Adaptive Passivation of a Class of Uncertain Nonlinear Systems

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Abstract— **We propose in this work an adaptive passivation procedure for affine nonlinear systems with constant but unknown parameters, which is achievable through an adaptive state-dependent input coordinate transformation. The unknown parameters are assumed to enter linearly into the *drift* vector fields of the dynamic equations defining the nonlinear system. It is shown that the update law designed for the unknown parameters qualifies as a "force which does not work" in the context of a general passivity canonical form for nonlinear systems. The design of passivity-based controllers, via energy shaping and damping injection, is addressed and an application of this approach for the adaptive stabilization of a gravity-tank/pipe system is also considered.**

Keywords— **Adaptive control, Nonlinear Systems, Uncertainty, Passivity**

I. Introduction

THE study of passive nonlinear systems and their properties have received much attention in recent years. Passive systems with nonnegative storage functions exhibit attractive stability properties. For instance, if a passive system satisfies a certain *detectability* condition, it is stabilizable by a simple static output feedback control law [1]. Passivity is a particular case of the more general concept of dissipativity, which was introduced by Willems in [10]. Basic and important contributions to this field, in the context of affine nonlinear systems, have been given in the work by Hill and Moylan [2], and Byrnes *et al* [1].

On the other hand, adaptive stabilization of nonlinear systems containing constant but unknown parameters has attracted attention of many researchers recently (see [5] [6], [7]). A common assumption in the context of adaptive control is associated with linear parameterization of uncertain systems which simplifies the problem of designing adaptive feedback control laws. We propose here a systematic procedure to obtain an adaptive state dependent input coordinate transformation which renders passive a class of nonlinear systems containing constant but unknown parameters. This adaptive feedback passivation procedure constitutes a generalization of the geometric approach proposed by Sira-Ramirez and Delgado [9], due to the incorporation of online update laws. It is shown that the parameter adaptation laws are "forces" which do not perform any work with respect to an augmented storage function including energy terms associated with the parameter estimation errors. An alternative general treatment of adaptive feedback passivation of uncertain nonlinear systems has been presented in the work of Seron *et al* [8]. Our approach differs from that one in that we do not impose any requirement regarding the relative degree of the system output and, moreover, an essential singularity is removed, by assuming constant nonzero equilibrium points where the storage function has nonzero gradient projection along the input vector field, which allows for direct passivation through an input coordinate transformation.

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Once the uncertain system has been made passive, we proceed to decompose the drift vector field of it, by straightforward factorization, in dissipative forces and workless forces. Then, a passivity-based feedback control law which achieves asymptotic stability of the closed loop system is synthesized by considering a modified storage function plus damping injection which preserves benefitial self-stabilizing nonlinearities.

This paper is organized as follows: Section II presents some basic definitions about passivity and revisits the fundamental ideas of the feedback passivation scheme proposed in [9] for single-input single-output (SISO) systems. Our adaptive feedback passivation scheme is described in Section III. An illustrative example is presented in Section IV. Section V contains some conclusions.

II. Passivation of SISO Nonlinear Systems

A. Basic Definitions

Consider the single input single output system,

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \qquad (1)$$

where $x \in \chi \subset \Re^n$ is the state vector, $u \in \mathcal{U} \subset \Re$ is the control input and the scalar function $y \in \mathcal{Y} \subset \Re$ is the output function of the system. The vector fields $f(x)$ and $g(x)$ are assumed to be smooth on χ. For simplicity, we assume the existence of an isolated nonzero equilibrium state, $x = x_e \in \chi$, where $f(x_e) + g(x_e)\overline{u} = 0$, for some nonzero constant $\overline{u}$. The region $\chi \subset \Re^n$ is the *operating region* of the system which strictly contains x_e. All our results are local for as long as χ cannot be assumed to be all of $\Re^n$. Associated with system (1) it is assumed to exist an energy storage function, $V : \chi \to \Re^+$ which may be zero outside of χ (at the origin, for instance). The *supply rate* function is defined as a function $s : \mathcal{U} \times \mathcal{Y} \to \Re$.

We introduce some well-known basic definitions about dissipative, lossless and passive systems (see Byrnes *et al* [1] for further details).

