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SLIDING MODE CONTROL WITHOUT STATE MEASUREMENTS

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Abstract

In this article we present, in a tutorial fashion, an introduction to the links between flatness, generalized PID control and sliding mode control of linear systems. Several simple illustrative examples are presented including computer simulations and experimental results.

1 Introduction

One of the main drawbacks of state space based control theory and, in particular, of sliding mode control, is constituted by the need to completely measure the state of the system, or to estimate it by means of either asymptotic observers or, as usually done in practise, to resort to on-line computer based calculations of time derivatives of output signals, obtained by suitable time discretizations. As it is widely known, either approach reduces the performance and the robustness of the chosen control scheme in an important manner. For the continuous regulation of linear time invariant systems, whether single or multi-input, the need for Luenberger observers, or time discretizations, has been recently side stepped in the work of Fliess *et al* [1], [2]. The main idea of this approach, which is theoretically based on localizations, module theory, and Mikusinsky's calculus, is to pursue an *integrated* state feedback controller design based on naïve estimates, or "structural estimates", of the state variables. These state estimates use only iterated integrals of the inputs and of the outputs of the system and, therefore, the controller can be easily synthesized by use of traditional, or modern, analog electronic circuits. The closed-loop system using the structural estimates of the state variables, in any given state feedback control law, only requires the use of additional integral action, based only on the output signal, in order to adequately compensate for the effect of the incurred structural state estimation errors. This control design technique, addressed as Generalized PID (GPID) control, has been used in the regulation of an experimental system with very good results [5].

In this article, we present, in a tutorial manner, an introduction to generalized PID control in the context of sliding mode regulation for some linear systems and a nonlinear system of general interest.

In section 2 we review the implications of GPID control in sliding mode control of an elementary SISO system. A sliding mode controller is synthesized by means of integral input-output parameterizations of the flat output variables. Section 3 presents the sliding mode control of a linear system of physical nature. We illustrate the obtained results by performing computer simulations to assess the properties of the proposed sliding mode regulators. Section 4 presents an experimental implementation of GPID control to a single link manipulator.

2 Generalized PID sliding mode control of linear systems

We begin this section by providing a motivation of our approach using a rather simple example of traditional sliding mode controller design on the phase plane.

2.1 Sliding on the phase plane

Consider the linear controllable second order integrator system

$$\ddot{y} = u \tag{1}$$

In traditional sliding mode control (See Utkin [6]) one considers the following linear *sliding surface*, defined in the phase plane $(y, \dot{y})$ of the system:

$$S = \{(y, \dot{y}) \in R^2 \mid \sigma = \dot{y} + ky = 0, \quad k > 0\} \qquad (2)$$

The choice of (2) is evidently motivated by the asymptotic exponential stability features of the first order dynamics: $\dot{y} + ky = 0$, associated with the ideal sliding condition $\sigma = 0$. We have then the following proposition

Proposition 2.1 *Given the linear system (1), the discontinuous feedback controller*

$$u = -W \operatorname{sign} \sigma \qquad (3)$$

locally creates a sliding motion on $\sigma = 0$, within the rectangular region of the phase space, bounded by:

$$|\, y \,| < \frac{W}{k^2}, \; |\, \dot{y} \,| < \frac{W}{k} \qquad (4)$$

Under ideal sliding conditions $\sigma = 0$, $\dot{\sigma} = 0$, the system is governed by the asymptotically stable dynamics $\dot{y} = -ky$. Moreover, the rectangular region (4) is reachable, in finite time, from any finite initial condition in the phase space. Once the region (4) is reached the controlled system trajectories never abandon this rectangular region.

The proof of the above proposition follows rather directly from the computation of the first order time derivative of the sliding surface coordinate function, σ. This results in

$$\dot{\sigma} = \ddot{y} + k\dot{y} = u + k\dot{y} \qquad (5)$$

The discontinuous feedback control, $u = -W \operatorname{sign} \sigma$, with $W > 0$, applied on the system, is guaranteed to globally force the system phase trajectories to reach the sliding surface $\sigma = 0$, in finite time, and, more importantly, to locally force the motions to stay on such a line. This assertion is valid provided the design constant W, "dominates" the quantity $k\,\dot{y}$, regardless of its sign, in the aim of forcing $\dot{\sigma}$ to be strictly negative. The rest of the proposition follows easily (see also Ukin [6]).

