Regulation of the PPR Mobile Robot with a Flexible Joint: A combined passivity and flatness approach [1]

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Abstract

A passivity based controller is proposed for the regulation of the PPR mobile robot equipped with an underactuated arm coupled to the main robot body by means of a flexible joint. The system, which happens to be differentially flat, can then be controlled using a combination of the passivity based approach and trajectory planning facilitated by the flatness property of the system.

1 Introduction

Passivity–based control (PBC) is a well established controller design methodology [4] that has shown to be very useful in solving, with a clear physical interpretation, control problems of a large class of nonlinear (physical) systems. Its main characteristic lies in the fact that for feedback purposes, it considers (passive) outputs that are "easy to control", leading to simple controller structures that in many cases resemble those currently used in industrial applications. In addition and from several perspectives (e.g. parameter uncertainty, unmodeled dynamics, etc.), the robustness properties of the proposed controllers have been proven not only in a theoretical context but by means of experimental evaluation.

One interesting feature of PBC is related with the systematic procedure that must be followed in developing a controller. In this sense, however, it must be pointed out that perhaps the main disadvantage of the approach, comes from the requirement to carry out some kind of system inversion at some of step of the design. This situation, on the one hand, has leading to a more deep knowledge about the applicability limitations of the method, but on the other hand, has motivated the idea of finding new simpler alternatives to solve this problem. In fact, roughly speaking, three general solutions can be currently identified: The application of well known inversion algorithms like the proposed in [9], the use of coordinate transformations in order to simplify the system structure [6] and the exploitation of the knowledge that the designer could have about the system behavior [8].

On the other hand, in the last decade the concept of differential flatness of dynamical systems has been introduced in a control context [1], [2]. In particular, it is said that a finite dimensional nonlinear multivariable system is differentially flat if it is equivalent, by means of endogenous feedback, to a linear controllable system in decoupled Brunovsky's form. The importance of this concept comes from the fact that for systems satisfying this property, there exist a set of variables, called flat outputs, which completely parameterize the system state variables and control inputs. Hence, given some (off-line) planned trajectories for the flat outputs, it is possible to find a (dynamic) control law that achieves the objective of tracking the prescribed trajectories. In other words, for situations when the objective is to control the flat outputs of a flat system, it is possible to carry out a system inversion in order to solve the problem.

The flatness approach has shown to be applicable to a large class of physical systems that satisfy

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the aforementioned property. Indeed, some problems that were not possible to solve with other approaches have been by the application of this technique. However, the main disadvantages of the proposed controllers are related with its complexity and its robustness properties.

The objective of this paper is twofold: First, to evaluate by means of a particular example (a Prismatic-Prismatic-Revolute (PPR) mobile robot), the possibility of combining the PBC with the flatness approach. Here, the aim is to solve the system inversion requirement of the former by the exploitation of the flatness property of the robot. Second, to compare, by means of simulations, the performances of the passivity + trajectory planning controller with that based solely on flatness considerations.

Specifically, a nonlinear multivariable passivity based controller is proposed for the trajectory tracking error regulation of the PPR mobile robot equipped with an underactuated arm which is coupled to the main robot body by means of a flexible joint. The PPR robotic system has been shown to be differentially flat in [3] and equivalent, by means of dynamic feedback, to a set of decoupled controllable linear systems. The flatness property is here exploited in the off-line computation of suitable trajectories for the passive outputs which are now expressed in terms of desired trajectories for the system's flat outputs. The passive outputs trajectories are required in the expressions of a passivity based controller developed in [7].

The paper is organized as follows: In section 2 the mathematical model of the PPR mobile robot is presented and its passivity properties are quickly revised. The PBC for this system is developed in section 3 while in section 4 the flatness properties of the robots are presented. Section 5 contains the results obtained from the evaluation of the proposed controller via digital simulations and the papers ends with some concluding remarks presented in section 6.

2 Mathematical model

In this section, by means of the Euler–Lagrange equations, it is developed the model of the PPR mobile robot with a flexible revolute joint shown in Figure 1[1]. The variables x and y denote the coordinates of the tip of the end–effector appendage. The base body can freely translate in the plane and the position of its center of mass is represented by a pair of coordinates (x_B, y_B). The control input forces are supposed to be applied at this point of the body and cause its translational motions. The control input torque, T, rotates the base body and provides the angular orientation of the robot main body, denoted by the angle θ.