Definition 1: System (1) is said to be *dissipative* with respect to the supply rate $s(u, y)$ if there exists a storage function $V : \chi \rightarrow \Re^+$, such that for all $x_0 \in \chi$ and for all $t_1 \geq t_0$, and all input functions u, the following relation holds

$$V(x(t_1)) - V(x(t_0)) \leq \int_{t_0}^{t_1} s(u(t), y(t))dt \qquad (2)$$

with $x(t_0) = x_0$ and $x(t_1)$ is the state resulting, at time t_1, from the solution of system (1) taking as initial condition x_0 and as control input the function $u(t)$. Inequality (2) is equivalent to

$$\dot{V} \leq s(u(t), y(t)) \qquad (3)$$

The system is *lossless* if the inequalities (2), or (3), are, in fact, equalities.

Definition 2: System (1) is *passive* if it is dissipative with respect to the supply rate $s(u, y) = uy$. The system is *strictly input passive* if there exists $\delta > 0$ such that the system is dissipative with respect to $s(u, y) = uy - \delta u^2$. The system is *strictly output passive* if there exists $\rho > 0$ such that the system is dissipative with respect to $s(u, y) = uy - \rho y^2$.

We shall be considering means of rendering a system of the form (1) passive by means of state feedback. We therefore give a definition of "passivifiable" systen in the following terms.

Definition 3: System (1) is said to be *passivifiable* with respect to the storage function V, if there exists a *regular affine feedback law* of the form

$$u = \alpha(x) + \beta(x)v \quad ; \quad \alpha(x), \beta(x) \in \Re \qquad (4)$$

where $\beta(x)$ is a nonzero scalar function in χ, and such that the closed loop system (1)-(4) becomes passive with the new scalar control input v.

B. Feedback Passivation

Consider system (1) with V being, locally in χ, a strict *relative degree* one function, i.e. $L_gV(x) \neq 0, \quad \forall x \in \chi$. Then, for any given control input u and any initial state $x_0 \in \chi$, the time derivative of the storage function V, along the solutions of (1), is given by

$$\dot{V} = \frac{\partial V}{\partial x} f(x) + \left(\frac{\partial V}{\partial x} g(x)\right) u = L_f V(x) + [L_g V(x)]u \quad (5)$$

with $L_fV(x)$ and $L_gV(x)$ being the Lie derivatives of the storage function $V(x)$ along the vector fields $f(x)$ and $g(x)$ respectively.

Suppose that the vector field $f(x)$ has natural components, $f_d(x), f_{nd}(x), f_I(x)$, with respect to the storage function V, i.e.

$$f(x) = f_d(x) + f_{nd} + f_I(x) \qquad (6)$$

such that,

$$\begin{array}{ll} L_{f_d}V(x) & \leq 0 \quad ; \quad \forall x \in \chi \\ L_{f_{nd}}V(x) & \left\{ \begin{array}{l} \text{is, either sign } - \text{ undefined in } \chi \\ \text{or, else, it is nonnegative in } \chi \end{array} \right. \\ L_{f_I}V(x) & = 0 \quad ; \quad \forall x \in \chi \end{array}$$

We address $f_d(x)$ as the *dissipative* component of $f(x)$. Similarly, $f_{nd}(x)$ will be termed the *non-dissipative* component of $f(x)$ and, $f_I(x)$ is the *invariant* component of $f(x)$.

The time derivative of the energy storage function, along the solutions of the system, is given by

$$\dot{V} = L_{f_d}V(x) + L_{f_{nd}}V(x) + [L_gV(x)]u \qquad (7)$$

Define the following state dependent input coordinate transformation,

$$u = \frac{1}{L_gV(x)}\left[h(x)v - L_{f_{nd}}V(x) - \delta h^2(x)\right] \qquad (8)$$

where δ is a strictly positive scalar.

It is seen, upon substitution of expression (8) into equation (7), that the time derivative of the energy storage function satisfies the following string of relations,

$$\dot{V} = L_{f_d}V(x) + h(x)v - \delta h^2(x) \leq yv - \delta y^2 \leq yv \qquad (9)$$

In other words, if the system is such that L_gV is locally nonzero, then the input coordinate of the system may be transformed in such a way that the partially closed loop system will exhibit a ***strictly output passive*** behaviour between the external control input v and the original scalar output $y = h(x)$. This result is summarized in the following proposition.

Proposition 1: System (1) is locally strictly output passivifiable with respect to the storage function V, by means of affine feedback of the form (4) if and only if

$$L_gV(x) \neq 0 \qquad \forall x \in \chi \qquad (10)$$

The affine feedback law, or state dependent input coordinate transformation, that achieves strict output passivation is given by the expression (8).

