Position Control of an Inertia-Spring DC-motor System without Mechanical Sensors: Experimental Results [1]

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Abstract

In this article, two Generalized Proportional-Integral (GPI) feedback control schemes are proposed for the stabilization of the angular position of a DC-motor-actuated rotation inertia load, fixed to a wall by means of a rotation spring. The feedback schemes, which are not based on asymptotic observers nor calculations based on samplings, use only electrical variables measurements.

Keywords: Generalized PID control, Sliding Modes.

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1 Introduction

Generally speaking, one of the main drawbacks of state space based control theory and of sliding mode control, in particular, is constituted by the need to completely measure the state of the system. Usually, the state is estimated by means of either asymptotic observers or, as it is usually done in practice, by resorting to on-line computer-based calculations of time derivatives of the available output signals. These derivatives are customarily obtained from suitable output samplings. As it is widely known, either approach reduces the performance and the robustness of the chosen control scheme in an important manner.

For the continuous feedback regulation of linear time invariant systems, whether single or multi-input, the need for asymptotic Luenberger observers, or for time discretizations, has been recently side stepped in the work of Fliess *et al* [1]-[2]. The main idea of the, so called, Generalized PID (GPI) approach, which is theoretically based on localizations, module theory, and Mikusinsky's calculus (See Fliess *et al* [3]), is to pursue an *integrated* state feedback controller design based on naïve estimates, or "structural estimates", of the state variables. These state estimates use only iterated integrals of the inputs and of the outputs of the system and, therefore, the controller can be easily synthesized by use of traditional, or modern, analog electronic circuits. The closed-loop system using the structural estimates of the state variables, in any given linear state feedback control law, only requires the use of additional compensating iterated integral output error control actions, in order to adequately compensate for the effect of the incurred structural state estimation errors. Such reconstruction errors arise from iterated integration of the unknown initial conditions. The GPI feedback control design technique has been already used in the angular velocity regulation of an experimental DC motor inertia system with very good results (see Marquez *et al* [5]). We use a similar experimental model to that described in [5] with an added linear torsion spring. This simple modification renders the system observable thus making it possible to tackle the angular position control problem via GPI, i.e., without using mechanical sensors.

In this article, we develop two feedback control schemes, which are based on the GPI feedback control approach, for the regulation of an inertia-spring DC motor system. The synthesis of the feedback control laws only require electrical variables measurements for the stabilization of the angular position, to a constant pre-specified value, of the rotation inertia load which is fixed to a "wall" by means of a rotation spring while being actuated by means of a DC motor. The first control scheme is based on Differential Flatness techniques (See Fliess *et al* [4]), and it basically entitles a pole placement technique, while the second scheme uses Sliding Mode feedback control (see Utkin [6]) in a manner which also uses no state measurements. In both cases, the feedback control laws are synthesized using only measurements of the DC motor armature circuit input voltage and the corresponding DC motor armature current.

In section 2 we present the mathematical model of the inertia-spring dc-motor system and establish the main properties of the model. Namely; flatness and constructibility. In section 3 we derive the pole placement based GPID controller directly derived from the controllability (i.e. flatness) of the system. In this section, we also present the computer simulation results as well as the experimental results. Section 4 is devoted to describe and report a new sliding mode control approach, which does not require state measurements, based on the GPI control design philosophy. The simulations and the satisfactorily carried out experimental results are also presented in this section. Section 5 contains the conclusions and suggestions for further research.

2 The Inertia-spring DC motor system

Consider the electro-mechanical system shown in Figure 1. The mathematical model of this system is readily obtained as:

$$\begin{aligned} L\dot{I} &= -RI - k_e\omega + u \\ J\dot{\omega} &= -B\omega - k\theta + k_m I \\ \dot{\theta} &= \omega \end{aligned} \tag{1}$$

where I is the DC motor armature circuit current, θ is the angular displacement of the motor axis, measured with respect to a fixed but arbitrary reference position, and ω is the corresponding angular velocity of the motor axis. The control input, denoted by u, represents the external variable voltage applied to the armature circuit terminals. The parameters L, R, k_e represent, respectively, the armature circuit inductance, the circuit resistance and the back electro-motive force constant of the DC motor. The parameters J, B, k and k_m denote, respectively, the combined rotor and load inertia, the viscous friction coefficient, the torsion spring coefficient and the DC motor torque constant.