2.2 Evading the need to measure $\dot{y}$

2.2.1 Modified sliding surface

If no reliable measurements of the phase velocity variable $\dot{y}$ are allowed, or available, then the above sliding mode controller cannot be implemented. It is also known that the performance, and the robustness, of the sliding mode control scheme is substantially lost when an asymptotic observer, such as a *reduced order* observer of the Luenberger type, is used to estimate the phase variable $\dot{y}$, based on the output measurement, y, and knowledge of the control input u.

Note, that from the system equations, and modulo an unknown constant initial condition $\dot{y}(0) = \dot{y}_0$, the quantity, $\int_0^t u(\tau)d\tau$, constitutes a "structural estimate", which we may denote by $\widehat{\dot{y}}$, of the phase velocity $\dot{y}$. In fact, the actual relation between $\widehat{\dot{y}}$ and $\dot{y}$ is given by the relation

$$\widehat{\dot{y}} = \int_0^t u(\tau)d\tau = \dot{y} - \dot{y}_0 \qquad (6)$$

Based on the previous observation, we propose the following modification of the sliding surface coordinate function, σ, which is now synthesized on the basis of the input, u, the output, y, and their integrals:

$$\begin{aligned} \widehat{\sigma} &= \int_0^t u(\tau)d\tau + k_2 y + k_1 \int_0^t y(\tau)d\tau \\ &= k_2 y + \int_0^t (u(\tau) + k_1 y(\tau))d\tau \end{aligned} \qquad (7)$$

The added integral of the output y, in the sliding surface coordinate expression (7), is aimed at compensating the constant initial condition off-set error existing between the structural estimate, $\widehat{\dot{y}}$, and the actual value of the variable $\dot{y}$, as it is shown below.

2.2.2 Conditions and domain of existence of sliding manifold

The time derivative of the new sliding surface coordinate function, $\widehat{\sigma}$, results in

$$\dot{\widehat{\sigma}} = u(t) + k_2\dot{y} + k_1 y \qquad (8)$$

The sliding mode controller $u = -W \operatorname{sign} \widehat{\sigma}$ forces the system phase trajectories to reach the sliding surface $\widehat{\sigma} = 0$, provided the following reaching conditions are satisfied

$$-W < k_2\dot{y} + k_1 y < W \qquad (9)$$

which now represents, in the original phase space $(y, \dot{y})$, an *infinite region* containing the origin, bounded by two parallel lines, and entirely containing the straight line: $\dot{y} = -k_1/k_2 y$. This fact is to be immediately contrasted with the nature of traditional sliding mode control in the previous proposition.

The equivalent control, obtained from the condition: $\dot{\widehat{\sigma}} = 0$, is clearly given by

$$u_{eq} = -k_1 y - k_2\dot{y} \qquad (10)$$

Evidently, under suitable choice of the design parameters k_1 and k_2, the controlled motions on, $\widehat{\sigma} = 0$, are associated with the following exponentially asymptotically stable *ideal sliding dynamics*:

$$\ddot{y} + k_2\dot{y} + k_1 y = 0 \qquad (11)$$

2.2.3 Analysis of sliding manifold dynamics and state estimation

Consider then the following extended system

$$\ddot{y} = u, \quad \dot{\xi} = y, \quad \frac{d}{dt}\widehat{\dot{y}} = u \qquad (12)$$

with initial conditions given by: $y(0) = y_0$, $\dot{y}(0) = \dot{y}_0$, $\widehat{y}(0) = 0$ and $\xi(0) = 0$. Notice that at least one state equation in (12) is redundant. This is due to the fact that $\widehat{y} = \dot{y} - \dot{y}_0$. In fact, then, we have a third order system. For the third order system (12), the sliding surface will be chosen as: $\widehat{s} = \widehat{y} + k_2 y + k_1 \xi$. Using the phase velocity structural estimate, $\widehat{y}$ in terms of the input u, we obtain the following equivalent value of the proposed sliding surface, written now in terms of the unknown initial condition, $\dot{y}_0$:

$$\begin{aligned} \widehat{s} &= \dot{y} + k_2 y + k_1 \xi - k_2 \dot{y}_0 \\ &= \dot{y} + k_2 y + k_1 \int_0^t y(\tau) d\tau - k_2 \dot{y}_0 \end{aligned} \tag{13}$$

Clearly, under ideal sliding motions, $\widehat{s} = 0$, the closed loop system results in the following integro-differential system excited by a constant unknown quantity:

$$\dot{y} + k_2 y + k_1 \int_0^t y(\tau) d\tau = k_2 \dot{y}_0 \tag{14}$$

It is evident that, for a suitable set of Hurwitz coefficients, k_2 and k_1, system (14) is equivalent to the exponentially asymptotically stable differential polynomial system: $\ddot{y} + k_2 \dot{y} + k_1 y = 0$.