C. Feedback Control Design from a Passivity Canonical Form

We revisit in this section a systematic procedure for the synthesis of passivity based feedback controllers. This procedure, based on storage function modification and damping injection through feedback, has been extensively used in the area of electro-mechanical systems.

Suppose that system (1) is passivifiable and assume $f_d(x), f_{nd}(x), f_I(x)$ are the natural components of $f(x)$ with respect to the storage function $V(x)$. Assuming that $L_gV(x) \neq 0$ in the operating region χ of the state space, then, the passivified system can be written as

$$\begin{array}{rcl} \dot{x} & = & f_d(x) + f_I(x) + \left[I - g(x)\dfrac{\triangle V(x)}{L_gV(x)}\right] f_{nd}(x) \\ & & + \dfrac{h(x)v}{L_gV(x)}g(x) - \delta\dfrac{h^2(x)}{L_gV(x)}g(x) \end{array} \qquad (11)$$

with $\triangle V(x) := \frac{\partial V}{\partial x}$.

As integrating parts of the time derivative of V one has the following terms

$$\begin{aligned}\triangle V(x)\left[f_d(x)-g(x)\frac{\delta h^2(x)}{L_gV(x)}\right] &\leq 0\\ \triangle V(x)\left\{f_I(x)+\left[I-g(x)\frac{\Delta V(x)}{L_gV(x)}\right]f_{nd}(x)\right\} &= 0\end{aligned}$$

It then follows, by straightforward factorization, that such a system may always be rewritten in the following form

$$\begin{aligned}\dot{x} &= -\mathcal{R}(x)(\Delta V(x))^T-\mathcal{J}(x)(\Delta V(x))^T+\mathcal{M}(x)v\\ y &= h(x)\end{aligned}\tag{12}$$

with $\mathcal{R}(x)$ being a positive semidefinite matrix in χ, and $\mathcal{J}(x)$ being a skew-symmetric matrix. This implies the following identifications,

$$\begin{aligned}f_d(x)-\delta\frac{h^2(x)}{L_gV(x)}g(x) &= -\mathcal{R}(x)(\Delta V(x))^T\\ f_I(x)+\left[I-g(x)\frac{\Delta V(x)}{L_gV(x)}\right]f_{nd}(x) &= -\mathcal{J}(x)(\Delta V(x))^T\\ \frac{h(x)}{L_gV(x)}g(x) &= \mathcal{M}(x)\end{aligned}$$

This factorization is particularly simple when the storage function is quadratic, i.e. $V(x)=1/2x^Tx$, as will be considered in the rest of this work. The expression (12) characterizes a *passivity canonical form* for which a feedback controller may be designed via energy shaping and damping injection, as illustrated below.

A passivity-based controller can be proposed for systems of the form (12) by considering the following storage function

$$V_d(x,x_d)=\frac{1}{2}(x-x_d)^T(x-x_d)\tag{13}$$

where x_d is an auxiliary state vector to be defined later.

Along the solutions of system (12), the function $V_d(x,x_d)$ exhibits the following time derivative

$$\dot{V}_d(x,x_d)=(x-x_d)^T[-\mathcal{R}(x)x-\mathcal{J}(x)x+\mathcal{M}(x)v-\dot{x}_d]\tag{14}$$

Completing squares in the right hand side and adding a *damping injection* term of the form $-\mathcal{R}_{di}(x)x$, so that $\mathcal{R}_m(x)=\mathcal{R}(x)+\mathcal{R}_{di}(x)$ is a positive definite matrix for all $x\in\chi$, one obtains

$$\begin{aligned}\dot{V}_d(x,x_d) &= (x-x_d)^T\Big[-(\mathcal{R}(x)+\mathcal{R}_{di}(x))(x-x_d)\\ &\quad -\mathcal{J}(x)(x-x_d)-\dot{x}_d-\mathcal{R}(x)x_d-\mathcal{J}(x)x_d\\ &\quad +\mathcal{R}_{di}(x)(x-x_d)+\mathcal{M}(x)v\Big]\end{aligned}\tag{15}$$