Note that the equilibrium of the system, corresponding to a constant desired value of the inertia load position $\theta = \overline{\theta}$, is obtained as

$$\overline{y} = \frac{k}{k_m}\overline{\theta}, \quad \overline{u} = \frac{Rk}{k_m}\overline{\theta} \tag{2}$$

The system is controllable and, hence, differentially flat, with flat output given by the inertia load, or motor axis, angular position θ. The flat output satisfies the following differential polynomial relation with the input,

$$J\theta^{(3)} + \left(B + \frac{RJ}{L}\right)\ddot{\theta} + \left(k + \frac{k_m k_e + RB}{L}\right)\dot{\theta} + \frac{Rk}{L}\theta = \frac{k_m}{L}u \quad (3)$$

As it can be easily determined from (1), the system model is observable for the output $y = I$. This fact establishes the *constructibility* of the system, which, in turn, implies that *all* system state variables are parameterizable in terms of inputs, outputs and iterated integrals of the input and the output variables (See Fliess *et al* [3]). Such an integral input-output parameterization of the system state variables is given, modulo initial conditions, by

$$\begin{aligned}
\widehat{\theta} &= -\frac{L}{k_e}y + \frac{1}{k_e}\int_0^t [u(\tau) - Ry(\tau)]d\tau \\
\widehat{\dot{\theta}} &= -\frac{k}{J}\int_0^t \widehat{\theta}(\tau)d\tau + \frac{k_m}{J}\int_0^t y(\tau)d\tau - \frac{B}{J}\widehat{\theta} \\
\widehat{\ddot{\theta}} &= -\frac{k}{J}\widehat{\theta} + \frac{k_m}{J}\,y - \frac{B}{J}\widehat{\dot{\theta}} \\
\widehat{I} &= I = y
\end{aligned} \quad (4)$$

The first expression in (4) is obtained by integration of the first equation in (1). The second expression is obtained by integration of the second equation in (1). The third relation is just the second equation in (1). Note that, for non-zero initial states, the relations linking the actual values of the angular position derivatives to the structural estimates in (4) are given by

$$\begin{aligned}
\theta &= \widehat{\theta} + \theta_0, \quad \dot{\theta} = \widehat{\dot{\theta}} + \dot{\theta}_0 - \frac{k}{J}\theta_0\, t - \frac{B}{J}\theta_0 \\
\ddot{\theta} &= \widehat{\ddot{\theta}} - \frac{B}{J}\dot{\theta}_0 + \frac{Bk}{J^2}\theta_0\, t + \left(\frac{B^2}{J^2} - \frac{k}{J}\right)\theta_0
\end{aligned} \quad (5)$$

where θ_0 and $\dot{\theta}_0$ denote the initial mass position and the initial mass velocity.

3 A pole placement based GPI controller

The differential relation (3) immediately suggests the following feedback controller for the stabilization of the motor shaft angular position around a desired constant equilibrium value, denoted by $\overline{\theta}$,

$$\begin{aligned}
u &= \frac{L}{k_m}\left[Jv + \left(\frac{JR}{L} + B\right)\ddot{\theta} + \left(k + \frac{k_m k_e}{L} + \frac{RB}{L}\right)\dot{\theta} + \frac{Rk}{L}\theta\right] \\
v &= -k_4\ddot{\theta} - k_3\dot{\theta} - k_2(\theta - \overline{\theta})
\end{aligned} \quad (6)$$