Note also that, under ideal sliding conditions, the equivalent control method allows one to conclude that the motions of the (reduced) second order ideal sliding dynamics are governed by the linear system,

$$\begin{aligned} \dot{y} + k_2 y + k_1 \xi &= k_2 \dot{y}_0 \\ \dot{\xi} &= y \end{aligned} \tag{15}$$

The equilibrium point of the linear system is clearly given by $y = 0$, $\dot{y} = 0$ and $\xi = (k_2/k_1)\dot{y}_0$. In other words, given the asymptotic exponential stability of the underlying second order equivalent system, $\ddot{y} + k_2 \dot{y} + k_1 y = 0$, the introduced variable, ξ, has been shown to converge towards a constant quantity which is proportional to the unknown initial condition, $\dot{y}_0$. The relation with the previous development is clear upon realizing that indeed, $\widehat{\sigma} = \widehat{s}$.

Proposition 2.2 *A second order system of the form: $\ddot{y} = u$, is globally asymptotically exponentially stabilized to the origin of phase coordinates $(y, \dot{y}) = (0, 0)$ by the following dynamic sliding mode controller:*

$$\begin{aligned} u &= -W \operatorname{sign} \widehat{\sigma} \\ \widehat{\sigma} &= \int_0^t u(\tau) d\tau + k_2 y + k_1 \xi \\ \dot{\xi} &= y, \quad \xi(0) = 0 \end{aligned} \tag{16}$$

for all initial values, $y(0)$, $\dot{y}(0)$, of the phase variables, y and $\dot{y}$. A sliding regime golobally exists in the infinite region of the phase space bounded by the linear relations:

$$-W \leq k_2 \dot{y} + k_1 y \leq W$$

The variable ξ asymptotically exponentially converges to the constant value $(k_2/k_1)\dot{y}_0$.

Figure 1 depicts the generalized PID sliding mode feedback control scheme for the regulation of the second order integrator system. The simulations presented in Figure 2 correspond to a typical response of the second order plant system to the proposed sliding mode controller scheme. In these simulations we have set $k_2 = 2\zeta\omega_n$, $k_1 = \omega_n^2$ with $\zeta = 0.8$ and $\omega_n = 1$. The switching gain W was set to the value of 2.

3 Controlling the position of a mass-spring system without mechanical sensors

In this section, we apply the GPID sliding mode control scheme to the problem of controlling the position of a mass-spring system, shown in Figure 3, with the sliding mass attached to a wall by a spring characterized by a stiffness constant of value k. Consider then the system model as:

$$\begin{aligned} M\ddot{x} + kx &= k_m I \\ L\frac{di}{dt} + Ri &= u - k_e \gamma \dot{x} \\ y &= i \end{aligned} \tag{17}$$

where M is the load mass, L is the motor's armature circuit inductance, R is the corresponding resistance, k is the stiffness constant of the spring linking the load to a fixed point in the wall, k_m is the motor torque constant, and k_e is the counter electromotive constant of the motor. The constant γ is a proportionality constant relating the motor's shaft angular velocity to the linear tangent velocity of the gear interacting with the mass. The variables x and i and u, respectively, denote the load position, the motor's armature circuit electric current and the applied input voltage. The only measured variable, aside from the externally applied voltage u, is the armature circuit current i, which we denote by y.