Note that, if we let the auxiliary vector $x_d(t)$ satisfies the following system of differential equations

$$\dot{x}_d=-\mathcal{R}(x)x_d-\mathcal{J}(x)x_d+\mathcal{R}_{di}(x)(x-x_d)+\mathcal{M}(x)v\tag{16}$$

the time derivative of $V_d(x,x_d)$ yields

$$\begin{aligned}\dot{V}_d(x,x_d) &= -(x-x_d)^T\mathcal{R}_m(x)(x-x_d)\\ &\leq -\frac{a}{b}(x-x_d)^T(x-x_d)\\ &\leq -\frac{a}{b}V(x,x_d)\leq 0\end{aligned}$$

where, in terms of the minimum and maximum eigenvalues $(\lambda_{min},\lambda_{max})$ of $\mathcal{R}_m(x)$, a and b are given by,

$$a=inf_{x\in\chi}\ \lambda_{min}(\mathcal{R}_m(x))>0$$

$$b=sup_{x\in\chi}\ \lambda_{max}(\mathcal{R}_m(x))>0$$

It follows that the vector $x(t)$ asymptotically converges towards the auxiliary vector trajectory $x_d(t)$. The feedback controller can be synthesized from the system of differential equations (16). Typically, one sets for a particular component of the vector x_d a desired constant equilibrium value. The objective of such a particularization is to obtain a feedback expression for the external control input v in terms of both the available state vector x and the rest of auxiliary variables in x_d. The differential equations defining the remaining auxiliary variables in x_d are to be regarded as "state" components of a *dynamical feedback compensator* [9].

III. Adaptive Feedback Passivation

We consider in this section an adaptive approach of the passivation procedure described above, for a class of SISO uncertain nonlinear systems.

Consider a SISO nonlinear system with linearly parameterized uncertainty in the form

$$\begin{aligned}\dot{x} &= f(x)+\sum_{i=1}^{p}f_i(x)\theta_i+g(x)u\\ y &= h(x)\end{aligned}\tag{17}$$

where $x\in\chi\subset\Re^n$ is the state; u,y the scalar input and output respectively; and θ_i, $i=1,\ldots,p$ is a set of constant unknown parameters. The drift vector fields $f(x)$, $f_1(x),\ldots,f_p(x)$, and the "input" vector field $g(x)$ are smooth n-dimensional vector fields.

System (17) may always be rewritten as follows

$$\begin{aligned}\dot{x} &= f(x)+\Phi(x)\theta+g(x)u\\ y &= h(x)\end{aligned}\tag{18}$$

where the matrix $\Phi(x)\in\Re^{n\times p}$ and the vector θ of unknown parameters are defined as follows

$$\Phi(x)=[f_1(x),\ldots,f_p(x)]\quad;\quad\theta=[\theta_1,\ldots,\theta_p]^T\tag{19}$$

By incorporating a parameter estimate vector $\hat{\theta}$ of the constant unknown parameters, (18) may be rewritten as follows

$$\begin{aligned}\dot{x} &= f(x)+\Phi(x)\hat{\theta}+g(x)u+\Phi(x)(\theta-\hat{\theta})\\ y &= h(x)\end{aligned}\tag{20}$$

We will initially consider the known *nominal* part of system (20)

$$\begin{aligned}\dot{x} &= f(x)+\Phi(x)\hat{\theta}+g(x)u\\ y &= h(x)\end{aligned}\tag{21}$$

and a quadratic Lyapunov function $V(x) = 1/2x^T x$. By decomposing the vector field $f(x)$ and, in addition, assuming that the vector field $\Phi(x)\hat{\theta}$ is non-dissipative with respect to $V(x)$, we can rewrite (21) in the way

$$\begin{aligned} \dot{x} &= f_d(x) + f_I(x) + \hat{f}_{nd}(x,\hat{\theta}) + g(x)u \qquad (22) \\ y &= h(x) \end{aligned}$$

with $\hat{f}_{nd}(x,\hat{\theta}) = f_{nd}(x) + \Phi(x)\hat{\theta}$. Thus, the time derivative of $V(x)$ can be written

$$\dot{V}(x) = L_{f_d}V(x) + L_{\hat{f}_{nd}}V(x,\hat{\theta}) + L_g V(x)u \qquad (23)$$

By using the following state-dependent input coordinate transformation,

$$u = \frac{1}{L_g V(x)}\left[h(x)v - L_{\hat{f}_{nd}}V(x,\hat{\theta}) - \delta h^2(x)\right] \qquad (24)$$

where δ is a positive scalar and v is an auxiliary control input, $\dot{V}(x,\hat{\theta})$ satisfies the following string of relations,

$$\dot{V} = L_{f_d}V(x) + h(x)v - \delta h^2(x) \leq yv - \delta y^2 \leq yv \qquad (25)$$

and the partially closed loop system yields

$$\begin{aligned} \dot{x} &= f_d(x) + f_I(x) + \left[I - g(x)\frac{\Delta V(x)}{L_g V(x)}\right]\hat{f}_{nd}(x,\hat{\theta}) \\ &\quad + \frac{h(x)v}{L_g V(x)}g(x) - \delta\frac{h^2(x)}{L_g V(x)}g(x) \qquad (26) \end{aligned}$$