In the flatness based controller (6) one proceeds to replace the angular position θ, its velocity, $\dot{\theta}$, and the angular acceleration, $\ddot{\theta}$, by its structural estimates, obtained from the expressions found in (4). This, however, implies that the closed loop system would be actually excited by constant values and by "ramp" functions. To suitably correct for the destabilizing effect of such structural estimation errors, one uses iterated integral error compensation as follows:

$$\begin{aligned}
u &= \frac{L}{k_m}[Jv + \left(\frac{JR}{L} + B\right)\widehat{\ddot{\theta}} + \left(k + \frac{k_m k_e}{L} + \frac{RB}{L}\right)\widehat{\dot{\theta}} + \frac{Rk}{L}\widehat{\theta}] \\
v &= -k_4\widehat{\ddot{\theta}} - k_3\widehat{\dot{\theta}} - k_2(\widehat{\theta} - \overline{\theta}) - k_1\xi - k_0\eta \\
\dot{\xi} &= y - \overline{y} \\
\dot{\eta} &= \xi
\end{aligned} \quad (7)$$

The closed loop system is obtained by substituting the GPI controller (7) in the expression (3). Taking the second order time derivative in the obtained expression one finds, after replacing the armature current variable by the expression $y = (J/k_m)\ddot{\theta} + (B/k_m)\dot{\theta} + (k/k_m)\theta$, one obtains the following closed loop dynamics:

$$\begin{aligned}
&\theta^{(5)} + k_4\theta^{(4)} + (k_3 + \frac{k_1 J}{k_m})\theta^{(3)} \\
&+(k_2 + \frac{k_1 B + k_0 J}{k_m})\ddot{\theta} + (\frac{k_1 k + k_0 B}{k_m})\dot{\theta} \\
&+\frac{k_0 k}{k_m}(\theta - \overline{\theta}) = 0
\end{aligned} \quad (8)$$

The desgin coefficients $k_4, ..., k_0$ are chosen so that the corresponding characteristic polynomial:

$$p(s) = s^{(5)} + k_4 s^{(4)} + (k_3 + \frac{k_1 J}{k_m})s^{(3)}$$

$$+(k_2+\frac{k_1B+k_0J}{k_m})s^2$$
$$+(\frac{k_1k+k_0B}{k_m})s+\frac{k_0k}{k_m} \qquad (9)$$

has all its roots located in the left portion of the complex plane.

3.1 Simulation and experimental results

The system parameters were found to be: $J = 11.12 \times 10^{-5}$ kg-m^2, $k = 0.345$ N-m/rad, $k_m = 56.37 \times 10^{-3}$ N-m/A, and the viscous friction coefficient $B = 0.6242 \times 10^{-3}$ N-m-s/rad, while the electrical parameters of the motor were determined to be $L = 43.31$ mH, $R = 30\ \Omega$ and $k_e = 0.076$ V-sec/rad.

Figure 2 presents the simulation results of the performance of the controlled system to the flatness based controller synthesized on the basis of using the integral input-output parameterizations of the flat output and its time derivatives with the added iterated integral compensation. The control objective, in this simulations, was to stabilize the system motions around the constant equilibrium value given by $\overline{\theta} = 0.03$ [rad]. The controller gains were chosen so that the closed loop system has a fifth order characteristic polynomial of the form: $(s+b)(s^2+2\zeta\omega_n s+\omega_n^2)^2$, with $b = 80$ and $\zeta = 0.707$, $\omega_n = 80$.

Figure (3) presents the experimental results obtained for the flatness based controller. It should be stated that in order to overcome the effects of the unmodeled, but ever present, mechanical friction and dead zone phenomenon, we added an nonlinear outer-loop friction compensation scheme based on feeding back the output stabilization error signal through a generated high gain term of the form $(6/\pi)\arctan[100(\overline{y}-y)]$. This choice arose from the "magnitude" of the measured dead zone which was found to be around ± 3 Volts. This added nonlinear compensation largely accounts for the differences between the simulated and the obtained experimental results.