By defining $\theta_2 = \theta + \phi$ and noting that

$$\begin{array}{ll} x = x_b + l\cos(\theta_2) & \Rightarrow \quad \dot{x} = \dot{x}_B - l\dot{\theta}_2 \sin(\theta_2) \\ y = y_B + l\sin(\theta_2) & \qquad \dot{y} = \dot{y}_B + l\dot{\theta}_2\cos(\theta_2) \end{array} \tag{2.1}$$

the generalized coordinates of the system are given by $q = [x_B, y_B, \theta, \theta_2]^T$, while the kinetic co-energy function can be written as $\mathcal{T}(q, \dot{q}) = \frac{1}{2}\dot{q}^T D(q)\dot{q}$, with

$$D(q) = \begin{bmatrix} (M+m)\mathcal{I}_2 & 0_{2\times 1} & f(\theta_2) \\ 0_{1\times 2} & I & 0 \\ f^T(\theta_2) & 0 & ml^2 \end{bmatrix} = D^T(q) > 0$$

$\mathcal{I}_2$ the 2×2 identity matrix and $f(\theta_2) = [-ml\sin(\theta_2), ml\cos(\theta_2)]^T$. On the other hand, assuming that the spring between θ and θ_2 is linear (with K being the torsional spring coefficient), the potential energy is given by $V(q) = \frac{1}{2}q^T\mathcal{K}q$ with

$$\mathcal{K} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & K & -K \\ 0 & 0 & -K & K \end{bmatrix}$$

Thus the Lagrangian is given by $\mathcal{L}(q,\dot{q}) = \frac{1}{2}\dot{q}^T D(q)\dot{q} - \frac{1}{2}q^T\mathcal{K}q$ while the vector of generalized external forces is $Q = [F_1, F_2, T, 0]^T$.

Applying the Euler-Lagrange equations, the model for the robot is

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + \mathcal{K}q = Q \tag{2.2}$$

where

$$C(q,\dot{q}) = \begin{bmatrix} 0_{2\times 2} & 0_{2\times 1} & \dot{f}(\theta_2) \\ 0_{1\times 2} & 0 & 0 \\ 0_{1\times 2} & 0 & 0 \end{bmatrix}$$

Notice that, as is well-known, matrix $\overline{D}(q,\dot{q}) = \dot{D}(q) - 2C(q,\dot{q})$ is skew-symmetric and the map $\begin{bmatrix} F_1 & F_2 & T \end{bmatrix}^T \mapsto \begin{bmatrix} \dot{x}_B & \dot{y}_B & \dot{\theta} \end{bmatrix}^T$ is passive.

3 PBC of the PPR mobil robot

Considering the model presented in the last section and following [7], in this section a PBC is developed

[1] This robot has been recently treated by Reyhanoglu *et al* [5] from a viewpoint of non-integrable acceleration constraints.

to achieve the objective of trajectory tracking of the robot coordinates. To this end, consider the variables defined as $s = \dot{q} - \xi$ and $\xi = \dot{q}_d - \Lambda(q - q_d)$ with $\Lambda = \Lambda^T > 0$. We will, specifically, chose Λ as a diagonal matrix given by $\text{diag}\{\lambda_x, \lambda_y, \lambda_\theta, \lambda_{\theta_2}\}$. Here $q_d = [x_{Bd}, y_{Bd}, \theta_d, \theta_{2d}]^T$ is the vector of desired behavior for the robot coordinates while $e = q - q_d$ is the tracking error. Notice that this error and the variable s are related by the following linear dinamical equation $\dot{e} = -\Lambda e + s$.

With these definitions, model (2.2) can be equivalently written as

$$D(q)\dot{s} + C(q, \dot{q})s = Q - \left\{ D(q)\dot{\xi} + C(q, \dot{q})\xi + \mathcal{K}q \right\} \tag{3.1}$$

Define the modified energy function as $H_d = \frac{1}{2} s^T D(q) s$. Then along the trajectories of (3.1) we have,

$$\frac{d}{dt} H_d = s^T \left[Q - \left\{ D(q)\dot{\xi} + C(q, \dot{q})\xi + \mathcal{K}q \right\} \right]$$

Thus if the controller is proposed as

$$Q = \left\{ D(q)\dot{\xi} + C(q, \dot{q})\xi + \mathcal{K}q \right\} - K_v s \tag{3.2}$$

with $K_v = \text{diag}\{K_{vx}, K_{vy}, K_{v\theta}, 0\} > 0$ then $\dot{H}_d = -s^T K_v s \leq 0$. Hence, invoking standar arguments it can be concluded that both s and e tend to zero as time tends to infinity.

