On the Control of Switched Reluctance Motors*

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Abstract

In this paper, the control of Switched Reluctance Motors is approached from a Passivity-based Control perspective. The proposed controller solves the torque/speed/position tracking problem by exploiting the passivity properties of the machine. The methodology design considers the feedback decomposition of the motor model into one electrical and one mechanical passive systems and is divided into the following three steps: Control of the electrical sub-system to achieve current tracking, definition of the desired current behaviour to assure torque tracking and design of a speed/position control loop. The contribution of the paper is threefold: The controller design is developed using energy-dissipation ideas, the mathematical formalization of the current engineering practice of controlling this kind of machines with a cascade approach, and the establishment of an extension to previously reported passivity-based controllers for electric machines in the sense that in this case Blondel-Park transformability properties are not required.

1 Introduction

Switched reluctance motors (SRM) establish a class of electric machines that has grown in popularity in the last years due to the absence of permanent magnets or winding in the rotor, making them a low cost and high reliable alternative. They belong to the class known as doubly salient electric motors and in addition of its simple structure, if they are designed with a large number of poles, these devices can produce high torque at low speeds, eliminating the use of gear boxes and establishing thus, that SRM can be considered a suitable candidate for direct-drive applications.

The main limitation for exploiting the advantages mentioned above, is the strong nonlinear electromechanical behavior exhibited by this kind of electrical devices, namely: It primarily operates with magnetic saturation in order to maximize torque/mass radio and the developed torque is a nonlinear function of stator currents and rotor position [1].

One way that has shown to be useful for dealing with the aforementioned limitation is, on the one hand, to assume a simplified structure for the magnetic circuit of the machine leading to a linear relationship between fluxes and currents and, on the other hand, to treat the torque generation problem by means of the so-called torque sharing approach [2]. The simplified structure greatly simplifies the controller design while the sharing approach, motivated by the (experimentally justified) assumption that the stator windings are decoupled, allows for the consideration that the generated torque is composed by the sum of the torque induced by each of the stator windings. Hence, a quite reasonable solution in order to produce a desired torque, is to define *sharing functions* whose basic objective is to scale phase torques in such a way that their sum achieves the desired value.

On the other hand, Passivity-based control (PBC) is a well established controller design methodology [3] that has shown to be very useful in solving control problems for a large class of nonlinear (physical) systems. For the particular case of electric machines, PBC was first applied to induction motors [5] and later on to a class of Blondel-Park transformable (both under and fully actuated) electric motors [4]. The main feature of this approach lies in the fact that a central role in the controller design is deserved to modelling aspects by focusing on the motor structural aspects that can be exploited to solve the problem. In particular, this technique takes advantage of the energy-dissipation (passivity) properties of the machine and simplifies the analysis by decomposing the model as a

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feedback interconnection of two passive sub–systems (one electrical and one mechanical). This approach allows the designer to control the electrical sub–system to achieve objectives like current/torque tracking, considering the mechanical subsystem as a "passive disturbance", and after this, to solve the speed/position control problem for the mechanical subsystem.

In this paper, the control of SRM is approached from a PBC perspective. The proposed controller states a novel contribution in the field since it solves the torque/speed/position tracking problem by exploiting the passivity properties of the machine. The methodology design considers the feedback decomposition of the motor model into one electrical and one mechanical passive systems, allowing in a natural way, the following three steps: Control of the electrical sub–system to achieve current tracking, definition of the desired current behavior to assure torque tracking and design of a speed/position control loop. It must be pointed out, that this controller structure evidently resembles the well–known cascade approach to control electrical machines. Therefore, a second contribution can be identified in the sense that a formal justification for this practice is provided.

A final contribution of the paper is related with the fact that the Blondel-Park transformability properties, required by previously reported passivity based controllers, are not required in this case.

The paper is organized as follows: Section 2 is devoted to present the general characteristics and the considered model of SRM. The passivity–based design followed in the controller design is introduced in section 3 while the proposed controller is presented in section 4 where some simulation results are included to validate its usefulness. Finally, some concluding remarks are included in section 5.