4 GPI Sliding mode control

A sliding surface coordinate function, σ, that ideally induces, by the sliding invariance condition $\sigma = 0$, an asymptotic exponential stabilization of the mass position, θ, towards a constant desired value, $\overline{\theta}$, is given by,

$$\sigma = \ddot{\theta} + k_3\dot{\theta} + k_2(\theta - \overline{\theta}) \qquad (10)$$

for suitable (Hurwitz) choices of k_3 and k_2. We propose, nevertheless, the following modified sliding surface coordinate function, which does not use the otherwise required measurements of the angular position, angular velocity and angular acceleration, but instead uses the structural estimates of these variables, as defined in the previous section.

$$\widehat{\sigma} = \widehat{\ddot{\theta}} + k_3\widehat{\dot{\theta}} + k_2(\widehat{\theta} - \overline{\theta}) + k_1\xi + k_0\eta \qquad (11)$$

with

$$\dot{\xi} = y - \frac{k}{k_m}\overline{\theta}, \quad \xi(0) = 0$$
$$\dot{\eta} = \xi, \quad \eta(0) = 0 \qquad (12)$$

The added iterated integral control action suitably compensates the constant and the linearly growing structural estimate errors with respect to the actual values of the flat output and its time derivatives. The underlying equivalent (but never used) expression of the sliding surface in terms of the actual (unmeasured) values of the load angular position variables, and its time derivatives, is obtained by replacing (5) into (11). The obtained expression is of the form:

$$\widehat{\sigma} = \ddot{\theta} + k_3\dot{\theta} + k_2(\theta - \overline{\theta}) + \left(k_1\int_0^t [y - \frac{k}{k_m}\overline{\theta}]d\tau - \alpha\right) + \int_0^t \left[k_0\int_0^\tau (y - \frac{k}{k_m}\overline{\theta})d\rho - \beta\right] d\tau \qquad (13)$$

where the constant parameters α and β depend on the initial conditions for θ and $\dot{\theta}$. The ideal sliding condition, $\widehat{\sigma} = 0$, is easily seen to be equivalent to the following forth order closed loop system, which is completely independent of any initial condition values.

$$\theta^{(4)} + (k_3 + k_1\frac{J}{k_m})\theta^{(3)} + (k_2 + \frac{k_1B}{k_m} + \frac{k_0J}{k_m})\ddot{\theta} + \left(\frac{k_1k}{k_m} + \frac{k_0B}{k_m}\right)\dot{\theta} + \frac{k_0k}{k_m}(\theta - \overline{\theta}) = 0 \qquad (14)$$

where, as before, use has been made of the flatness-based differential parameterization linking the system output, $y = I$, to the flat output θ. It is clear that a suitable choice of the design parameter set $\{k_3, k_2, k_1, k_0\}$ yields an exponentially asymptotically stable closed loop dynamics for the mass position θ.

We propose the following discontinuous sliding mode feedback controller:

$$u = \overline{u} - W \text{ sign } \widehat{\sigma}, \quad W > 0 \qquad (15)$$

4.1 Simulation and experimental results

Figure 4 depicts the sliding mode simulated controlled evolution of the mass position, the sliding surface time evolution, $\widehat{\sigma}(t)$, the motor current and the externally applied input voltage. The desired objective was to stabilize the system motions to $\overline{\theta} = 0$ [rad] from an initial position of $\theta_0 = 0.03$ [rad] and an initial current $I(0) = 0.183$ [Amp]. The design constants were chosen to be the coefficients of a fourth order polynomial, in the complex variable s, of the form: $(s^2 + 2\zeta\omega_n s + \omega_n^2)^2$, with $\zeta = 0.707$, $\omega_n = 80$. We also set, $W = 10$.

Figure 5 depicts the system variables responses obtained from the experimental results.