The explicit expression for the components of the passivity based controller (3.2) are written here, for the purpose of later reference

$$\begin{array}{c} F_1 = (M+m)(\ddot{x}_{Bd} - \lambda_x(\dot{x}_B - \dot{x}_{Bd})) \\ -ml\sin(\theta_2)(\ddot{\theta}_{2d} - \lambda_{\theta_2}(\dot{\theta}_2 - \dot{\theta}_{2d})) \\ -ml\dot{\theta}_2\cos(\theta_2)(\dot{\theta}_{2d} - \lambda_{\theta_2}(\theta_2 - \theta_{2d})) \\ -K_{vx}(\dot{x}_B - \lambda_x(x_B - x_{Bd})) \\ F_2 = (M+m)(\ddot{y}_{Bd} - \lambda_x(\dot{y}_B - \dot{y}_{Bd})) \\ +ml\cos(\theta_2)(\ddot{\theta}_{2d} - \lambda_{\theta_2}(\dot{\theta}_2 - \dot{\theta}_{2d})) \\ -ml\dot{\theta}_2\sin(\theta_2)(\dot{\theta}_{2d} - \lambda_{\theta_2}(\theta_2 - \theta_{2d})) \\ -K_{vy}(\dot{y}_B - \lambda_y(y_B - y_{Bd})) \\ T = I(\ddot{\theta}_d - \lambda_\theta(\dot{\theta} - \dot{\theta}_d)) + K(\theta - \theta_2) \\ -K_{v\theta}(\dot{\theta} - \lambda_\theta(\theta - \theta_d)) \end{array} \tag{3.3}$$

where must be noticed that this controller requires the specification of the desired trajectories, $x_{Bd}(t), y_{Bd}(t), \theta_d(t), \theta_{2d}(t)$, of the adopted generalized coordinates.

The main point to be noticed in the development above, is the fact that, due to the structure of the generalized external forces vector, the last equation in (3.2), given by

$$\begin{array}{c} -ml\sin(\theta_2)(\ddot{x}_{Bd} - \lambda_x(\dot{x}_B - \dot{x}_{Bd})) \\ +ml\cos(\theta_2)(\ddot{y}_{Bd} - \lambda_y(\dot{y}_B - \dot{y}_{Bd})) \\ +ml^2(\ddot{\theta}_{2d} - \lambda_{\theta_2}(\dot{\theta}_2 - \dot{\theta}_{2d})) \\ +K(\theta - \theta_2) = 0 \end{array}$$

establishes a constraint that must be satisfied for all time. In this sense, the desired behavior for the robot coordinates q_d, must be chosen such that this constraint is satisfied. Indeed, this requirement states the critical step in the design of the PBC and leads to the necessity of inverting the system, as mentioned in the introcution of the paper. As was also mentioned, the solution to this problem will be given (in the next section) by considering the flatness properties of the system. However and with the aim of simplifying this task, it is useful to notice that since $q \to q_d$, the constraint takes the following form

$$\begin{array}{c} -ml\sin(\theta_{2d})\ddot{x}_{Bd} + ml\cos(\theta_{2d})\ddot{y}_{Bd} + ml^2\ddot{\theta}_{2d} \\ +K(\theta_d - \theta_{2d}) = 0 \end{array} \tag{3.4}$$

which corresponds identically to the last system equation under ideal reference trajectory tracking conditions. Thus, the only condition that must be imposed to the desired variables q_d, is that they must satisfy the natural dynamic behavior of the system.

4 Flatness Control

With the aim of solving the system inversion problem posed in the last section, in this section it is presented the flatness property of the PPR robot recently reported in [3]. The purpose of this presentation is to show how the exploitation of this property allow us to obtain the desired behavior q_d, required by the PBC, that satisfy the constraint (3.4).

The PPR robot model (2.2) is easily seen to be differentially flat. Indeed, the flat outputs are given by the main body center of gravity position coordinates, x_B, y_B, and the orientation angle, θ_2, of the robot arm. Hence, all variables in the system (i.e. states and control inputs) are expressible as *differential functions* of the flat coordinates, (x_B, y_B, θ_2).

