &= K_m\rho\cos(N_r\theta+\phi) - B\omega \\ \frac{d}{dt}\theta &= \omega \end{aligned} \quad (2.9)$$

The model (2.9) of the PM stepper motor clearly exhibits the differential flatness property of the system, since all its variables can be completely parameterized in terms of *differential* functions of the independent flat outputs. Notice that the transformed state variable $\phi = \arctan(i_a/i_b)$ and the angular position, θ, also qualify as flat outputs. For the flatness of the simpler "d-q coordinates model" of the permanent magnet stepper motor, the reader is referred to the articles by [4], [2] and [5].

The flatness property allows one to express the phase A and phase B currents, in terms of the flat outputs. From (2.8) we obtain,

$$\begin{aligned} i_a &= F_1\sin\left[\arccos\left(\frac{J\ddot{F}_2 + B\dot{F}_2}{K_m F_1}\right) - N_r F_2\right] \\ i_b &= F_1\cos\left[\arccos\left(\frac{J\ddot{F}_2 + B\dot{F}_2}{K_m F_1}\right) - N_r F_2\right] \end{aligned}$$

Other important properties, such as constant equilibrium state detectability, specially useful when output feedback regulation schemes are sought, can also be assessed from the differential parameterization provided by flatness. This issue is not pursued in this article.

2.6 A dynamic controller combining passivity and flatness

The passivity based controller, exploiting the flatness property of the system, is given by,

$$\begin{aligned} \vartheta_a &= L\frac{d}{dt}i_a^*(t) - K_m\zeta_1\sin(N_r\theta) + Ri_a^*(t) + \gamma\frac{\omega}{i_a}\zeta_2 \\ \vartheta_b &= L\frac{d}{dt}i_b^*(t) + K_m\zeta_1\cos(N_r\theta) + Ri_b^*(t) \end{aligned} \quad (2.10)$$

with ζ_1 and ζ_2, satisfying,

$$\begin{aligned} J\dot{\zeta}_1 &= -B\zeta_1 - K_m i_a^*(t)\sin(N_r\theta) \\ &\quad + K_m i_b^*(t)\cos(N_r\theta) + R_B(\omega - \zeta_1) \\ \gamma\dot{\zeta}_2 &= \gamma\frac{\omega}{i_a}i_a^*(t) + R_\theta(\theta - \zeta_2) \end{aligned} \quad (2.11)$$

with the current reference trajectories $i_a^*(t)$ and $i_b^*(t)$, given by

$$\begin{aligned} i_a^*(t) &= F_1^*(t)\sin\left[\arccos\left(\frac{J\ddot{F}_2^*(t) + B\dot{F}_2^*(t)}{K_m F_1^*(t)}\right)\right. \\ &\qquad \left. - N_r F_2^*(t)\right] \\ i_b^*(t) &= F_1^*(t)\cos\left[\arccos\left(\frac{J\ddot{F}_2^*(t) + B\dot{F}_2^*(t)}{K_m F_1^*(t)}\right)\right. \\ &\qquad \left. - N_r F_2\right] \end{aligned} \quad (2.12)$$

The advantages of the proposed combination are manifold. First, if a passivity based controller has been designed, on the basis of physical energy dissipation considerations, the controller actions tend to take advantage of the beneficial nonlinearities by enhancing their dissipation properties while neutralizing the locally destabilizing fields. This yields a controller which requires less authority to achieve stabilization or trajectory tracking. Secondly, the flat outputs are fundamental system outputs which are devoid of internal dynamics and correspond to the linear controllability properties of the system. Hence, indirectly forcing these outputs to track pre-specified trajectories does not, *per se*, yield any internal stability problems. The prescribed passive outputs trajectories already contemplates that the corresponding angular position be forced to adopt a final constant value with corresponding zero angular velocity.

3 Simulation Results

We consider a PM stepper motor with the following parameters

$$R = 8.4\ \Omega\ L = 0.010\ \text{H},\ K_m = 0.05\ \text{V}-\text{s/rad}$$

$$J = 3.6 \times 10^{-6}\ \text{N}-\text{m}-\text{s}^2/\text{rad}$$

$$B = 1 \times 10^{-4}\ \text{N}-\text{m}-\text{s/rad},\quad N_r = 50\quad R_b = 0.05\Omega$$

It is desired to transfer the angular position θ from the initial value of $\overline{\theta}^1$ [rad], towards the final value $\overline{\theta}^2$ [rad], following a trajectory specified by means of an interpolating time polynomial of the form $\psi(t, t_0, t_f)$ satisfying

$$\psi(t_0, t_0, t_f) = 0, \quad \psi(t_f, t_0, t_f) = 1$$

Thus,

$$\theta^*(t) = \overline{\theta}^1 + \psi(t, t_0, t_f)\left[\overline{\theta}^2 - \overline{\theta}^1\right]$$

The flat output variable, ρ, was also made to follow a similar time trajectory $\rho^*(t)$, taking this coordinate from the value $\rho(t_0) = \overline{\rho}^1$, towards the final value $\rho(t_f) = \overline{\rho}^2$, during the same time interval, $[t_0, t_f]$, used for the angular position change. In other words, we specified $\rho^*(t)$ as

$$\rho^*(t) = \overline{\rho}^1 + \psi(t, t_0, t_f)\left(\overline{\rho}^2 - \overline{\rho}^1\right)$$

The initial and final values for the motor shaft angular position were taken to be $\overline{\theta}^1 = 0$ rad and $\overline{\theta}^2 = 0.02$ rad. The proposed angular position transfer makes the phase angle ϕ take the initial and final values $\overline{\phi}^1 = \pi/2 = 1.5707$ rad and $\overline{\phi}_2 = \pi/2 - N_r\overline{\theta}^2 = 0.5707$ rad. This planning avoids the condition $\sin\phi = 0$, which is equivalent to $i_a = 0$, as it is required in order to avoid a singularity in the passivity based controller (2.10), (2.11).