Note that the equilibrium of the system, corresponding to a constant desired value of the mass position $x = \overline{x}$, is obtained as

$$\overline{y} = \frac{k}{k_m}\overline{x}, \quad \overline{u} = \frac{Rk}{k_m}\overline{x} \tag{18}$$

The system is controllable and, hence, differentially flat, with flat output given by the load position x. The flat output satisfies the following differential polynomial relation with the input,

$$Mx^{(3)} + \frac{MR}{L}\ddot{x} + \left(k + \frac{k_m k_e \gamma}{L}\right)\dot{x} + \frac{Rk}{L}x = \frac{k_m}{L}u \tag{19}$$

The system is also observable, and hence, constructible. An integral input-output parameterization of the system state variables is given by

$$\widehat{x} = -\frac{L}{k_e \gamma} y + \frac{1}{k_e \gamma} \int_0^t [u(\tau) - Ry(\tau)] d\tau$$

$$\begin{aligned}\widehat{x} &= -\frac{k}{M}\int_0^t \widehat{x}(\tau)d\tau + \frac{k_m}{M}\int_0^t y(\tau)d\tau \\ \widehat{\dot{x}} &= -\frac{k}{M}\widehat{x} + \frac{k_m}{M}\, y \\ \widehat{I} &= I = y \end{aligned} \tag{20}$$

The first expression in (20) is obtained by integration of the second equation in (17). The second expression is obtained by integration of the first equation in (17). The third relation is just the first equation in (17). Note that the relation linking the actual values of the position derivatives to the structural estimates in (20) is given by

$$x = \widehat{x} + x_0, \quad \dot{x} = \widehat{\dot{x}} + \dot{x}_0 - \frac{k}{M}x_0\, t, \quad \ddot{x} = \widehat{\ddot{x}} - \frac{k}{M}x_0 \tag{21}$$

where x_0 and $\dot{x}_0$ denote the initial mass position and the initial mass velocity. A sliding surface coordinate function, σ, that induces, by the ideal sliding condition $\sigma = 0$, an asymptotic exponential stabilization of the mass position, x, towards a constant desired value, $\overline{x}$, is given by,

$$\sigma = \ddot{x} + k_3\dot{x} + k_2(x - \overline{x}) \tag{22}$$

for suitable (Hurwitz) choices of k_3 and k_2. We propose, nevertheless, the following modified sliding surface coordinate function

$$\widehat{\sigma} = \widehat{\ddot{x}} + k_3\widehat{\dot{x}} + k_2(\widehat{x} - \overline{x}) + k_1\xi + k_0\eta \tag{23}$$

with

$$\begin{aligned}\dot{\xi} &= y - \frac{k}{k_m}\overline{x}, \ \ \xi(0) = 0 \\ \dot{\eta} &= \xi, \ \ \eta(0) = 0\end{aligned} \tag{24}$$

The added iterated integral control action suitably compensates the constant and the linearly growing structural estimate errors with respect to the actual values of the flat output and its time derivatives. The underlying equivalent expression of the sliding surface in terms of the actual (unmeasured) values of the mass position variables, and its time derivatives, is of the form

$$\begin{aligned}\widehat{\sigma} &= \ddot{x} + k_3\dot{x} + k_2(x - \overline{x}) + \left(k_1\int_0^t [y - \frac{k}{k_m}\overline{x}]d\tau - \alpha\right) \\ &\quad + k_0\int_0^t \left[\int_0^\tau (y - \frac{k}{k_m}\overline{x})d\rho - \beta\right] d\tau\end{aligned} \tag{25}$$

where the constant parameters α and β depend on the initial conditions for x and $\dot{x}$. The ideal sliding condition, $\widehat{\sigma} = 0$, is easily seen to be equivalent to the following forth order closed loop system,

$$x^{(4)} + (k_3 + k_1\frac{M}{k_m})x^{(3)} + (k_2 + \frac{k_0 M}{k_m})\ddot{x} + \frac{k_1 k}{k_m}\dot{x} + \frac{k_0 k}{k_m}(x - \overline{x}) \tag{26}$$

where use has been made of the flatness-based differential parameterization linking the system output, $y = I$, to the flat output x, which, from (17), is given by $y = (M/k_m)\ddot{x} + (k/k_m)x$. It is clear that a suitable choice of the design parameter set $\{k_3, k_2, k_1, k_0\}$ yields an exponentially asymptotically stable closed loop dynamics for the mass position x.