The time derivative of V has the following terms

$$\begin{aligned} \Delta V(x)\left[f_d(x) - g(x)\frac{\delta h^2(x)}{L_g V(x)}\right] &\leq 0 \\ \Delta V(x)\left\{f_I(x) + \left[I - g(x)\frac{\Delta V(x)}{L_g V(x)}\right]\hat{f}_{nd}(x,\hat{\theta})\right\} &= 0 \end{aligned}$$

with $\Delta V(x) := \frac{\partial V}{\partial x}$. Then, by straightforward factorization, this system may always be rewritten in the following *adaptive passivity canonical form*

$$\begin{aligned} \dot{x} &= -\mathcal{R}(x)(\Delta V(x))^T - \mathcal{J}(x,\hat{\theta})(\Delta V(x))^T + \mathcal{M}(x)v \\ y &= h(x) \qquad (27) \end{aligned}$$

with $\mathcal{R}(x)$ being a positive semidefinite matrix in χ, and $\mathcal{J}(x,\hat{\theta})$ being a skew-symmetric matrix. This implies the following identifications,

$$\begin{aligned} f_d(x) - \delta\frac{h^2(x)}{L_g V(x)}g(x) &= -\mathcal{R}(x)(\Delta V(x))^T \\ f_I(x) + \left[I - g(x)\frac{\Delta V(x)}{L_g V(x)}\right]\hat{f}_{nd}(x,\hat{\theta}) &= -\mathcal{J}(x,\hat{\theta})(\Delta V(x))^T \\ \frac{h(x)}{L_g V(x)}g(x) &= \mathcal{M}(x) \end{aligned}$$

When the storage function is quadratic, system (27) can be rewritten in the following way

$$\dot{x} = -\mathcal{R}(x)x - \mathcal{J}(x,\hat{\theta})x + \mathcal{M}(x)v \quad ; \quad y = h(x) \qquad (28)$$

By considering the modified storage function

$$V_d(x,x_d) = \frac{1}{2}(x - x_d)^T(x - x_d) \qquad (29)$$

with x_d an auxiliary state vector to be defined, and following the energy shaping and damping injection procedure described in Section II-C, we obtain the dynamical compensator

$$\dot{x}_d = -\mathcal{R}(x)x_d - \mathcal{J}(x,\hat{\theta})x_d + \mathcal{R}_{di}(x)(x - x_d) + \mathcal{M}(x)v \quad (30)$$

which achieves that the time derivative of $V_d(x,x_d)$ satisfies

$$\begin{aligned} \dot{V}_d(x,x_d) &= -(x - x_d)^T\mathcal{R}_m(x)(x - x_d) \\ &\leq -\frac{a}{b}(x - x_d)^T(x - x_d) \\ &\leq -\frac{a}{b}V(x,x_d) \leq 0 \end{aligned}$$

However, the design of the adaptive controller has not been completed yet, because the actual system (20) contains an estimate error term and, consequently, the passivity canonical form, resulting of applying the procedure above, is

$$\begin{aligned} \dot{x} &= -\mathcal{R}(x)x - \mathcal{J}(x,\hat{\theta})x + \mathcal{M}(x)v + \Phi(x)(\theta - \hat{\theta}) \\ y &= h(x) \qquad (31) \end{aligned}$$

Thus, $\dot{V}_d(x,x_d)$ has an additional term

$$\dot{V}_d(x,x_d) = -(x - x_d)^T\mathcal{R}_m(x)(x - x_d) + (x - x_d)^T\Phi(x)(\theta - \hat{\theta})$$

Therefore, we must find an update law for the unknown parameters and eliminate the destabilizing effect of the estimate error in $\dot{V}_d$. To this end, we extend the storage function as follows

$$W(x,x_d,\hat{\theta}) = V_d(x,x_d) + \frac{1}{2}(\theta - \hat{\theta})^T\Gamma^{-1}(\theta - \hat{\theta}) \qquad (32)$$