5 Conclusions

In this article we have presented two feedback control schemes for the regulation of the angular position in an inertia-spring DC motor system. The desired objective is to bring the inertia load towards a desired constant equilibrium value. A pole placement approach and a sliding mode control approach were used in a real laboratory experimental set-up. The control laws, which were obtained from the GPI control approach, are solely based on online processing the input and the output electrical signals associated with the DC motor. Namely, the applied input voltage to the armature circuit and the resulting armature circuit current. No other signals, such as those arising from mechanical sensors, were used in the feedback law. The load position was measured only for the purpose of verifying the effectiveness of the proposed feedback control scheme. The proposed GPI feedback control schemes have, thus, been shown to successfully control the angular position of electromechanical systems without requiring any mechanical sensors. All the system parameters in the experimental set-up were identified in an off-line fashion. An interesting work for the future will be to obtain a similar feedback "sensorless" feedback control scheme which identifies the crucial parameters in an on-line fashion.

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FIGURES

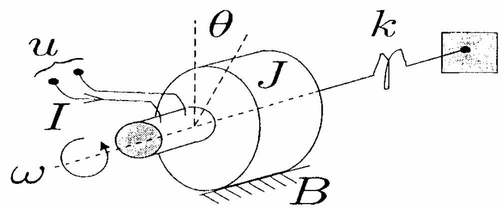

Figure 1: Inertia-spring DC motor system

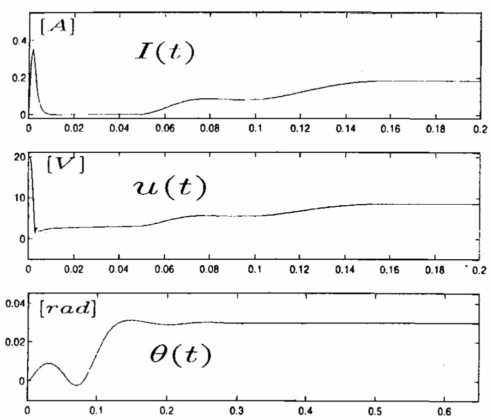

Figure 2: Simulation results for the pole placement based controller

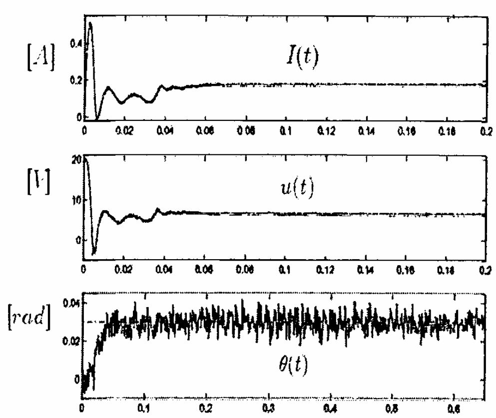

Figure 3: Experimental results for the pole placement based controller

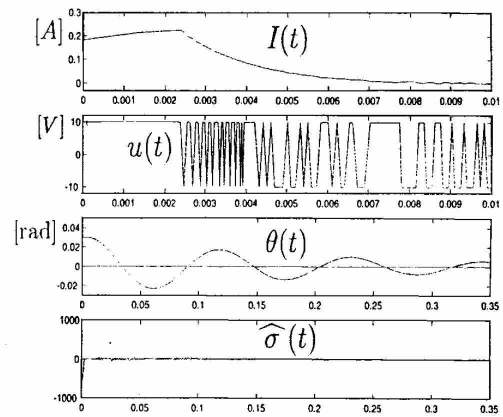

Figure 4: Simulation results for the sliding mode based controller

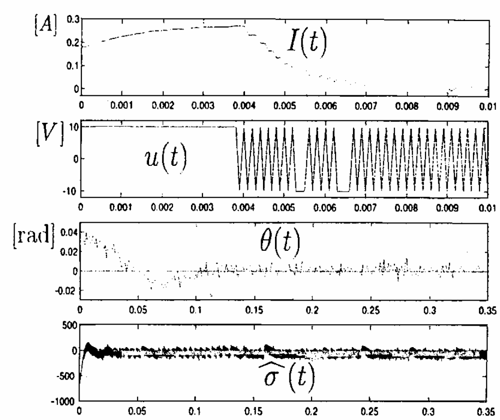

Figure 5: Experimental results for the sliding mode based controller