2 SRM: General characteristics and Modeling

SRM differs from standard motors because forces are generated by reluctance instead of induction actions, i.e. torque production mechanism is identical to the observed when two displaced opposite polarized magnets are aligned. This structure allows for concentration of windings in the stator part of the machine while the rotor does not exhibit neither permanent magnets nor windings, leading to several attractive features: Simple construction and brushless structure and generated torque many times greater than those obtained by excitation–type motors.

Paradoxically, it can be considered that the main disadvantage of this kind of electric machines comes also from its simple structure, since in order to generate torque, the stator currents must be sequentially switched on and off in accordance with the rotor position.

One way that has shown to be very useful for dealing with the problem of smooth torque generation, is the approach known as *torque sharing*. In general terms, the basic characteristic of this approach is to consider that not only one phase winding produces torque at the same time, but instead that this task is shared with the adjacent winding. Thus, instead of generating an "instantaneous" switching between phases, this transition is accomplished by blending the applied currents to the two considered windings. In this paper this approach is followed and will be formalized below in the mathematical definition of the electronic commutator.

In order to state the differential equations that describe the dynamical behavior of a 3ϕ SRM[1], in this paper it is considered the (experimentally justified and widely accepted) assumption that the three stator phases are magnetically decoupled, i.e. that the mutual inductance between stator phases can be neglected. Hence, the aforementioned differential equations are given by

$$\begin{array}{c}\dot{\psi}_j + r i_j = u_j; \;\; j = 1,2,3 \\ J\ddot{\theta} = T_e(\theta, i_j) - T_L(\theta, \dot{\theta})\end{array} \qquad (2.1)$$

where u_j is the voltage applied to the stator terminals of phase j, i_j is the stator current of phase j, $\psi_j(\theta, i_j)$ is the flux linkage of phase j, r is the stator winding resistance, θ is the angular position of the rotor, $T_L(\theta, \dot{\theta})$ is the load torque and J is the total rotor and load inertia. Notice that the mechanical torque of electrical origin $T_e(\theta, i_j)$, depends on both the angular position of the rotor and all of the three stator currents.

In this paper, following [2], it is assumed that fluxes linkage have the following structure $\psi_j(\theta, i_j) = L_j(\theta) i_j$ where phase inductances are given by

$$L_j(\theta) = l_0 - l_1 \cos\left[N_r\theta - (j-1)\frac{2\pi}{3}\right] \qquad (2.2)$$

where l_0 and l_1 are constants that make this inductance a strictly positive function.

Under this condition, the model (2.1) can be written as

$$\begin{array}{c}L_j(\theta)\frac{di_j}{dt} + K_j(\theta)\dot{\theta} i_j + r i_j = u_j; \;\; j = 1,2,3 \\ J\ddot{\theta} = T_e(\theta, i_j) - T_L(\theta, \dot{\theta})\end{array} \qquad (2.3)$$

where

$$K_j(\theta) = \frac{\partial L_j(\theta)}{\partial \theta} = N_r l_1 \sin\left(N_r\theta - (j-1)\frac{2\pi}{3}\right) \quad (2.4)$$

is the phase inductance variation with respect the angular rotor position.

[1] Without loss of generality and for ease of presentation, in this paper it will be considered the case of a 3ϕ motor.

It is important to remark that the mechanical torque of electrical origin takes the form

$$T_e(\theta, i_1, i_2, i_3) = \sum_{j=1}^{3} T_j(\theta, i_j) = \sum_{j=1}^{3} \frac{1}{2} K_j(\theta) i_j^2$$

where it can be observed that each phase torque appears as a quadratic function on phase currents and that its sign is determined by the partial derivative of the phase inductance with respect rotor position. This property is fundamental in the sense that considering it, the mathematical definition of the electronic commutator and the torque sharing mechanism can be developed, following ideas of [2], in the following way:

Given two sets

$$\Theta_j^+ = \{\theta : K(\theta) \geq 0\} \quad \Theta_j^- = \{\theta : K(\theta) < 0\}$$

where the superscript + and − stand for required positive and negative torque, respectively, choose any functions m_j^+ and m_j^- such that

$$\begin{aligned} m_j^+(\theta) > 0 \, \forall \theta \in \Theta_j^+; \qquad & \sum_{j=1}^{3} m_j^+(\theta) = 1 \, \forall \theta \\ m_j^-(\theta) > 0 \, \forall \theta \in \Theta_j^-; \qquad & \sum_{j=1}^{3} m_j^-(\theta) = 1 \, \forall \theta \end{aligned} \tag{2.5}$$

Then, these *sharing functions* can scale each phase torque in order to generate a total desired torque by assigning

$$m_j(\theta) = \begin{cases} m_j^+(\theta), & T_d \geq 0 \\ m_j^-(\theta), & T_d < 0 \end{cases} \tag{2.6}$$

with T_d the desired torque to be delivered.

It must be pointed out, that the presented structure for the electronic commutator has the advantage of leaving to the designer the freedom to choice the sharing functions, provided conditions (2.5) are satisfied. This characteristic is important in the sense that this degree of freedom can be used to solve additional optimization problems like torque rippley minimization. Moreover, the relevance of this structure is greater in a PBC context, since (as will be clear in the development of the proposed controller) it establishes the argument for avoiding the requirement of Blondel–Park transformability properties for the machine model.

3 Passivity–based design

Passive systems establish a class of nonlinear systems that enjoy several properties that are attractive from a control perspective [6]. This class of systems is characterized by the following basic

Definition. The dynamical system $\Sigma : u \rightarrow y$ with input $u \in \mathcal{R}^m$, output $y \in \mathcal{R}^m$ and state vector $x \in \mathcal{R}^n$ is passive if there exist a function $\mathcal{H}(x) \geq 0 \in C^1$, $\mathcal{H}(0) = 0$, such that, for all $u \in \mathcal{L}_{2e}^m$ and for all $t \geq 0$ the following inequality holds

$$\int_0^t u^T(\tau) y(\tau) d\tau \geq \mathcal{H}[x(t)] - \mathcal{H}[x(0)] \tag{3.1}$$

The importance of this definition for control purposes is revealed if inequality (3.1) is rewritten as $\dot{\mathcal{H}}[x(t)] \leq u^T(t)y(t)$ where it can be noticed that, if $\mathcal{H}(x)$ is interpreted as a Lyapunov function, then the unforced system, i.e. considering $u \equiv 0$, is stable as well as the zero dynamic of the system, i.e. those dynamic behavior obtained by considering $y \equiv 0$. Moreover, these (very attractive by itself) properties become more relevant if it is considered that the negative feedback interconnection of two passive systems is also passive.

In [4], the properties presented above are exploited to propose a controller design methodology with the aim to solve the torque/speed/position tracking problem for a wide class of electric machines. The developed result is based in the feature that the generalized model of the considered class of machines, can be decomposed as the negative feedback interconnection of two passive (one electrical Σ_e and one mechanical Σ_m) systems as

$$\begin{aligned} \Sigma_e : & \begin{bmatrix} u_s \\ -\dot{\theta} \end{bmatrix} \rightarrow \begin{bmatrix} i_s \\ T_e(\theta, i_s, i_r) \end{bmatrix} \\ \Sigma_m : & \ T_L - T_e(\theta, i_s, i_r) \rightarrow -\dot{\theta} \end{aligned}$$

with u_s input voltages, i_s stator and i_r rotor currents, respectively. Under these conditions the mentioned control problem can be solved by following the next three steps:

1. **Control of the electric subsystem.** In this step an output feedback controller, (i.e., using only stator currents for feedback) is developed in such a way that its closed loop with the electric subsystem is passive. Hence, invoking the passivity preservation property for feedback interconnected passive systems, the mechanical (passive) subsystem can be viewed as a "passive" disturbance that does not destroy the passivity (and then the stability) properties of the complete system.