The time derivative of $W(x,x_d,\hat{\theta})$ yields

$$\begin{aligned} \dot{W}_d &= -(x - x_d)^T\mathcal{R}_m(x)(x - x_d) \\ &\quad + (\theta - \hat{\theta})^T\Gamma^{-1}\left[-\dot{\hat{\theta}} + \Gamma\Phi^T(x)(x - x_d)\right] \qquad (33) \end{aligned}$$

Thus, by using the update law

$$\dot{\hat{\theta}} = \Gamma\Phi^T(x)(x - x_d) \qquad (34)$$

we eliminate the destabilizing effect of the estimate error and achieve

$$\begin{aligned} \dot{W}_d(x,x_d,\hat{\theta}) &= -(x - x_d)^T\mathcal{R}_m(x)(x - x_d) \\ &\leq -\frac{a}{b}(x - x_d)^T(x - x_d) \leq 0 \end{aligned}$$

The asymptotic convergence of the state vector $x(t)$ to the desired trajectory $x_d(t)$ is guaranteed. These results are summarized in the following proposition.

Proposition 2: Consider a nominal SISO nonlinear system of the form (21). The system is passivifiable with respect to a quadratic storage function $V(x)$, through a state dependent and parameter estimate dependent input coordinate trasnformation, if and only if the condition

$$L_g V(x) \neq 0 \tag{35}$$

in χ. The input coordinate trasnformation that strictly passivifies the uncertain system (21) is given by

$$u(x, \hat{\theta}, v) = \frac{1}{L_g V(x)} \left[-L_{\hat{f}_{nd}} V(x, \hat{\theta}) + h(x) v - \delta h^2(x) \right] \tag{36}$$

where $\hat{\theta}$ is the estimate of the unknown parameter vector θ, obtained from (34). By extending the storage function as (29) and applying the dynamical control law (30), we stabilize the system (28) in passivity canonical form. Then, by considering the estimate error, system (21) adopts the extended form (20) which can be stabilized by using the adaptive control law (36) together with the update law (34), with respect to the extended storage function (32).

IV. Passivity-Based Regulation of a Gravity-Flow Tank/Pipeline

Consider the following gravity-flow tank/pipeline taken from Karjala and Himmelblau (see [3]) which includes an elementary static model for an "equal percentage valve"

$$\begin{aligned} \dot{x}_1 &= \frac{A_p g}{L} x_2 - \frac{K_f}{\rho A_p^2} x_1^2 \\ \dot{x}_2 &= \frac{1}{A_t} \left(F_{Cmax} \alpha^{-(1-u)} - x_1 \right) \\ y &= x_2 \end{aligned} \tag{37}$$

where x_1 is the volumetric flow rate of liquid leaving the tank, x_2 is the height of the liquid in the tank, and u is the valve position (control input), taking values in the closed interval [0,1]. The system's parameters are F_{Cmax}: maximum value of the volumetric rate of fluid entering the tank, g: gravitational acceleration constant, L: length of the pipe, K_f: friction factor, ρ: density of the liquid, A_p: cross sectional area of the pipe, A_t: cross sectional area of the tank, and α: rangeability parameter of the valve.

For a constant value $U \in [0,1]$ of the control input u, the system has an equilibrium point given by

$$X_1 = F_{Cmax} \alpha^{-(1-U)} \quad ; \quad X_2 = \frac{L K_f}{A_p^3 g \rho} X_1^2$$

In order to avoid unnecessary complications, we consider the control input term via the following auxiliary control input w

$$w = F_{Cmax} \alpha^{-(1-u)} \tag{38}$$

We also assume that the friction factor K_f is constant but unknown, thus obtaining

$$\begin{aligned} \dot{x}_1 &= \frac{A_p g}{L} x_2 - \theta \frac{x_1^2}{\rho A_p^2} \\ \dot{x}_2 &= \frac{1}{A_t}(w - x_1) \\ y &= x_2 \end{aligned} \tag{39}$$

where $\theta = K_f$. The operating region for system (37) is given by points strictly located in the first quadrant of $\Re^2$. From (39), we may identify the following relations

$$f(x) = \begin{pmatrix} \frac{A_p g}{L} x_2 \\ -\frac{1}{A_t} x_1 \end{pmatrix} \qquad g(x) = \begin{pmatrix} 0 \\ \frac{1}{A_t} \end{pmatrix}$$

$$\Phi(x) = \begin{pmatrix} -\frac{x_1^2}{\rho A_p^2} \\ 0 \end{pmatrix}$$

and incorporating the parameter estimate $\hat{\theta}$, we obtain the nominal system

$$\begin{aligned} \dot{x}_1 &= \frac{A_p g}{L} x_2 - \hat{\theta} \frac{x_1^2}{\rho A_p^2} \\ \dot{x}_2 &= \frac{1}{A_t}(w - x_1) \\ y &= x_2 \end{aligned} \tag{40}$$