$$\begin{array}{c} \theta = \theta_2 + \left[\frac{ml^2\ddot{\theta}_2 - ml\ddot{x}_B\sin(\theta_2) + ml\ddot{y}_B\cos(\theta_2)}{K} \right] \\ F_1 = (M+m)\ddot{x}_B - ml\sin(\theta_2)\ddot{\theta}_2 - ml\dot{\theta}_2^2\cos(\theta_2) \end{array} \tag{4.1}$$

$$F_2 = (M+m)\ddot{y}_B + ml\cos(\theta_2)\ddot{\theta}_2 - ml\dot{\theta}_2^2\sin(\theta_2)$$

$$T = -ml\ddot{x}_B\sin(\theta_2) + ml\ddot{y}_B\cos(\theta_2) + (I+ml^2)\ddot{\theta}_2 + \tfrac{ml^2I}{K}\theta_2^{(4)} + \tfrac{mlI}{K}\left(y_B^{(4)} - \dot{\theta}_2^2\ddot{y}_B - \ddot{\theta}_2\ddot{x}_B - 2\dot{\theta}_2 x_B^{(3)}\right)\cos(\theta_2) - \tfrac{mlI}{K}\left(x_B^{(4)} - \dot{\theta}_2^2\ddot{x}_B + \ddot{\theta}_2\ddot{y}_B + 2\dot{\theta}_2 y_B^{(3)}\right)\sin(\theta_2)$$

In the context of this paper, the importance of the differential parameterization (4.1), specially the first equation, lies in the fact that it allows one to explicitly compute a suitable reference trajectory for all state variables in the system. In particular, the required body angular orientation, $\theta_d(t)$, can be directly expressed in terms of the desired off-line planned trajectories for the body position coordinates, $(x_{Bd}(t), y_{Bd}(t))$, and the desired arm angular position trajectory, $\theta_{2d}(t)$, in the following way

$$\theta_d = \theta_{2d} + \tfrac{-ml\sin(\theta_{2d})\ddot{x}_{Bd} + ml\cos(\theta_{2d})\ddot{y}_{Bd} + ml^2\ddot{\theta}_{2d}}{K} \tag{4.2}$$

giving as a result that all the information required for the PBC is now at disposition. Notice that the end effector position is also expressible in terms of the flat outputs, by means of (2.1).

The differential parameterization (4.1) allows one to explicitly off-line compute suitable reference trajectories for all state variables in the system in terms of desired flat output reference trajectories.

The structure of the dependence of the control inputs on the flat outputs time derivatives reveals that while the torque control input T depends up to fourth order time derivatives of all the flat outputs, the forces F_1 and F_2 at most involve second order time derivatives of only two flat outputs. The multivariable input-flat output relation is, therefore, not an invertible one in the sense that the higher order derivatives of the flat outputs are not in a one to one relationship with a possible set of independent control inputs. A *dynamic extension* is, therefore, needed on the first two control inputs in order to achieve a desirable input-flat output "decoupling". This extended relation is

$$\ddot{F}_1 = (m+M)x_B^{(4)} - ml\left[\theta_2^{(4)} - 6(\dot{\theta}_2)^2\ddot{\theta}_2\right]\sin(\theta_2) - ml\left[4\dot{\theta}_2\theta_2^{(3)} + 3(\ddot{\theta}_2)^2 - (\dot{\theta}_2)^4\right]\cos(\theta_2) \tag{4.3}$$

$$\ddot{F}_2 = (m+M)y_B^{(4)} - ml\left[4\dot{\theta}_2\theta_2^{(3)} + 3(\ddot{\theta}_2)^2 - (\dot{\theta}_2)^4\right]\sin(\theta_2) + ml\left[\theta_2^{(4)} - 6(\dot{\theta}_2)^2\ddot{\theta}_2\right]\cos(\theta_2)$$

$$T = (I+ml^2)\ddot{\theta}_2 + \tfrac{mlI}{K}\Big[l\theta_2^{(4)} - \left(x_B^{(4)} + \left[\tfrac{K}{I} - \dot{\theta}_2^2\right]\ddot{x}_B + \ddot{\theta}_2\ddot{y}_B + 2\dot{\theta}_2 y_B^{(3)}\right)\sin(\theta_2) + \left(y_B^{(4)} + \left[\tfrac{K}{I} - \dot{\theta}_2^2\right]\ddot{y}_B - \ddot{\theta}_2\ddot{x}_B - 2\dot{\theta}_2 x_B^{(3)}\right)\cos(\theta_2)\Big]$$

which is globally invertible with respect flat outputs, implying that a suitable (global) state-dependent input coordinate transformation reduces the system to the following decoupled set of linear controllable systems in Brunovsky's canonical form $x_B^{(4)} = v_1$, $y_B^{(4)} = v_2$, $\theta_2^{(4)} = v_3$ with v_1, v_2 and v_3 defined in the obvious way.