We propose the following discontinuous sliding mode feedback controller

$$u = \overline{u} - W \text{ sign } \widehat{\sigma}, \quad W > 0 \tag{27}$$

Figure 4 depicts the sliding mode controlled evolution of the mass position, its velocity, the motor current and the externally applied input voltage. The desired objective was to stabilize the system motions to $\overline{x} = 0$. The system parameters were chosen as: $M = 0.5$ kg, $k = 1$ N/m, $k_m = 0.6$ N/A, $k_e\gamma = 0.06$ V-s/m. The design constants were chosen to be the coefficients of a fourth order polynomial, in the complex variable s, of the form $(s^2 + 2\zeta\omega_n s + \omega_n^2)^2$ with $\zeta = 0.8$, $\omega_n = 5$.

4 Regulation of an inverted pendulum

Consider the non-linear system constituted by a pendulum driven by a DC motor. The system model is given by:

$$\begin{aligned} L\frac{dI}{dt} + RI + k_e\dot{q} &= v \\ J_m\ddot{q} + B_m\dot{q} &= k_m I - \tau_L \\ mL^2\ddot{q} + B_L\dot{q} + mgL\sin(q) &= \tau_L \end{aligned} \tag{28}$$

where I is the current in the armature circuit, v is the applied external voltage and q denotes the angular position of the motor axis. The parameter m represents the mass of the pendulum, assumed to be concentrated at its bob. The rest of the parameters were already defined in the previous example. It is assumed that the only variables measured are the armature current I and the applied voltage v. No mechanical variables q, $\dot{q}$, etc. are assumed to be used for feedback purposes.

Eliminating the torque τ between the mechanical equations yields:

$$\begin{aligned} L\frac{dI}{dt} + RI + k_e\dot{q} &= v \\ J\ddot{q} + B\dot{q} + mgL\sin(q) &= k_m I \end{aligned}$$

where

$$J = (J_m + mL^2), \ \ = (B_m + B_L)$$

It is desired to have the flat output $F = q$ to follow a prespecified trajectory,

$$F \to F^*(t)$$

The system is found to be locally observable, from the DC motor current variable I for the values $\frac{-\pi}{2} < q < \frac{\pi}{2}$. Nevertheless, the system is globally constructible, since the system variables are expressible in terms of inputs and outputs.

The differential parameterization of the control input v is found to be given by:

$$\begin{aligned} v &= \frac{JL}{k_m}F^{(3)} + \left(\frac{LB}{k_m} + \frac{RJ}{k_m}\right)\ddot{F} + \frac{R}{k_m}mgL\sin(F) \\ &\quad + \left(\frac{mgL^2}{k_m}\cos(F) + \frac{RB}{k_m} + k_e\right)\dot{F} \end{aligned}$$

An integral reconstructor of the angular position is given by

$$\widehat{F}(t) = -\frac{L}{k_e} I(t) + \int_0^t \left[\frac{1}{k_e} v(\tau) - \frac{R}{k_e} I(\tau) \right] d\tau$$

which is a linear reconstructor.

The relation between the reconstructed state $\widehat{F}$ and its actual value F is given by

$$F = \widehat{F} + F_0$$

For regulation of the system we propose the following sliding surface

$$\sigma = (I - I^*(t)) + k_1(\widehat{F} - F^*(t)) + k_0 \int_0^t (I - I^*(\tau)) d\tau$$

Flatness of the system allows one to compute $I^*(t)$ as:

$$I^*(t) = \frac{1}{k_m} \left[J\ddot{F}^*(t) + B\dot{F}^*(t) + mgL \sin(F^*(t)) \right]$$

Taking the first order time derivative of σ leads to:

$$\begin{aligned} \dot{\sigma} &= \frac{v}{L} - \frac{R}{L} I - k_1 \dot{F}^*(t) + k_0(I - I^*(t)) \\ &\quad + (k_1 - \frac{k_e}{L}) \dot{F} \end{aligned}$$

The natural selection of the discontinuous feedback control law, based only on measured or estiimated quantities is given by:

$$v = L \left[\frac{R}{L} I + k_1 \dot{F}^*(t) - k_0(I - I^*(t)) \right] - W \text{sign}(\sigma)$$

which results in the following closed loop dynamics for the sliding surface coordinate function.

$$\dot{\sigma} = (k_1 - \frac{k_e}{L}) \dot{F} - W \text{sign}\ (\sigma)$$

Evidently, for a sufficiently large value of W, a sliding mode is guaranteed to exist on $\sigma = 0$. We define:

$$\begin{aligned} e &= F - F^*(t) \\ \nu &= -[\sin(F) - \sin(F^*(t))] \end{aligned}$$