2. **From current to torque tracking.** Once the current tracking control problem has been solved, the second step is to identify a desired current behavior compatible with the system dynamic but that at the same time generate a pre-specified reference for the generated torque. This step can be formulated as a system inversion requirement in the sense that for a given desired torque, the designer must be able to compute a current behavior that generate the desire torque. Evidently, the definition of this current behavior must be complemented with the proof of the asymptotic achievement of the objective, i.e.

it must be proved that current convergence implies torque convergence.

3. **Control of the mechanical subsystem.** In this step it is defined the desired torque structure that achieves the control of the mechanical variables. This definition involves the feedback of mechanical speed or position, depending on the control objectives, and must be done in such a way that the stability of the overall control system can be achieved.

The following remarks are in order about the refereed controller design methodology:

• Notice that the three steps presented above resemble the well-known and widely used cascade control [7]. The main difference in this case is that the stability properties of closed loop are formally stated.

• The system inversion requirement imposed in second step, is solved in [4], by exploiting the Blondel-Park transformability properties of the class of motors considered in that paper. As is well known, SRM do not enjoy this property, but this problem will be solved (as it will be clear in the next section) by following the torque sharing approach.

4 Main result

In this section the main result of the paper is presented, namely, a passivity-based controller for the considered model of SRM. Although the particular case of speed control is treated in detail, the required modifications for the proposed controller to deal with torque and position control are also stated.

The passivity properties exhibited by model (2.3) that makes suitable the approach described in the last section, are presented in the following

Fact. The model (2.3) of a 3ϕ SRM can be decomposed as the feedback interconnection of the following two passive systems

$$\Sigma_e: \quad \begin{bmatrix} u \\ -\dot{\theta} \end{bmatrix} \rightarrow \begin{bmatrix} i \\ T_e(\theta, i) \end{bmatrix}$$
$$\Sigma_m: \quad T_L(\theta,\dot{\theta}) - T_e(\theta, i) \rightarrow -\dot{\theta}$$

where $u = [u_1, u_2, u_3]^T$ and $i = [i_1, i_2, i_3]^T$.

Proof. In order to prove the passivity properties of the model, consider the magnetic co-energy function and the kinetic co-energy function of the machine given respectively by $W'(\theta, i) = \frac{1}{2} i^T D(\theta) i$ and $\mathcal{K}'(\dot{\theta}) = \frac{1}{2} J\dot{\theta}^2$.

The first part of the fact is proved by taking the time derivative of $W'(\theta, i)$ given by

$$\dot{W}'(\dot{\theta}, \theta, \frac{di}{dt}, i) = i^T D(\theta)\frac{di}{dt} + i^T C(\theta) i\dot{\theta}$$

Its evaluation along the trajectories of the electrical subsystem gives as a result

$$\dot{W}'(\dot{\theta}, \theta, \frac{di}{dt}, i) = -\frac{1}{2} i^T C(\theta) i\dot{\theta} - i^T R i + i^T u$$

Thus, after time integration, the passivity of this subsystem is proved. The second part of the fact is proved in a similar way by considering the time derivative of the kinetic co-energy function, along the trajectories of the mechanical subsystem, given by

$$\dot{\mathcal{K}}'(\dot{\theta}) = -\dot{\theta}\,(T_L - T_e)$$

Again time integration of this last expression completes the proof. △△△

Once the application of the passivity-based approach has been justified and with the aim of simplify the presentation of the proposed controller, it will be useful to point out that motor model can be written in matrix form as

$$D(\theta)\frac{di}{dt} + C(\theta)\dot{\theta} i + Ri = u \quad (4.1)$$
$$J\ddot{\theta} = T_e(\theta, i) - T_L(\theta, \dot{\theta}) \quad (4.2)$$

where $D(\theta) = diag\{L_1(\theta), L_2(\theta), L_3(\theta)\}$ is the inductance matrix, $C(\theta) = diag\{K_1(\theta), K_2(\theta), K_3(\theta)\}$ and $T_e(\theta, i) = \frac{1}{2} i^T C(\theta) i$ is the generated torque.