In the proposition below, the above invertibility property of the system is exploited with the aim of desinging a feedback linearization controller, which will be used to achieve the second objective of this work, i.e. the comparison between PBC and flatness control.

Proposition 4.1 *Given a prescribed trajectory, $\{x_B^*(t), y_B^*(t), \theta_2^*(t)\}$ for the flat outputs of the PPR robot (2.2), the* dynamic *feedback controller obtained from (4.3) by substituting $x_B^{(4)} = v_x$, $y_B^{(4)} = v_y$, $\theta_2^{(4)} = v_{\theta_2}$, achieves closed loop asymptotic exponential tracking of the given path with $v_x = p_x(\frac{d}{dt})e_x - \frac{d^4}{dt^4}x_B$, $v_y = p_y(\frac{d}{dt})e_y - \frac{d^4}{dt^4}y_B$ and $v_{\theta_2} = p_{\theta_2}(\frac{d}{dt})e_{\theta_2} - \frac{d^4}{dt^4}\theta_2$. The variables e_x, e_y and e_{θ_2} denote, respectively, the tracking errors, $x_B - x_B^*(t)$, $y_B - y_B^*(t)$ and $\theta_2 - \theta_2{}^*(t)$ while $p_{(\cdot)}(\frac{d}{dt}) = \frac{d^4}{dt^4} + \beta_{3(\cdot)}\frac{d^3}{dt^3} + \beta_{2(\cdot)}\frac{d^2}{dt^2} + \beta_{1(\cdot)}\frac{d}{dt} + \beta_{0(\cdot)}$ where the real constant coefficients $\{\beta_{0(\cdot)}, \beta_{1(\cdot)}, \beta_{2(\cdot)}, \beta_{3(\cdot)}\}$ constitute a Hurwitz set, i.e. they are associated with* stable, *monic, fourth order polynomials.*

Proof. The proposed controller yields the following closed loop tracking error dynamics

$$\begin{bmatrix} (m+M)\mathcal{I}_2 & f(\theta_2) \\ f^T(\theta_2) & ml^2 \end{bmatrix} \begin{bmatrix} [p_x(\frac{d}{dt})]\, e_x \\ [p_y(\frac{d}{dt})]\, e_y \\ [p_{\theta_2}(\frac{d}{dt})]\, e_{\theta_2} \end{bmatrix} = 0$$

Since the above matrix is non-singular, it follows that e_x, e_y and e_{θ_2} asymptotically exponentially converge to zero. □

The linearizing controller derived above is based on exact cancellation of the systems non-linearities. The performance of the linearizing controller is seriously deteriorated when unmodelled external perturbation inputs and uncertain parameter perturbations affect the system.

5 Simulation results

Simulations were performed for the derived PPR mobile robot model controlled by the passivity based controller (3.3). The desired trajectories, $q_d(t)$, for the chosen generalized coordinate variables, $q = [x_B, y_B, \theta, \theta_2]^T$, which are needed in the static controller expressions (3.3), were specified in terms of the desired trajectories for the flat outputs, $x_{Bd}(t), y_{Bd}(t), \theta_{2d}(t)$. These trajectories determine the required trajectory for the body orientation $\theta_d(t)$ in accordance with the differential parameterization (4.2).

Consider the motion represented by a straight line segment with prescribed initial point given by the coordinates, (x_0, y_0), and terminal point given by, (x_f, y_f). The robot starts with zero velocity and is required to reach the end of the line segment and proceed to park at the specified final point. The arm is required to point along the same direction of the robot movement.

For the required maneuver, we prescribe a polynomial spline for each of the flat output coordinates $x_{Bd}(t)$, $y_{Bd}(t)$ and also prescribe a constant value trajectory for the angular position $\theta_{2d}(t)$ which imposes a zero value for the arms angular deviation, $\phi(t) = \theta_2 - \theta$.

The nominal displacement variables are specified as,

$$
\begin{aligned}
x_{Bd}(t) &= x_0 + (x_f - x_0) g^5(t) \left[\sum_{j=1}^{6} g^{j-1}(t)\right] \\
y_{Bd}(t) &= y_0 + (y_f - y_0) g^5(t) \left[\sum_{j=1}^{6} g^{j-1}(t)\right] \\
\theta_{2d}(t) &= 0.78539 \ \ (= \tfrac{\pi}{4}) \\
g(t) &= \left(\frac{t-t_i}{t_f - t_i}\right)
\end{aligned}
$$

with $r_1 = 252$, $r_2 = 1050$, $r_3 = 1800$, $r_4 = 1575$, $r_5 = 700$, $r_6 = 126$ and $t_i = 10$ [s], $t_f = 30$ [s], $(x_f, y_f) = (1.5, 1.5)$, $(x_0, y_0) = (0.5, 0.5)$.