The nominal initial value of θ, chosen as $F_2^*(t_0) = \overline{\theta}^1 = 0$ implies, according to (2.2), that $i_b(t_0) = \overline{i}_b^1 = 0$ with $\overline{i}_a^1$ being arbitrary. We choose, just for convenience, the initial phase A current to be strictly positive ($\overline{i}_a^1 = 0.4$ A). The planned trajectory for $F_1^*(t) = \rho^*(t)$ must also evade the condition $i_a(t) = 0$, at any time $t \in [t_0, t_f]$. We choose the following initial value, $\overline{\rho}^1$ for $\rho^*(t)$,

$$F_1^*(t_0) = \overline{\rho}^1 = \overline{i}_a^1 = 0.4A,\quad \overline{i}_b^1 = 0$$

The final value $\overline{\rho}^2$ of ρ can be deduced from the following equilibrium relations

$$\tan\left(N_r\overline{\theta}^2\right) = \overline{i}_b^2/\overline{i}_a^2\ ;\ \overline{\rho}^2 = \sqrt{(\overline{i}_a^2)^2 + (\overline{i}_b^2)^2}$$

which yield

$$\overline{\rho}^2 = \overline{i}_a^2 \sec\left(N_r\overline{\theta}^2\right)$$

Choosing $\overline{i}_a^2 = 3.0587$ A, the final value of $\overline{F}_1^*(t)$ at time t_f is found to be, $\overline{F}_1^*(t_f) = \overline{\rho}^2 = 5.6547$ A, and the singularity condition is thus avoided. The initial and terminal times for the equilibrium transfer were set to be $t_0 = 0.02$ s and $t_f = 0.04$ s.

The controller design constants R_B, R_θ and γ, were set to be

$$R_B = 0.2,\quad R_\theta = 10,\quad \gamma = 0.05$$

Figure 1 shows the simulations of the ideal closed loop performance of the stepping motor mechanical and electrical variables, in the $a-b$ coordinates, commanded by the designed passivity based controller with passive outputs reference trajectories planned in terms of the flat outputs. Figure 2 shows the performance of the passivity plus flatness based controller in the presence of constant, but unknown, load torque perturbations occurring at time $t = 0.01$ [s]. As it can be seen the designed controller fails to stabilize the highly oscillatory perturbed response of the system. A simple modification of the proposed controller allows the use of an outer loop classical PID controller to account for unmodeled load torque pertubations. The results are shown in Figure 3.

4 Conclusions

In this article, we have proposed a combination of "passivity and flatness" for the feedback regulation of a (nontrivial) nonlinear multi-variable system constituted by the PM stepper motor. As a result, a dynamic feedback controller which requires less control effort, or authority, is obtained as compared, for instance, with a feedback linearizing controller. The controlled system comfortably tracks these trajectories, thanks to their intimate relation with the hidden linear controllability properties of the system. The control scheme can be easily modified to include a traditional ***outer loop*** PID controller which effectively accounts for the unmodeled presence of constant, but unknown, load torque perturbations.

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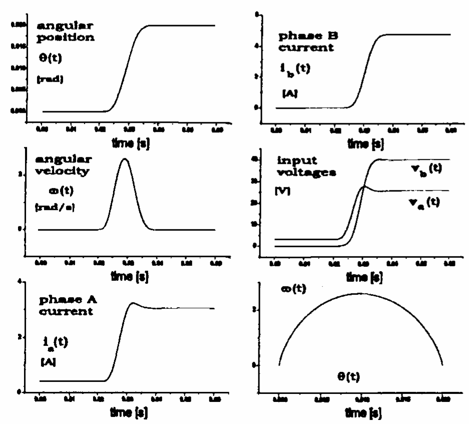

Figure 1: PM Stepper motor ideal closed loop response to Passivity + Flatness based controller (a-b coordinates)

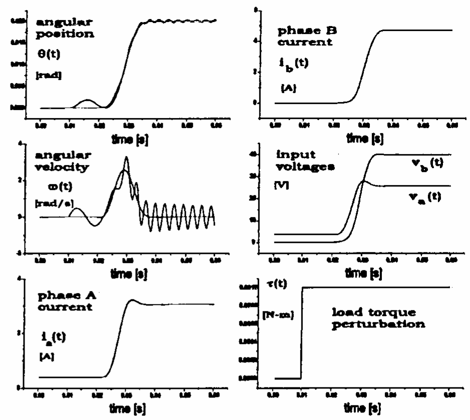

Figure 2: Closed loop response to unmodeled load torque perturbation

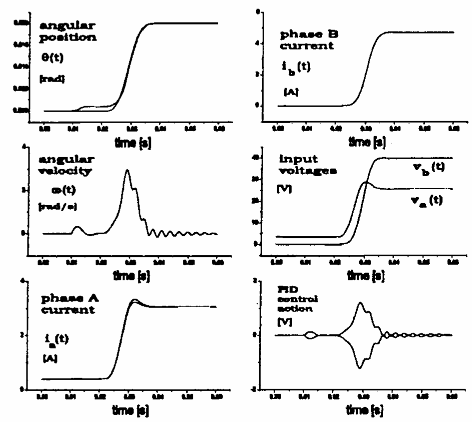

Figure 3: PM Stepper motor closed loop response to Passivity + Flatness based controller with outer loop PID compensation of load torque perturbation at t=0.01 [s]