By considering initially the storage function $V(x) = \frac{1}{2} x^T x$ the condition (10) is given by $L_g V(x) = \frac{x_2}{A_t} \neq 0$. and the time derivative of $V(x)$ along the trajectories of (40), is given by

$$\dot{V}(x) = -\frac{x_1^3}{\rho A_p^2} \hat{\theta} - \frac{1}{A_t} x_1 x_2 + \frac{A_p g}{L} x_1 x_2 + \frac{1}{A_t} x_2 w \tag{41}$$

This manner, (41) satisfies the following inequality

$$\dot{V}(x) \leq -\frac{x_1^3}{\rho A_p^2} \hat{\theta} + \frac{A_p g}{L} x_1 x_2 + \frac{1}{A_t} x_2 w \tag{42}$$

which is obtained under the assumption that the system evolution takes place on the operating region χ and, hence, x_1 and x_2 are strictly positive for all times. This allows one to decompose the vector field $f(x)$ as follows

$$f_d(x) = \begin{pmatrix} 0 \\ -\frac{1}{A_t} x_1 \end{pmatrix} \quad f_{nd}(x) = \begin{pmatrix} \frac{A_p g}{L} x_2 - \frac{x_1^2}{\rho A_p^2} \hat{\theta} \\ 0 \end{pmatrix} \tag{43}$$

Then, using the state dependent input coordinate trasnformation

$$w = \frac{x_1^3 A_t}{\rho A_p^2 x_2} \hat{\theta} - \frac{A_p g}{L} A_t x_1 + A_t v - A_t \delta x_2 \qquad \delta > 0 \tag{44}$$

we obtain the passivity inequality

$$\dot{V}(x) \leq x_2 v - \delta x_2^2 \leq y v \tag{45}$$

Thus, the passivity canonical form yields

$$\dot{x} = -\mathcal{R}(x) x - \mathcal{J}(x, \hat{\theta}) x + \mathcal{M}(x) v \tag{46}$$

with

$$\mathcal{R}(x) = \begin{pmatrix} 0 & 0 \\ 0 & \delta + \frac{1}{A_t} \frac{x_1}{x_2} \end{pmatrix}; \quad \mathcal{M}(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathcal{J}(x, \hat{\theta}) = \begin{pmatrix} 0 & -\frac{A_p g}{L} + \frac{x_1^2}{\rho A_p^2} \hat{\theta} \\ \frac{A_p g}{L} - \frac{x_1^2}{\rho A_p^2} \hat{\theta} & 0 \end{pmatrix}$$

A. Controller Design

Considering the modified energy function $W(x, x_d, \hat{\theta})$,

$$W(x,x_d,\hat{\theta}) = V_d(x,x_d) + \frac{1}{2\gamma}(\theta-\hat{\theta})^2 = \frac{1}{2}(x-x_d)^T(x-x_d) + \frac{1}{2\gamma}(\theta-\hat{\theta})^2 \quad (47)$$

and applying the energy shaping and damping injection procedure, we obtain the following set of auxiliary differential equations

$$\dot{x}_{1d} = -\frac{x_1^2}{\rho A_p^2 x_2}\hat{\theta}x_{2d} + \frac{A_p g}{L}x_{2d} + R_1(x_1 - x_{1d}) \quad (48)$$

$$\dot{x}_{2d} = -\frac{A_p g}{L}x_{1d} - (\delta + \frac{x_1}{A_t x_2})x_{2d} + \frac{x_1^2}{\rho A_p^2 x_2}\hat{\theta}x_{1d} + R_2(x_2 - x_{2d}) + v$$

where R_1 and R_2 are the diagonal components of the positive definite matrix $\mathcal{R}_{di}(x)$. The update law for the unknown parameter yields

$$\dot{\hat{\theta}} = \gamma\left(\frac{x_1^2}{\rho A_p^2}\right)(x_1 - x_{1d}) \quad (49)$$