The main result of the paper is presented in the following

Proposition. Consider the model of a SRM given by (4.1)-(4.2) in closed loop with the control law

$$u = D(\theta)\frac{di_d}{dt} + C(\theta)\dot{\theta} i_d + Ri_d - K_v e \quad (4.3)$$

where the gain matrix $K_v = diag\{K_{1v}, K_{2v}, K_{3v}\}$ is given by $K_v = c_1|\dot{\theta}|\mathcal{I}_3$ with $c_1 > N_r l_1$ and $\mathcal{I}_3$ the identity matrix.

The desired current phase behavior i_{jd} is

$$i_{jd} = \begin{cases} \sqrt{2m_j(\theta) T_d K_j^{-1}(\theta)} & \text{if} \quad K_j(\theta) \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.4)$$

with T_d the desired torque and $m_j(\theta)$ the sharing functions introduced in section 2. Also, the desired torque is related to the speed error $\dot{\tilde{\theta}} = \dot{\theta} - \dot{\theta}_d$ by

$$T_d(z) = J\ddot{\theta}_d - z + T_L(\theta, \dot{\theta}) \quad (4.5)$$

with controller state

$$\dot{z} = -az + b\dot{\tilde{\theta}} \quad (4.6)$$

Under these conditions, asymptotic speed tracking is insured, i.e. $\lim_{t\to\infty} \dot{\tilde{\theta}} = 0$, with all internal signals bounded.

Proof. If the current error is defined as $e = i - i_d$, model (4.1) can be equivalently written as

$$D(\theta)\frac{de}{dt} + C(\theta)\dot{\theta}e + Re = \Phi \tag{4.7}$$

where

$$\Phi = u - \left\{ D(\theta)\frac{di_d}{dt} + C(\theta)\dot{\theta}i_d + Ri_d \right\}$$

Then, considering the proposed controller (4.3), expression (4.7) takes the form

$$D(\theta)\frac{de}{dt} + \left[C(\theta)\dot{\theta} + R + K_v\right]e = 0$$

Since $D(\theta)$ is a strictly positive definite matrix, this last equation can be re-arranged as

$$\frac{de}{dt} = -D^{-1}(\theta)\left[C(\theta)\dot{\theta} + R + K_v\right]e$$

which, due to the diagonal structure of the matrices, defines a set of three decoupled linear time-varying differential equations of the form

$$\frac{de_j(t)}{dt} = -a_j(t)e_j(t); \quad j = 1,2,3$$

where it is sufficient to show that $a_j(t)$ always remains positive to insure the exponential convergence of the current error to zero. This positivity condition is guaranteed if the inequality

$$N_r l_1 \sin\left(N_r\theta - (j-1)\frac{2\pi}{3}\right)\dot{\theta} + r + K_{vj} > 0; \quad j = 1,2,3$$

holds, condition that is satisfied with the proposed value for the gain matrix. Thus, under these conditions, the first step of the passivity-based design is achieved.

In order to prove that current tracking implies torque tracking, consider the definition of the desired torque given by

$$T_d(\theta, i_d) = \frac{1}{2}i_d^T C(\theta)i_d$$

Torque convergence is proved by considering that in this case the difference between the generated and the desired torque is given by

$$T_e - T_d = \frac{1}{2}i^T C(\theta)i - \frac{1}{2}i_d^T C(\theta)i_d$$

This expression leads, after some easy calculations, to

$$|T_e - T_d| \leq \alpha_1\|e\|^2 + \alpha_2\|i_d\|\|e\|$$

with α_1, α_2 positive constants, and where it can be seen that current tracking implies torque tracking provided the desired value of currents i_d is bounded.

In order to obtain the desired behavior for the current vector, a torque sharing approach is followed. First, notice that the desired torque is obtained from the desired torque defined for each phase in the following way

$$T_d = \frac{1}{2}i_{1d}^T K_1(\theta)i_{1d} + \frac{1}{2}i_{2d}^T K_2(\theta)i_{2d} + \frac{1}{2}i_{3d}^T K_3(\theta)i_{3d}$$

If the torque sharing approach is adopted, the expression above must be equivalent to

$$T_d = m_1(\theta)T_d + m_2(\theta)T_d + m_3(\theta)T_d$$

which is satisfied with the proposed value for the desired current behaviour.