The ideal sliding dynamics associated with the invariance conditions of the sliding surface, $\sigma = 0, \dot{\sigma} = 0$ results in:

$$\begin{aligned} e^{(3)} + \left(\frac{B + Jk_0}{J} \right) \ddot{e} + \left(\frac{k_1 k_m + Bk_0}{J} \right) \dot{e} \\ = \frac{mgL}{J} [\dot{\nu} + k_0 \nu] \end{aligned}$$

The ideal sliding dynamics associated with the closed loop system is then constituted by a *linear system* with a stable transfer function of the form:

$$\frac{e}{\nu} = \frac{\gamma_3 s + \gamma_4}{s\,(s^2 + \gamma_1 s + \gamma_2)}$$

This linear system is negatively fedback by a nonlinear function, $\psi(e)$, of the tracking error e, specifically given by: $\psi(e) = \sin F - \sin F^*(t)$. This is a bounded nonlinearity which monotonically grows on the interval, $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Thus, the nonlinearity qualifies as a *sector nonlinearity*. According to classical absolute stability results, (See Khalil [4]) the system can be shown to be locally asymptotically stable for a suitable choice of the design parameters.

Figure 5 shows the experimental results of a stabilization task.

5 Conclusions

In this article, we have extended the generalized PID control technique, introduced in [1], to the sliding mode regulation of elementary linear and nonlinear dynamic systems of the SISO type. As in the continuous counterpart of GPID control, the need for asymptotic observers, or time discretizations, devised to compute needed states in the feedback law, is evaded, or sidestepped. The traditional state based sliding surface design is to be maintained, but now expressed through a suitable integral input-output parameterization of the unmeasured states, complemented with added iterated integral compensation in order to avoid all possible structural de-stabilizing effects of estimation errors and external perturbations. It can be shown that the GPID sliding mode controller design is quite robust with respect to a large class of externa perturbations and significantly large unmodelled parameter variations. Many other interesting issues, related to GPID control, will be discussed and illustrated in a forthcomming publication, from a general theoretical viewpoint [3].

The multivariable case offers no particular difficulty, specially if the controller design task is approached from a differential flatness viewpoint.

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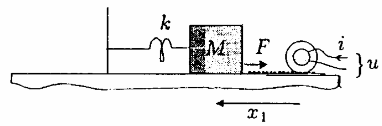

Figure 3: Mass-spring-DC motor system

FIGURES

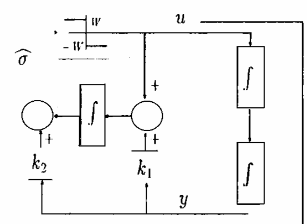

Figure 1: The generalized PID sliding mode control scheme for a second order integrator system

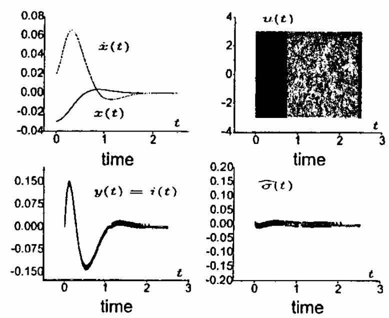

Figure 4: GPID Sliding mode controlled responses of the mass-spring-DC motor system

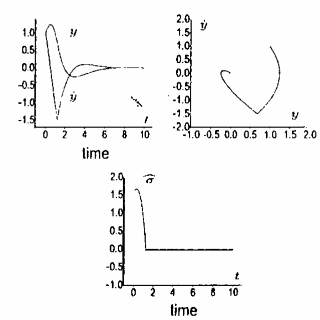

Figure 2: Response of GPID sliding-mode controlled second order integrator system

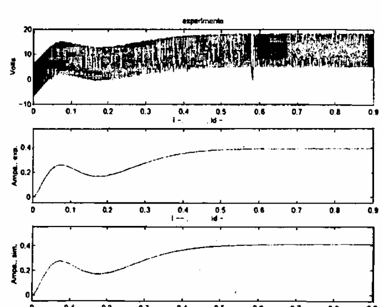

Figure 5: GPID Sliding mode controlled responses of the pendulum-DC motor system