Figure 2 depicts computer simulations illustrating the performance of the previously designed feedback controller for significant initial deviations from the prescribed path and from the prescribed orientation angle. These initial conditions were set to be $x(t_0) = 0.4$[m], $y(t_0) = 0.4$[m], $\theta_2 = 0.6$[rad] and $\theta = 0.6$[rad].

As can be seen in this figure, although the tracking objective is achieved, coordinate θ_2 shows a very poor performance. This behavior must be expected since, due to the structure of the system, it is not possible to inyect damping to this coordinate (notice the zero entry in matrix K_v in the equation of the controller (3.2)). This problem has been a topic of research that has deserved a great attention and current research is developed with the aim to solve this problem in this new context.

On the other hand, the relatively simple structure of the proposed controller is reflected in the fact that the control effort is also relatively small, as can be seen in figure 3.

In order to carry out a comparision between the PBC and the flatness controller, the same control objective than in the previous section imposed to the former was imposed to the flatness controller presented in Proposition 1. In this case, in addition to the nominal displacement variables, the nominal displacement for the coordinate θ was defined as $\theta^*(t) = 0.78539$. Figures 4 depicts the results obtained under the conditions above for the robot coordinates and the control inputs, respectively. As can be seen, the disadvantage that comes from the more complex structure of the flatness controller with respect PBC, is compensated by a considerably better performance in coordinate θ_2. However, it must be noticed that the control effort required under the flatness approach is almost twice of the required under the passivity approach.

6 Concluding remarks

A solution to the system inversion requirement in passivity–based control design was proposed in this paper. The particular example of the Prismatic–Prismatic–Revolute mobile robot with flexible link was approached. The inversion was carried out by exploiting the flatness properties of the system, task that was simplified by the fact that in this case the flat outputs coincide with the system variables to be controlled. The effectiveness of the proposed controller was evaluated via digital simulations where the lack of damping inyection on the underactuated coordinates was evident. The analysis of the presented solution was complemented by comparing it with a controller produced by considering a purely flatness approach. Although a clear superiority was obtained with the flatness controller regarding the transient response of the underactuated coordinates, the passivity-based control law showed some advantanges concerning computational requirements and control effort. It is the authors believe that this approach of combining passivity and flatness design methodologies, could lead to more systematic procedures for designing a controller. Nevertheless, current research is developing in evaluating important topics like robustness and performance improvement.

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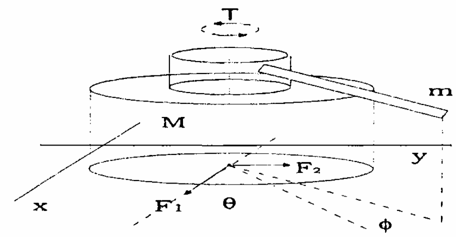

Figure 1: The PPR mobile robot

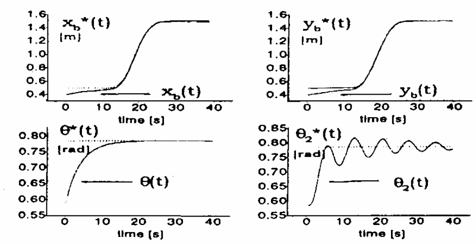

Figure 2: State variable responses for PBC

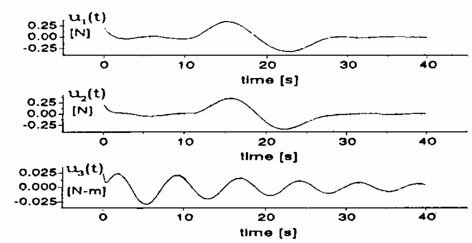

Figure 3: Control inputs for PBC

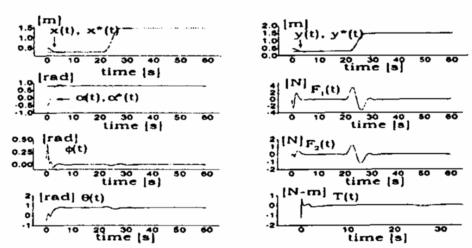

Figure 4: State variable and control inputs under flatness control