The final step in the proof is to show that speed error also tends to zero and that the internal stability of the overall system is guaranteed. This can be done by following exactly the procedure developed in [8] for the case of induction motors. △△△

The following remarks are in order about the proposed controller:

- It is interesting to note, that finding an expression for the desired current behaviour does not require any Blondel-Park transformability property for the machine. In this sense and at least for this kind of systems, the result presented in [4] is extended here.
- Regarding torque control, it must be noticed that this problem is in fact solved when the second step of the followed design methodology is achieved. On the other hand, the position control problem can be solved, again following ideas of [8], by modifying (4.5) in the following way

$$T_d(z) = J\ddot{\theta}_d - z - f\tilde{\theta} + T_L(\theta, \dot{\theta})$$

where $f > 0$ and $\tilde{\theta} = \theta - \theta_d$ is the position error.

The performance of the proposed control scheme was investigated by digital simulations. The considered motor parameters are the same as in [2], $N_r = 4$, $l_0 = 30mH$, $l_1 = 20mH$, $r = 5\Omega$, $J = 10^{-3}kg - -m^2$ and the load torque, for simplicity of presentation, was set to zero. With the aim to illustrate the global properties of the control, the motor was initially at standstill. Figure 1 shows the speed behavior when a square wave of $\pm 100 rad/sec$ of amplitude was used as a speed reference. The electric gain was set to $K_{vj} = 5$ while filter values were varied as $a = 75, 150, 175$, $b = 10$ each cycle of the reference. In this figure it can be observed how the mechanical transient response is improved as the damping inyected by the controller is increased. On the other hand, to illustrate the electrical performance improvement when the electric gain is increased, in Figure 2 the current error for phase one is shown for three different values of this parameter $K_{vj} = 1, 5, 10$. Also concerning with the electrical performance, in Figure 3 it is shown the actual behavior for the torque error under several values of the electric gain $K_{vj} = 1, 5, 10$.

5 Conclusions

The control problem of a nonlinear model for SRM was approached in this paper. The main result is related with the proposition of a control law that solves the torque/speed/position tracking control problem approaching the torque generation problem from a torque sharing perspective. The control law is designed exploiting the energy–dissipation (passivity) properties of the motor model by decomposing it as a negative feedback interconnection of the electrical and the mechanical (passive) subsystems. The methodology design resemble the well–known cascade control and does not lie in Blondel-Park transformability arguments, as previously reported passivity–based designs.

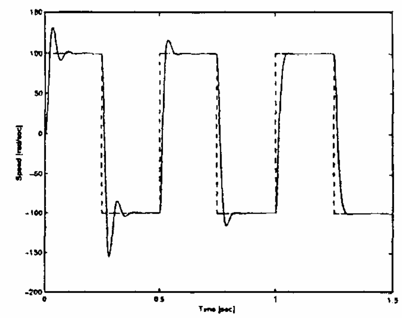

Figure 1: Speed behavior with increasing damping ($a = 75, 150, 175$).

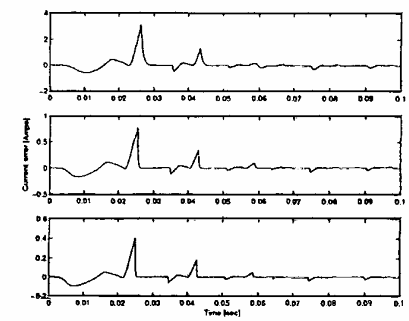

Figure 2: Current error with increasing electric gain ($K_{vj} = 1, 5, 10$).

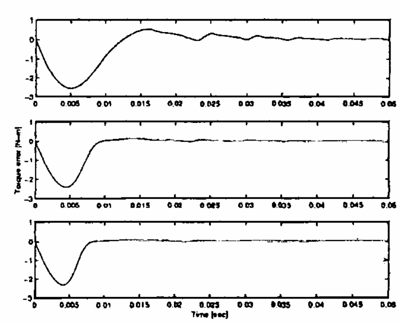

Figure 3: Torque error with increasing electric gain ($K_{vj} = 1, 5, 10$).

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