Then, letting $x_{2d} = X_2 =$ *constant*, one obtains the following dynamical controller expression, where x_{1d} has been substituted by the controller state variable ξ,

$$v = \frac{A_p g}{L}\xi + \left(\delta + \frac{x_1}{A_t x_2}\right)X_2 - \frac{x_1^2}{\rho A_p^2 x_2}\hat{\theta}\xi - R_2(x_2 - X_2)$$

$$\dot{\xi} = -\frac{x_1^2}{\rho A_p^2 x_2}\hat{\theta}X_2 + \frac{A_p g}{L}X_2 + R_1(x_1 - \xi) \quad (50)$$

which, together with the control input transformation (44), define the actual control input corresponding to the valve position

$$u = 1 + \frac{1}{\log\,\alpha}\log\left(\frac{w}{F_{Cmax}}\right) \quad (51)$$

We used the following parameter values for the simulation of the controlled gravity-tank/pipe system: $g = 9.81\ m/s^2$; $L = 914\ m$; $\rho = 998\ Kg/m^2$; $A_p = 0.653\ m^2$; $A_t = 10.5\ m^2$; $\alpha = 10$, and $F_{Cmax} = 2\ m^3/s$. The unknown parameter was set to be $K_f = 4.41\ N - s^2/m^3$. The required equilibrium point for $y = x_2$ was chosen to be $X_2 = 5\,m$, while that of x_1 was set to be $X_1 = 1.8360\,m^3/s$. This corresponds with a steady state value of the control input $U = 0.9627$. The design parameters were chosen to be $R_1 = 0.1$; $R_2 = 0.08$; $\gamma = 20$, and $\delta = 0.8$. Figure 1 shows the closed loop response of the gravity-tank / pipe system with good stabilization features: no overshoot and a settling time of less than 200 seconds.

V. Conclusions

In this work, we have proposed an adaptive passivity-based approach for the regulation of a large class of affine nonlinear systems with constant but unknown parameters. The adaptive passivation was shown to be achievable through a parameter-estimate-dependent control input transformation that renders the partially closed loop system passive. This is achieved under the condition that the storage function of the system has a nonzero directional derivative with respect the input vector field in the operating region. An illustrative example was used to show the performance of the proposed approach.

References

[1] C. I. Byrnes, A. Isidori, and J. C. Willems, "Passivity, Feedback Equivalence and the Global Stabilization of Minimum Phase Nonlinear Systems", *IEEE Transactions on Automatic Control*, Vol. 36, pp. 1228–1240, 1991.

[2] D. Hill and P. J. Moylan, "The Stability of Nonlinear Dissipative Systems", *IEEE Transactions on Automatic Control*, Vol. 21, pp. 708–711, 1976.

[3] T. Karjala and D. Himmelblau, *AIChE Journal*, Vol. 42, pp. 2225–2239, 1996.

[4] C. Kravaris and S. Palanki, *AIChE Journal*, Vol. 34, pp. 1119–1127, 1988.

[5] M. Krstić, I. Kanellakopoulos, and P.V. Kokotović, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, 1995.

[6] R. Marino and P. Tomei, *Nonlinear Control Design*, Prentice-Hall, 1995.

[7] M. Rios-Bolívar, "Adaptive Backstepping and Sliding Mode Control of Uncertain Nonlinear Systems", Ph.D. Thesis, University of Sheffield, 1997.

[8] M. Seron, D. Hill and A. Fradkov, "Nonlinear Adaptive Control of Feedback Passive Systems," *Automatica*, Vol. 31, No. 7, pp. 1053–1060, 1995.

[9] H. Sira-Ramírez and M. Delgado, "Passivity Based Regulation of Nonlinear Continuous Processes", in *Advances in Control* C. Leondes (Ed.) The Gordon and Breach Publishing Group, Newark, N.J., (to appear).

[10] J. C. Willems, *The analysis of Feedback Systems*, MIT Press, Cambridge, MA., 1971.

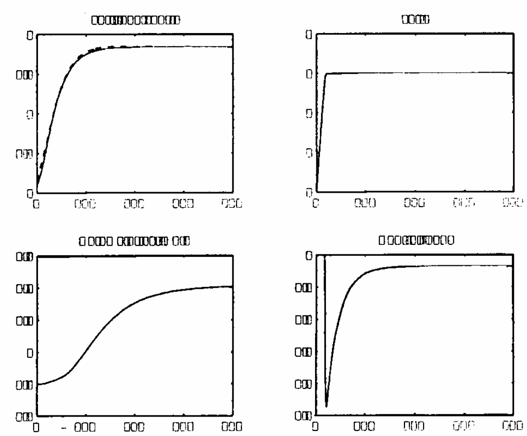

Fig. 1. Adaptively controlled responses of a gravity-tank/pipe system